MathDox - A System for Interactive Mathematics

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Abstract: We discuss the MathDox system, a system for presenting highly interactive mathematical documents. The system consists of both an XML format for interactive mathematical documents as well as a server for these documents called the MathDox Player.

Many of today's web pages are dynamic and interactive in the sense that they adapt to the user and offer the user some control over the contents of the page. Examples of such web pages are eBanking, auction and eLearning sites. At mathematical websites, however, the dynamics and interactivity are often limited. We see three main reasons for this. First of all, interactivity in mathematical content often requires non-trivial computations that are not easily performed without specialized mathematical software. Secondly, displaying mathematics properly on a web page can be challenging. Modern browsers have only limited support for rendering mathematics. Third, the input of mathematical expressions is a difficulty. It requires specialized software that is not yet readily available.

To address these issues we have developed the MathDox system. It consists of a format and a player. The MathDox format is an XML based language for interactive mathematical documents. MathDox documents can be transformed into web pages by the MathDox Player. These web pages are dynamic and interactive, support rendering of mathematics, offer easy access to computer algebra systems, and are equipped with a convenient mathematical input system. MathDox shows its potential when demonstrating algorithms, testing skills with exercises, or explaining new concepts with dynamic on-screen calculations. However, it can also be used for publishing static mathematical documents on paper or on the web.

In the following sections we will introduce the MathDox system. First we describe the design goals we have formulated for our system. Then we examine the different XML formats that are combined in the MathDox format. Section 3 “MathDox Tools” discusses how the MathDox system functions and how interactivity and rendering of mathematics in browsers and on paper is achieved. We end by giving some examples of MathDox used in practice.

Goals

In this section, we briefly discuss the three main design goals we have set for our Mathdox system and discuss how we have set up the MathDox system to reach these goals.

Interactivity

A user needs means to interact with the mathematical content of an interactive document. This can vary from performing computations, checking answers to exercises or creating and manipulating graphs of functions. MathDox combines existing interactivity of web pages with the power of mathematical computations, resulting in interactive mathematics. This is achieved by making use of a scripting language and interfaces with external software, notably computer algebra systems. After user actions, pages are updated with new parameters and web services will be triggered to provide output needed for preparation of the new page. This results in adaptable pages and on the fly computations.

One Source, Many Views

An electronic format for interactive mathematics is best served in an electronic environment. However, there is no reason to limit documents to just one presentation form. It will often be useful to have such a document available in various formats, ranging from, for example, HTML, Word, ODF or PDF to LaTeX. Therefore we have chosen to separate the sources from the presentation of the document. We have done so by choosing XML as our source format. By the use of XSL transformations MathDox sources can be translated into various other formats, including HTML, PDF and LaTeX, and into various styles.

Semantic Representation of Mathematics

In order to achieve interactivity of the user with various mathematical services, the mathematics in an interactive document has to be represented in a unambiguous way. When mathematical notions appear, one should be able to trace their definition, when variables appear, it should be clear what their types, values, and dependencies are. For this reason, MathDox offers the possibility to represent the mathematical formulae and expressions in OpenMath (http://www.openmath.org/) or (content) MathML (http://www.w3.org/TR/MathML/), semantically rich XML-encodings of mathematics.

The MathDox Format

MathDox combines in a modular way various existing XML formats best suited for our purposes. Each format contributes a useful facet for interactive documents. The MathDox format itself is XML based, which makes it possible to use existing XML-tools on MathDox documents. The XML formats used in MathDox are:

- DocBook (http://www.oasis-open.org/docbook/), used for the global structure of documents.
- OpenMath (http://www.openmath.org/), for semantic encoding of mathematics.
- XForms (http://www.w3.org/MarkUp/Forms/), for user driven interactivity.
- Jelly (http://commons.apache.org/jelly/), a programming and scripting format.
- XInclude (http://www.w3.org/TR/xinclude/), for separating functionality in to several files.

These formats will now be discussed individually.

