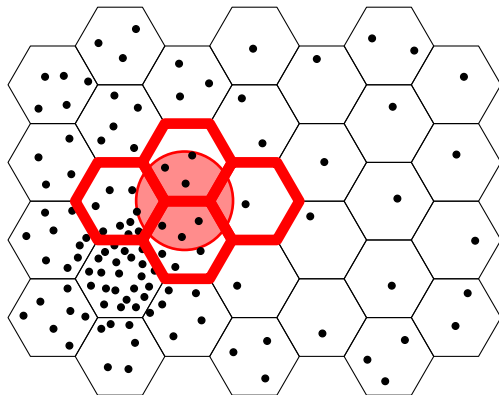


Circular range query: report all points inside query circle  $Q$

non-recursive tiling

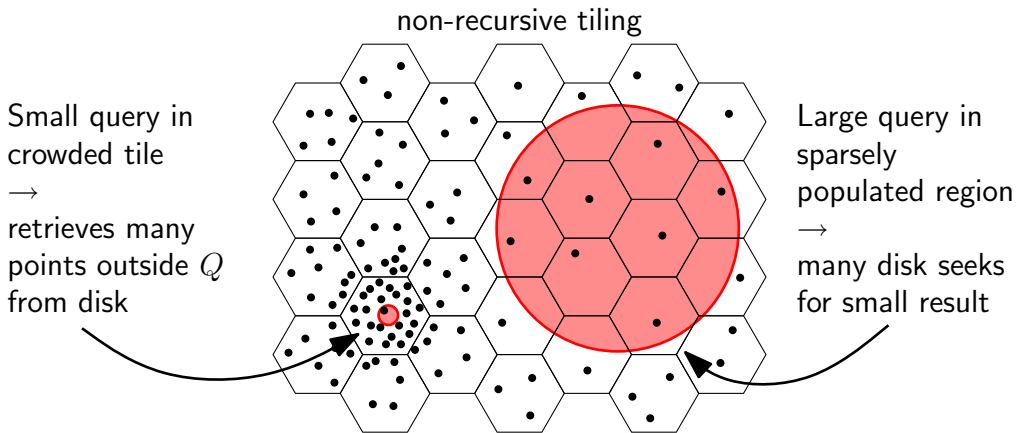


Data structure: store points tile by tile; each tile = contiguous block on disk

Circular range query: report all points inside query circle  $Q$

Query algorithm: retrieve every tile intersecting  $Q$  (one disk seek per tile)

# Recursive tilings and space-filling curves with little fragmentation



Data structure: store points tile by tile; each tile = contiguous block on disk

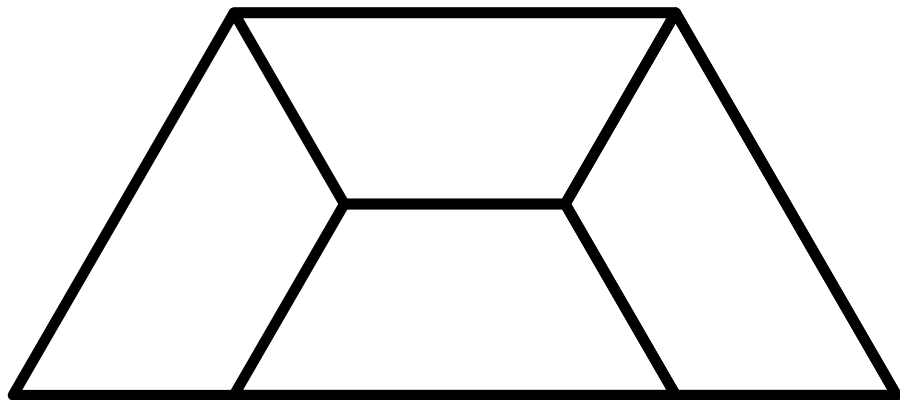
Circular range query: report all points inside query circle  $Q$

Query algorithm: retrieve every tile intersecting  $Q$  (one disk seek per tile)