DocBook

DocBook is a well-known documentation standard. Its logical structure facilitates searching and parsing of specific elements, enabling easy translation into other formats including HTML and PDF. This makes DocBook suitable for our needs.

OpenMath, Semantic Encoding of Math

Within MathDox mathematics appears in various forms: Mathematics as used in computations by software packages or mathematics solely meant for the user to read or sometimes for both the computer and the user. For each type of usage one wants mathematics to have specific properties. In general a user will be able to grasp the meaning behind a, possibly ambiguous, mathematical expression. Computer software, however, will need its mathematics to be completely unambiguous, since it cannot benefit from the context in the way a mathematically skilled reader does in order to solve gaps caused by incompleteness and ambiguity.

OpenMath has been chosen as the main format for mathematics within MathDox documents. It is well suited because it is semantically rich, unambiguous, XML-based and can easily be transformed into other formats, such as MathML and LaTeX, which are better suited for presentation. These transformations allow MathDox documents to use OpenMath where semantics are important and switch to MathML or LaTeX, when the mathematics need to be presented to the user either on screen or on paper.

The semantic information available from the OpenMath expressions enables easy translations between OpenMath and application specific syntax. At the moment MathDox supports interaction with the computer algebra systems Mathematica (http://www.wolfram.com/), Maple (http://www.maplesoft.com/), GAP (http://www.gap-system.org/), Maxima (http://maxima.sourceforge.net/), Wiris (http://www.mathsformore.com/), Magma (http://magma.maths.usyd.edu.au/) and Singular (http://www.singular.uni-kl.de/) and the dynamic geometry systems GeoGebra (http://www.geogebra.org/) and Cinderella (http://cinderella.de/). See also (Caprotti, Cohen, Cuypers, Riem & Sterk, 2000).

XForms: User Driven Interactivity

XForms in MathDox documents supplies a means of interaction with the reader of the MathDox document. It allows for many interaction (form) elements which can be used for user input, such as text fields, radio buttons, and drop down menus. In MathDox the XForms elements are used to trigger interaction with mathematical services like computer algebra systems.

XForms is a recommendation of W3C and a successor to HTML-Forms. XForms is included into the XHTML 2.0 draft specifications. At this moment XForms is not yet properly supported by browsers, but work in this direction is in progress. For now the XForms code in MathDox documents is translated automatically by the MathDox Player into HTML forms supplemented by JavaScript. This translation is done by Orbeon Forms (http://www.orbeon.com/).

Jelly: Programming and Scripting

To specify and fine-tune reactions of a MathDox document to user input, an author needs programming constructs. For this purpose Jelly (http://commons.apache.org/jelly/) has been included in the MathDox format. Jelly is a JSP-like XML-language (http://java.sun.com/products/jsp/), and has been developed as part of the Apache project (http://www.apache.org/). Jelly can be used for conditional statements, loops, variables, calls to Java (http://java.sun.com/) objects and calls to web services. For example, the input of a user can be inspected with the use of the Jelly to see whether it has the needed structure, setting certain parameters to trigger specific reactions.

XInclude: Separating Functionality in MathDox Files

MathDox documents do not need to be a single big file. There are cases in which it is advantageous to split documents into smaller parts. The resulting set of parts will increase possibilities for reuse and support maintainability. XInclude is used to include and group together XML components from different files. XInclude is a W3C recommendation.

MathDox Software

So far we only discussed the MathDox format and just briefly mentioned the server-side software responsible for processing of this format. In this section we discuss the MathDox software.

The MathDox Player

The MathDox Player is responsible for making MathDox documents accessible over the web. Its task is similar to that of a web server in the sense that both a web server and the MathDox Player offer stored documents from the server to the outside world. The main difference between a normal web server and the MathDox Player is that a web server offers ready-made HTML files, whereas the MathDox Player dynamically creates these HTML files. On request of the user the MathDox server collects data from the source document, the user and from mathematical back engines providing services and creates, by applying a number of XSL transformations on this input, a new view on the document. The MathDox Player is implemented in Orbeon Forms (http://www.orbeon.com/), a Java Servlet application (http://java.sun.com/) that runs in a Java Servlet container like Apache's Tomcat and JBoss.