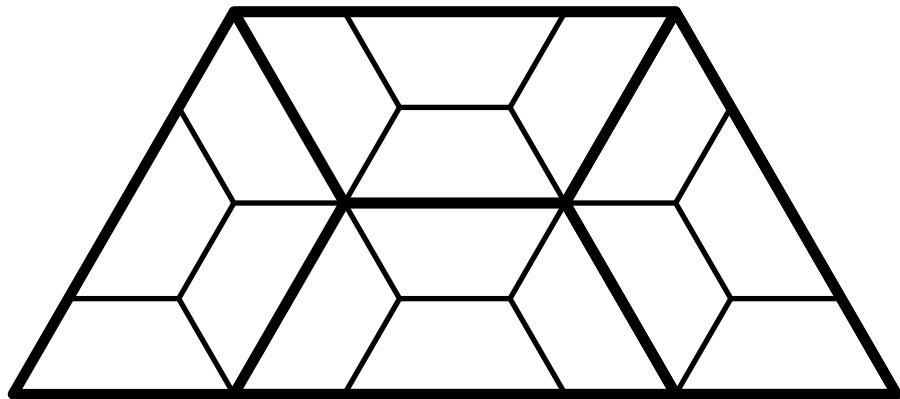
recursive tiling



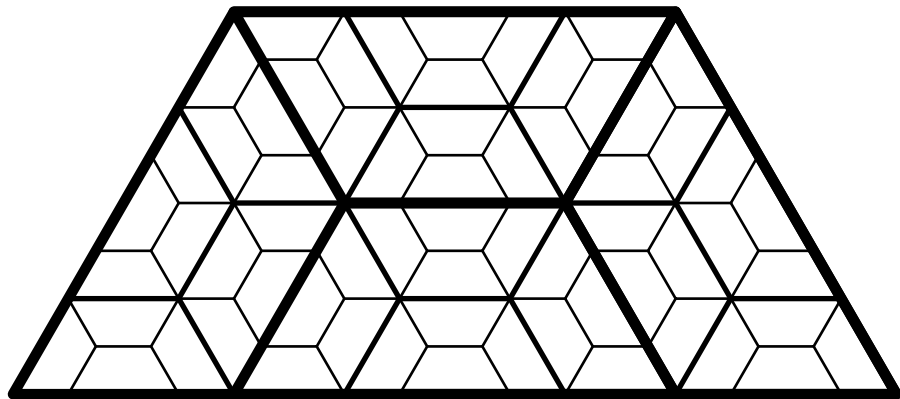
recursive tiling



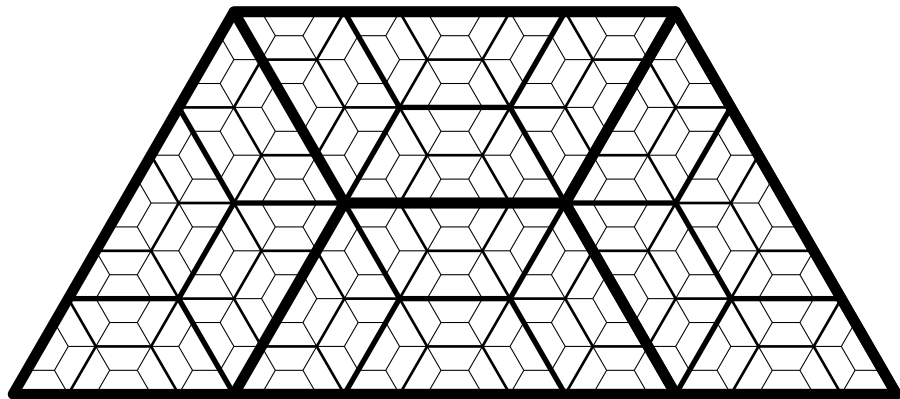
recursive tiling



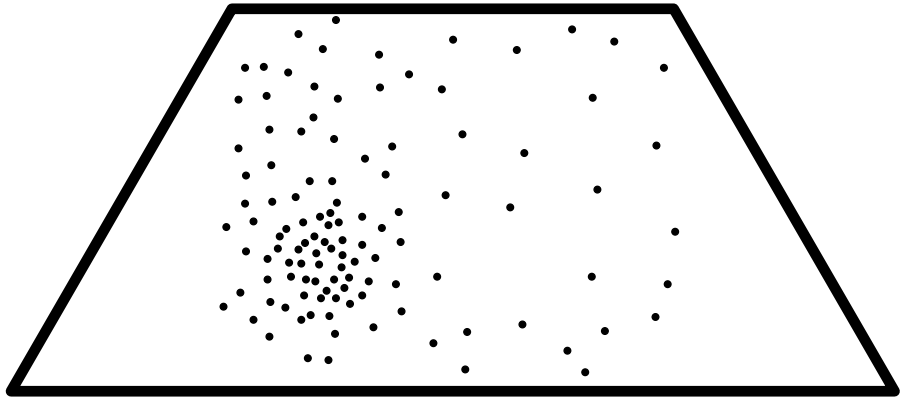
recursive tiling



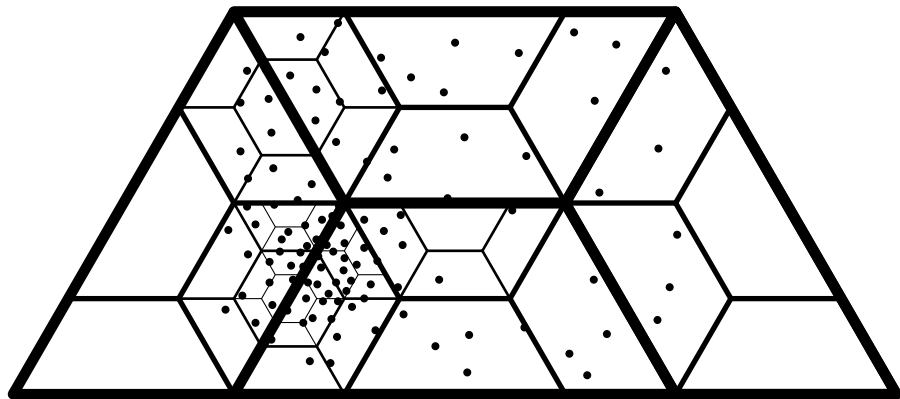
recursive tiling



recursive tiling

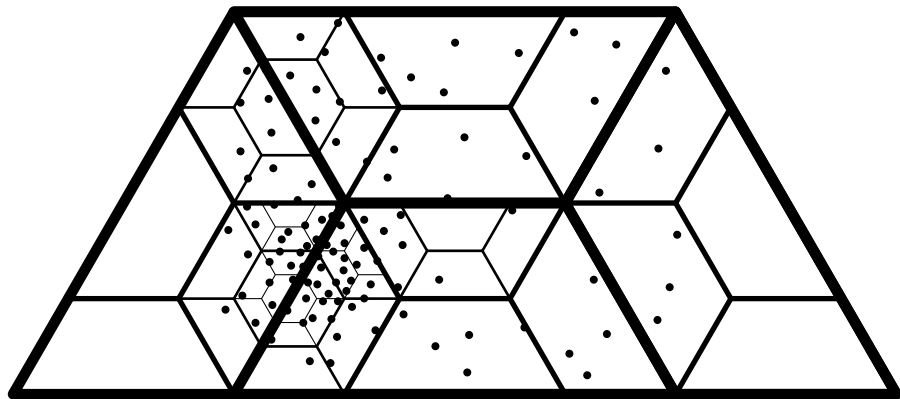


recursive tiling



Data structure: store points tile by tile; each tile = contiguous block on disk  
(similar to linear quadtree)

recursive tiling



Data structure: store points tile by tile; each tile = contiguous block on disk

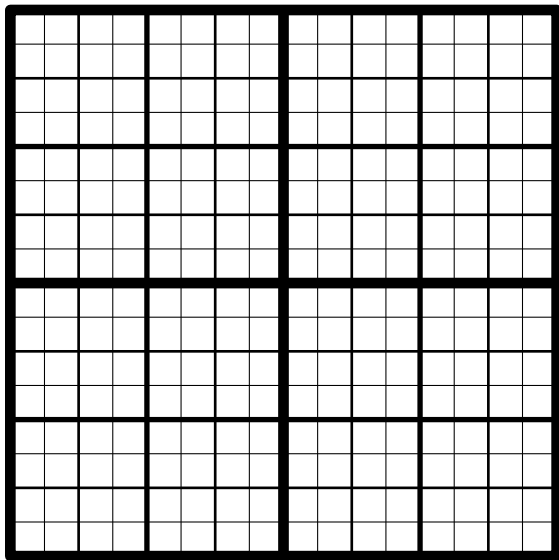
Goal 1:  $a$  tiles suffice to cover any disk-shaped query range  $Q$  (seek time)

Goal 2: the tiles that cover  $Q$  have total area at most  $c \cdot \text{area}(Q)$  (read time)

**Arrwid number** =

smallest  $a$  such that there is a constant  $c$  such that  $\forall Q$  both goals achieved

## The Arrwid number of a tiling with squares

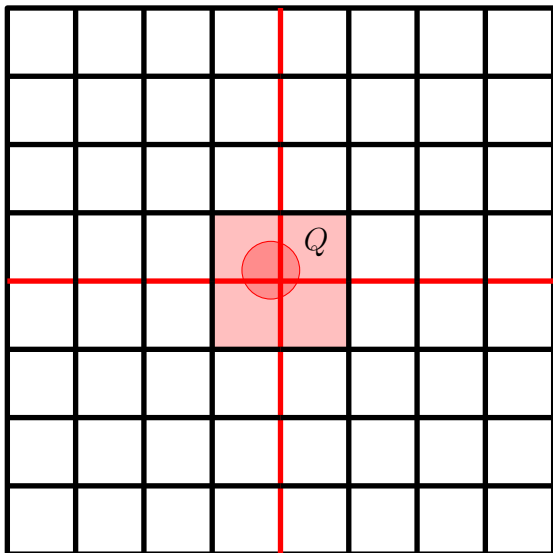


Arrwid nr. of square tiling  $\leq 4$ :

Arrwid nr. of square tiling  $> 3$ :

**Arrwid number** = smallest  $a$  such that  $\exists$  constant  $c$  such that any disk  $Q$  is covered by  $\leq a$  tiles of total area  $\leq c \cdot \text{area}(Q)$

## The Arrwid number of a tiling with squares



Arrwid nr. of square tiling  $\leq 4$ :

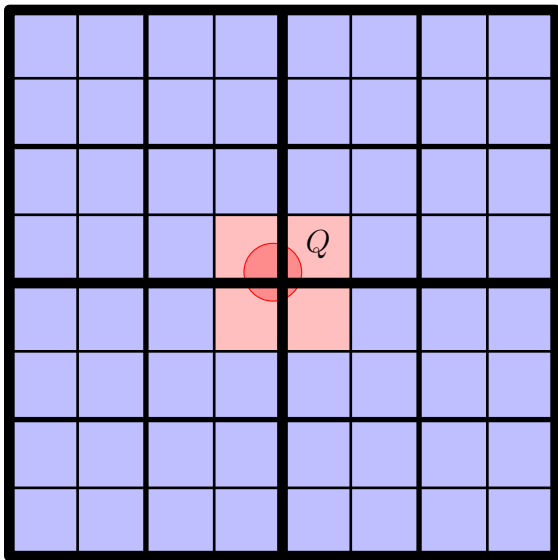
For disk with radius  $r$ , consider grid with tile width  $\geq 2r$ ,  $< 4r$ :

disk intersected by  $\leq 2$  grid lines;  
cover by  $\leq 4$  tiles meeting there.

Arrwid nr. of square tiling  $> 3$ :

**Arrwid number** = smallest  $a$  such that  $\exists$  constant  $c$  such that any disk  $Q$  is covered by  $\leq a$  tiles of total area  $\leq c \cdot \text{area}(Q)$

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Arrwid nr. of square tiling  $\leq 4$ :

For disk with radius  $r$ , consider grid with tile width  $\geq 2r, < 4r$ :

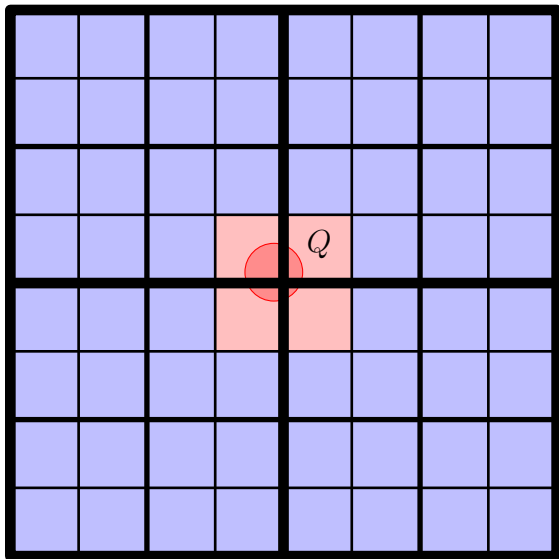
disk intersected by  $\leq 2$  grid lines;  
cover by  $\leq 4$  tiles meeting there.

Arrwid nr. of square tiling  $> 3$ :

To cover with  $\leq 3$  tiles,  
need **common ancestor** of red tiles,  
can be much bigger than  $Q$   
 $\rightarrow$  no constant  $c$ .