The MathDox Formula Editor

To enable the users of MathDox documents to interact with the system, HTML offers various options, like buttons, text areas and links. However, there is no standard way to communicate mathematics in a meaningful way. Especially since we want to enable the interaction with various mathematical services offered by the system, we require the user to provide semantically rich mathematical expressions. To this end we have developed a formula editor featuring:

- A two-dimensional WYSIWYG interface.
- Semantic representation of the formulae in OpenMath.
- No plugins need to be installed in the browser to use the editor.
- It can be easily integrated into existing HTML pages.

The editor is written in JavaScript and uses the HTML5 canvas element in combination with jsMath sprite fonts (http://www.math.union.edu/jsmath/) to render mathematics.

MathDox at Work

In this section some practical examples of MathDox will be presented. MathDox has been used in various projects including:

- Wortel TU/e (http://wortel.tue.nl/) is a project that runs from 2006-2009. Within the project interactive mathematical tests and exercises are offered to students helping them to detect and repair mathematical deficiencies in mathematical knowledge.
- Algebra Interactive (Cohen, Cuypers & Sterk 1999), an interactive algebra course for undergraduates. Algebra Interactive is not only available in electronic format, but also in on paper. This shows that interactive content can be printed and used outside the electronic environment.
- MathAdore (http://www.mathadore.nl/), a web based mathematics text book for Dutch high schools under development by Pragma-ADE (http://www.pragma-ade.nl/), Math4all (http://www.math4all.nl/) and the MathDox group at the TU/e. The projects' aim is to produce a new generation of high quality teaching material incorporating various forms of interactivity. MathDox is used to provide presentation and interactivity in both examples and exercises.
- LeActiveMath (http://www.leactivemath.org/) is a European research project that focused on web-based, multilingual e-learning. MathDox tools were used for a repository of interactive exercises together with a web-based editor.
- WebALT (http://www.webalt.com/), a European project. aiming at the creation and maintenance of interactive multilingual mathematical exercises. Within the WebALT project an exercise editor was used to create these multilingual mathematical exercises. The editor was built upon MathDox, see (Cohen, Cuypers, Poels, Spanbroek & Verrijzer 2006).

MathDox Example

We present a small example that demonstrates a few practical applications of what can be done in MathDox. The example presented here, aimed at high school students, explains the concept of a tangent line to the graph of a function. The explanation uses an example function *f* given by $f(x)=x^2+2x-5$ and shows how to calculate the tangent line at x=1. The resulting tangent line is shown together with the graph of the function *f* in a graph on the page. At the very bottom of the page input fields allow users to enter other values for $f(x)$ and x. If a value is entered, the page will be recalculated with the new values.