**Arrwid number** = smallest  $a$  such that  $\exists$  constant  $c$  such that  
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# The Arrwid number of a tiling with squares



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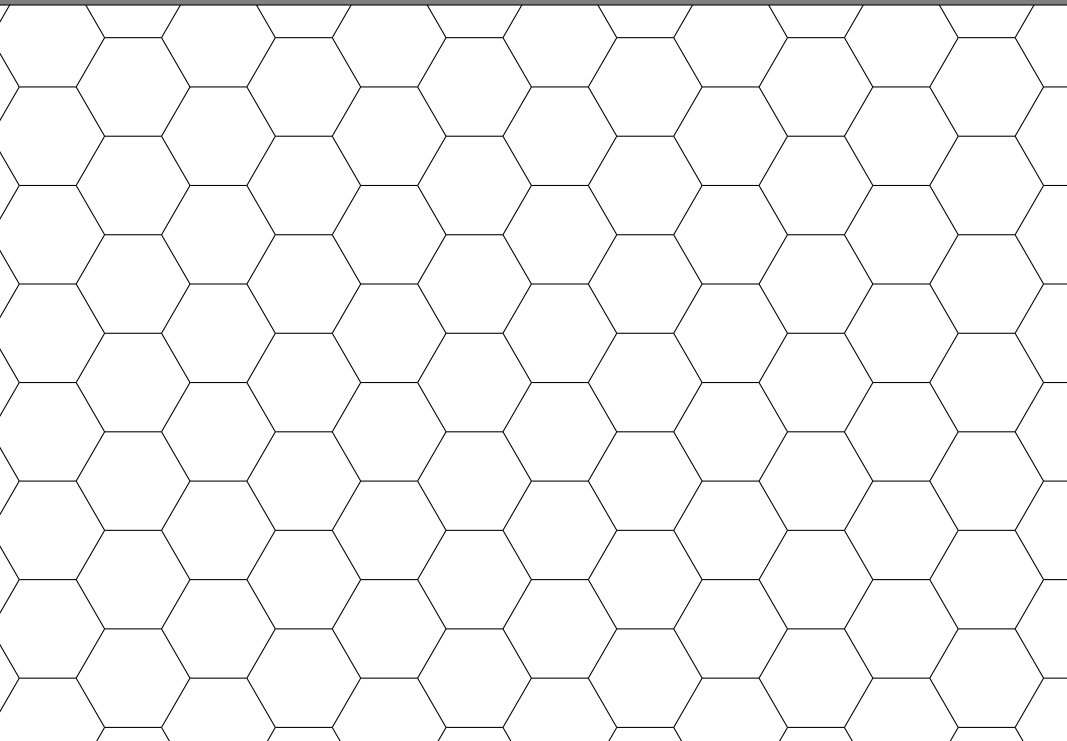
To cover with  $\leq 3$  tiles,  
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Arrwid nr.  $\approx$  degree of vertices

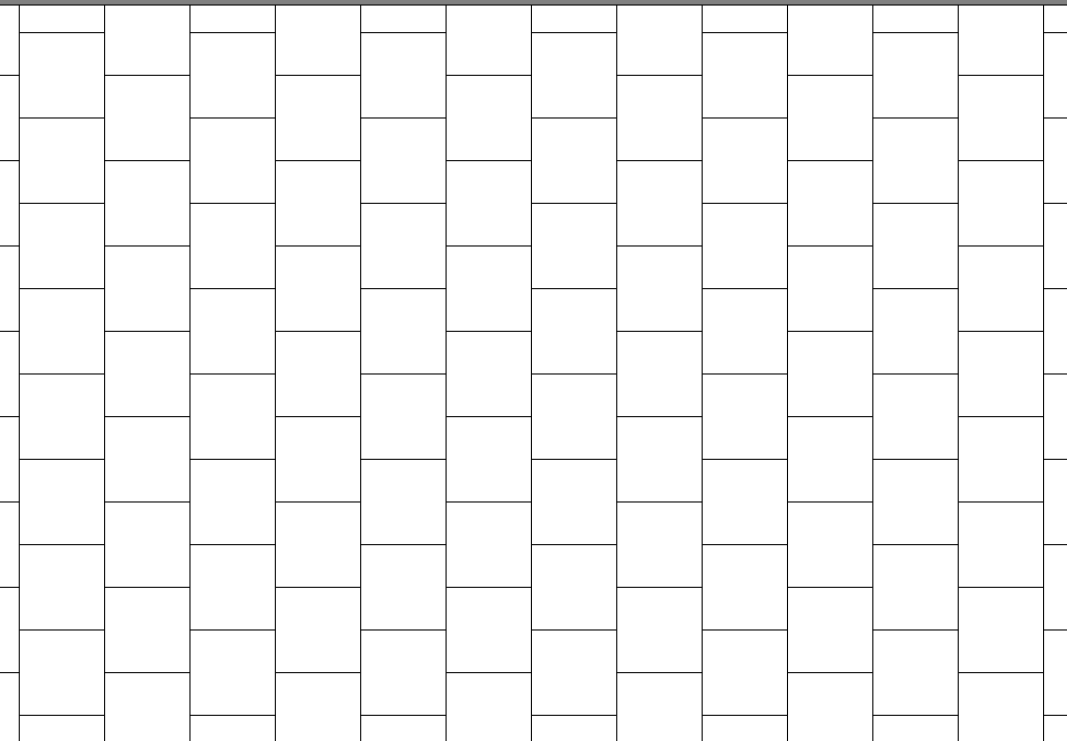
**Are there recursive tilings with vertex degree 3?**

**Arrwid number** = smallest  $a$  such that  $\exists$  constant  $c$  such that any disk  $Q$  is covered by  $\leq a$  tiles of total area  $\leq c \cdot \text{area}(Q)$

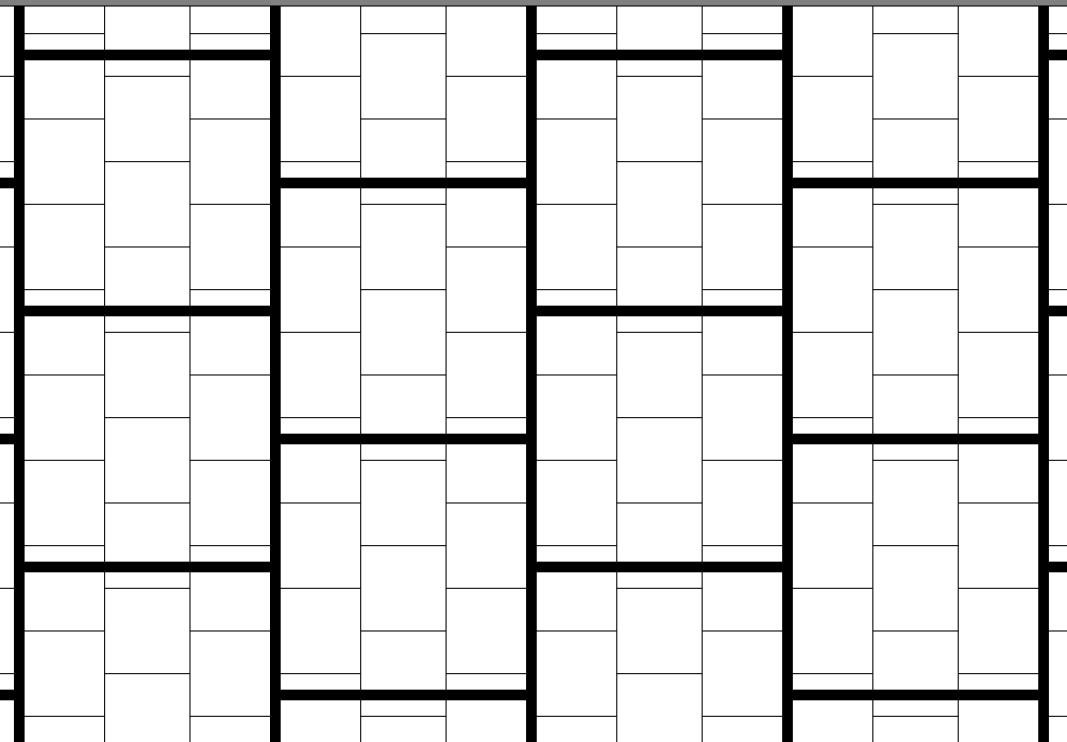
From non-recursive tilings...



From non-recursive tilings...