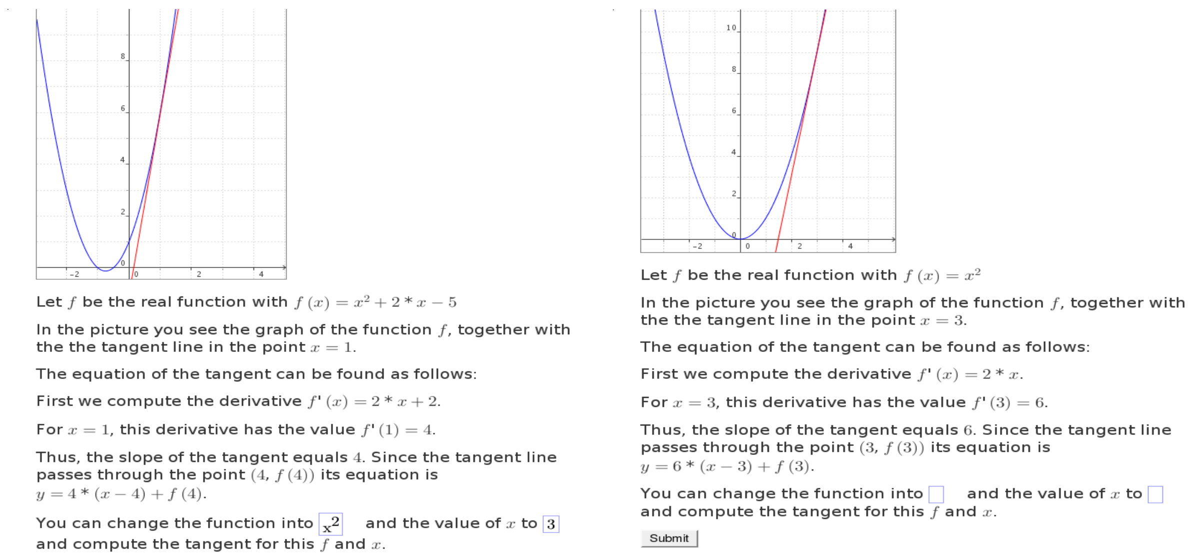

Figure 1: Two views on the same MathDox document

The resulting page, when the original function *f* is changed to $f(x)=x^2$ and the tangent is computed at the point x=3, is shown in the second figure.

Exercises in MathDox

MathDox documents can also be used to create exercises which allow users to test their skills. Basically there are two types of questions possible; 'multiple choice' (where the user has to choose his answer from a list of options) and 'open questions' (where the user has to enter his own answer).

Within MathDox it is possible, with the use of a computer algebra system, to compare the answers given by the user to that of the author of the exercise. The computer algebra system will also eliminate the problem that mathematical expressions can be written in many different ways while the semantic meaning remains the same. This allows for more complex answers and variations of the same answer as entered by the author. For instance, if the answer given by the reader is $(x+1)^2$ where the author's answer to the question had been x^2+2x+1, the answer given by the reader will not be marked as incorrect. The computer algebra system will evaluate the specified answer of the author and the answer of the reader as equivalent. Another advantage of MathDox is the possibility of randomizing values used in the question and expected answer. In this way it is possible to generate a near infinite amount of exercises.

MathDox also supports multiple step exercises. These exercises can be designed to capture mistakes and address them or correct the understanding of the user. Feedback provided in the exercise may vary from a hint, a reference to the theory, or by simplifying the question. Of course, it is also possible to increase the difficulty of an exercise as long as the user's answers are correct. In this way exercises adapt to the skills of the user.

MathDox Exercises and Scorm

To enable the use of MathDox exercises in an LMS like Blackboard (http://www.blackboard.com/), Moodle (http://moodle.org/), or Sakai (http://www.sakaiproject.org/), we developed a scoring system and made it is possible to include MathDox documents in SCORM (Shareable Content Object Reference Model) (http://www.adlnet.gov/scorm/) packages. In this way it is possible to track the student's performance.

Conclusions and Future Work

The MathDox format promises to be a good solution for interactive mathematics. It offers access to computer algebra systems of choice, takes care of rendering the mathematics in browsers without forcing users to install extra software, and it offers a mathematical editor that does not take up too much space on a page. The MathDox system is open source software. We will continue our work on MathDox and give special attention to:

Performance

The performance of the current MathDox is not yet optimal. Although some performance improvements have already been made, the search for further improvements will continue.

Presentation/Semantic Mathematics

Presentation mathematics is focused on how the mathematics should look like, while semantic mathematics often tends to be imprecise on how to present the mathematics. In the MathDox Player we have almost reached the ideal mixture of presentation and semantic mathematics. However rendering OpenMath in a page gives the user sometimes a mechanical impression. This is mainly caused by the lack of presentation information in the OpenMath format. Work is done to solve this kind of problems, by adding extra attributes to the OpenMath elements.

Context

At the moment the MathDox Player is without any notion of context. Adding a context to the MathDox system would extend the capabilities of MathDox considerably. It would then become possible to keep track of a users skill level, preferences, and history and adapt content, notations, and used software on a MathDox page accordingly. The communication of a larger set of MathDox pages could also be optimized to benefit the user. Investigations of adding context to the MathDox system are ongoing.

References

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