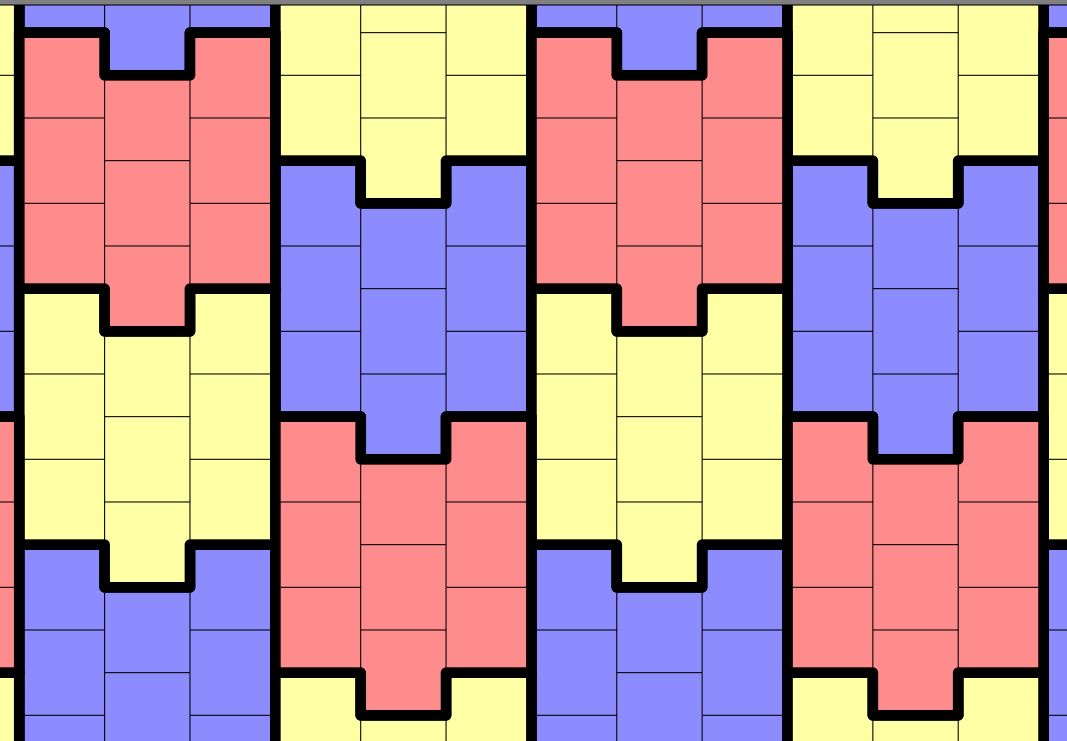


# From non-recursive tilings to recursive tilings

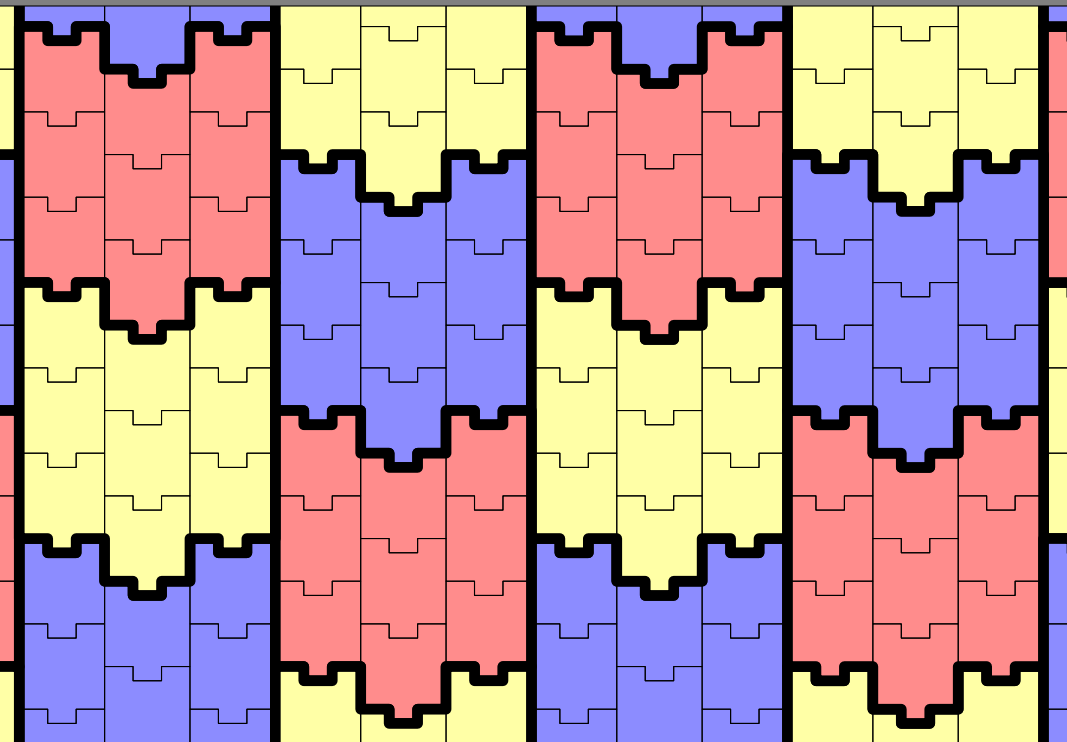




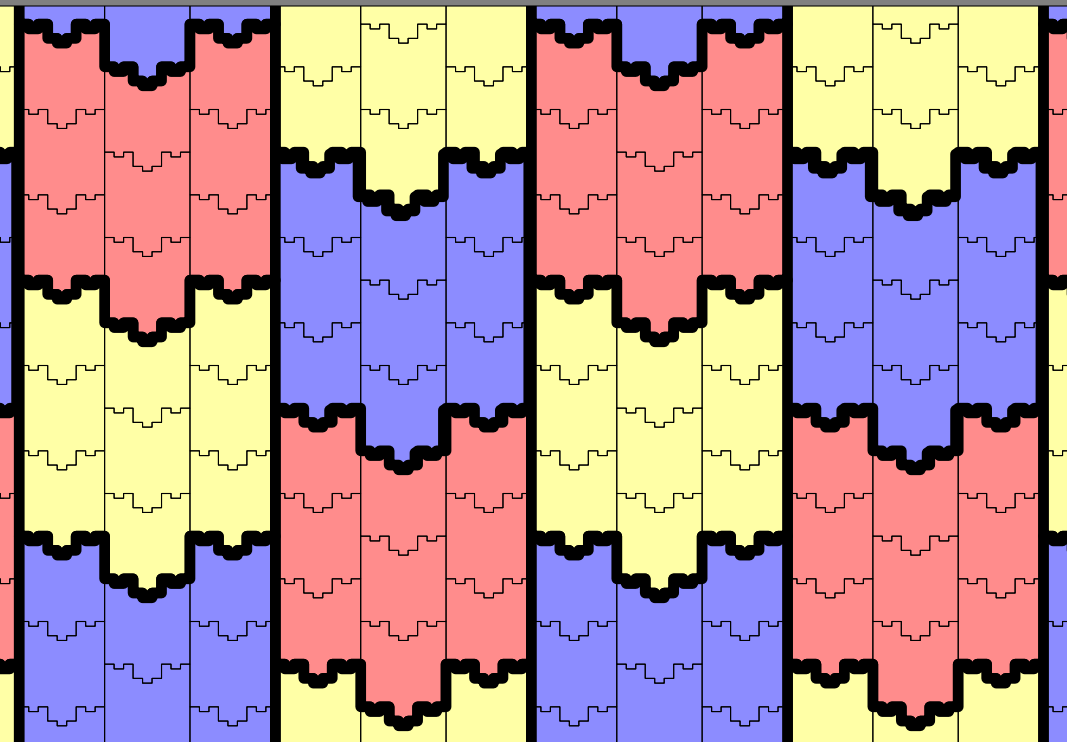
# From non-recursive tilings to recursive tilings



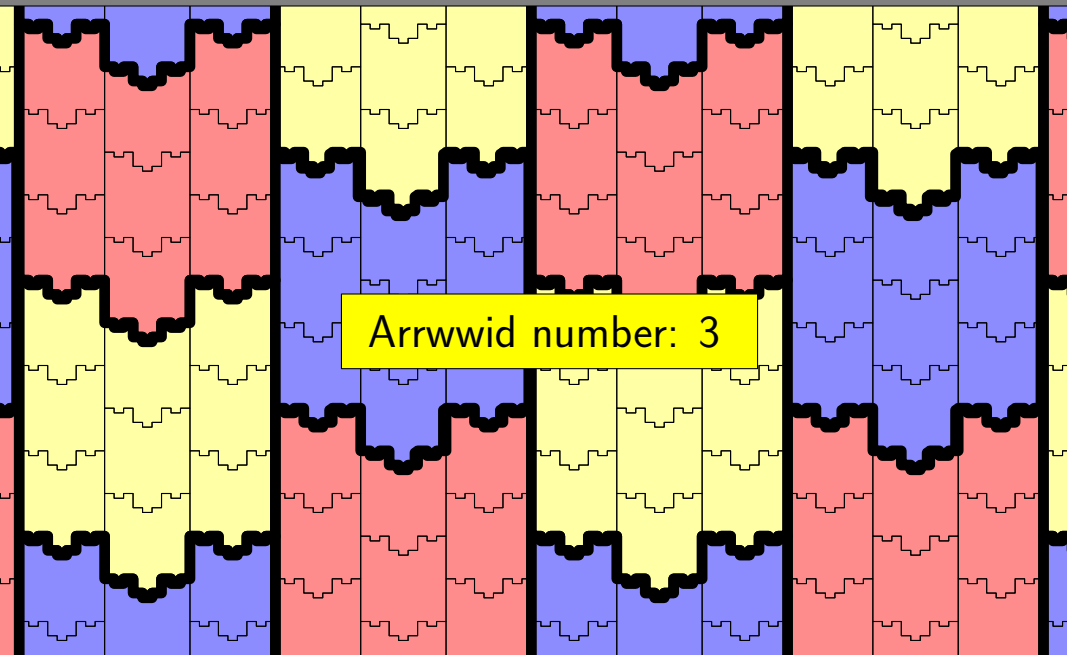
# From non-recursive tilings to recursive tilings



# From non-recursive tilings to recursive tilings



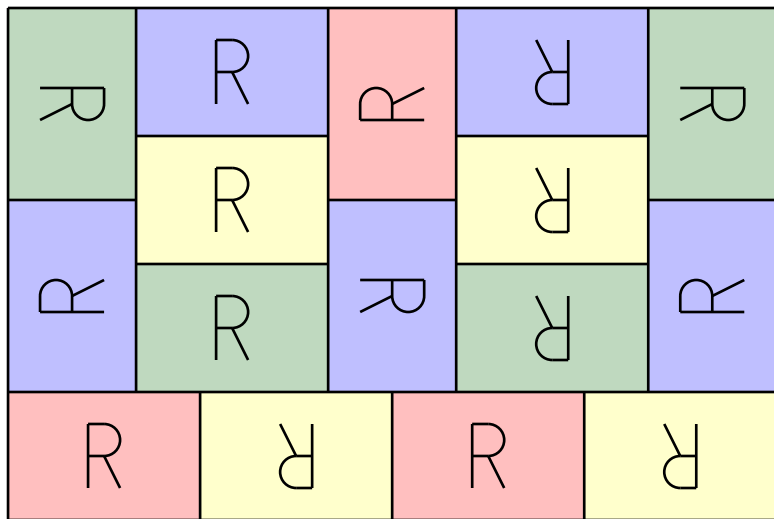
## From non-recursive tilings to recursive tilings



**Arrwwid number** = smallest  $a$  such that  $\exists$  constant  $c$  such that any disk  $Q$  is covered by  $\leq a$  tiles of total area  $\leq c \cdot \text{area}(Q)$

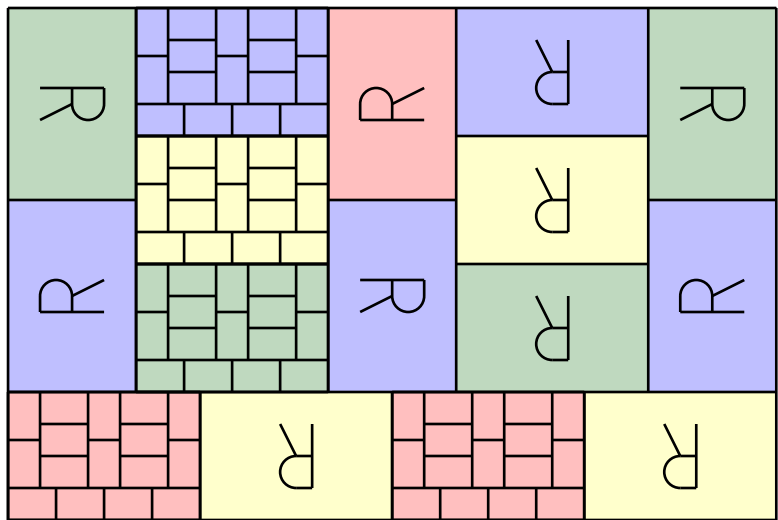
# Daun tiling: Arrwwid nr. 3 and "easy" tiles

R:



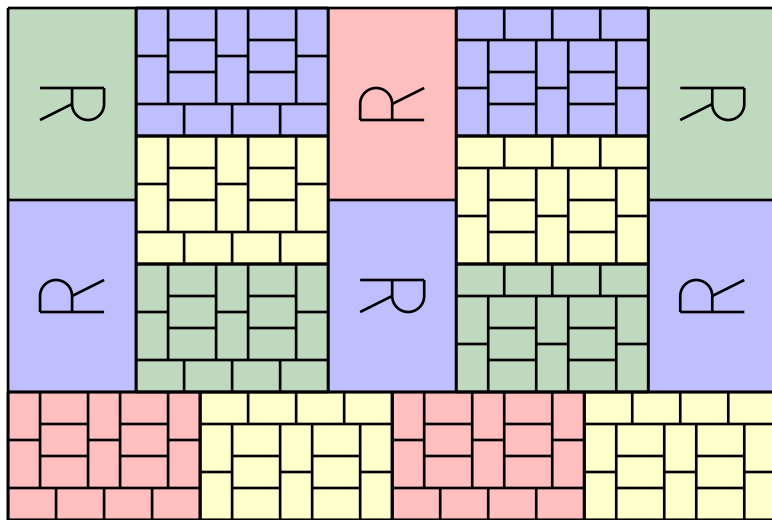
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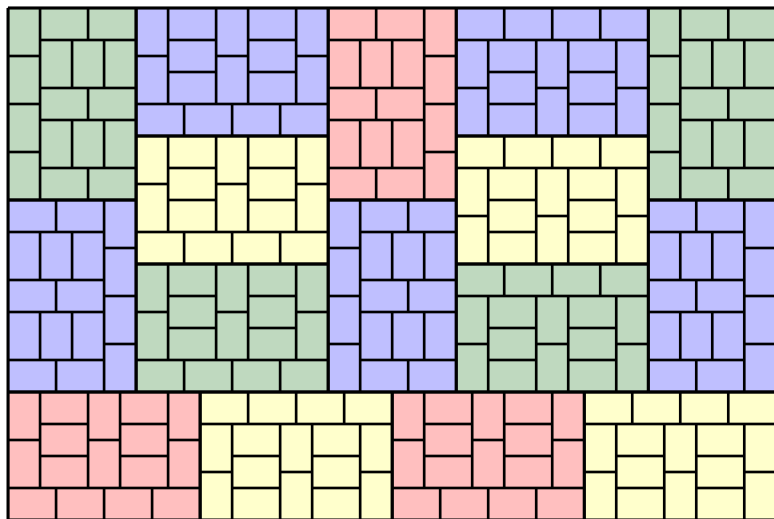
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R:



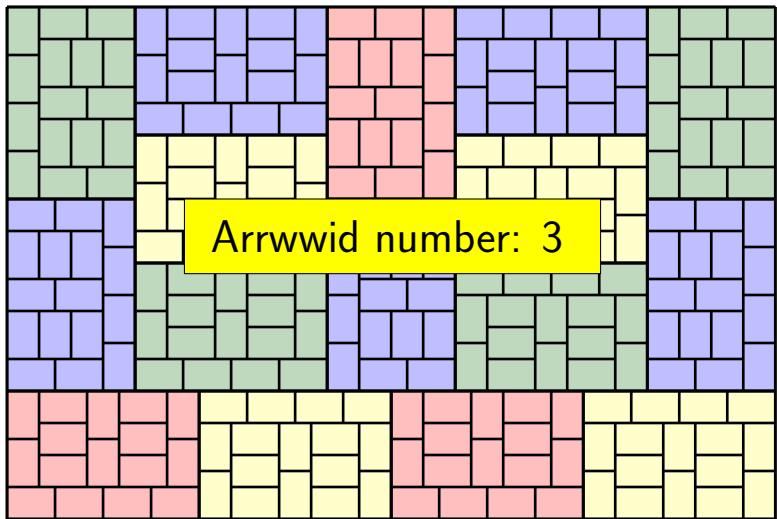
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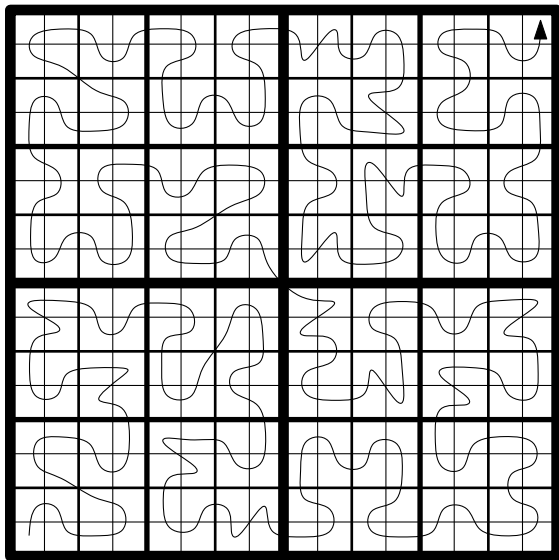
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## The Arrwid number of a space-filling curve



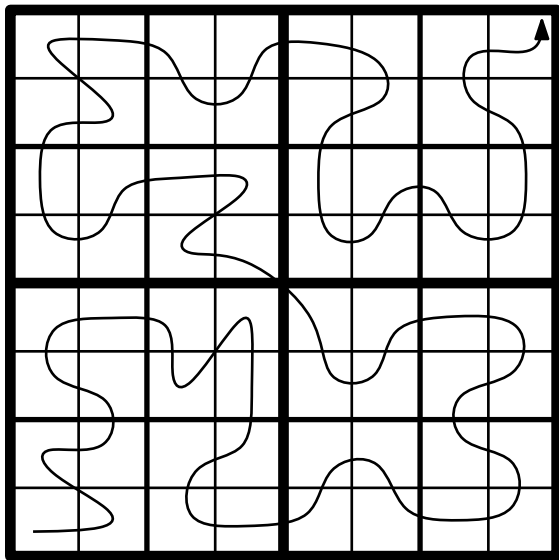
space-filling curve =  
rec. tiling + ordering rule s.t.

all subtiles within any tile are  
consecutive

← Example:  $AR^2W^2$  order

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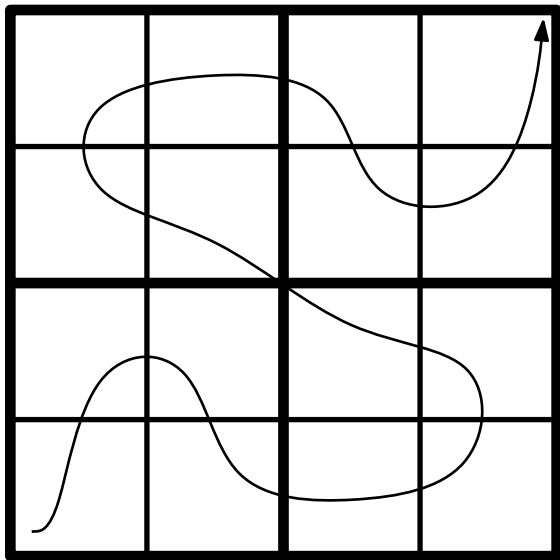
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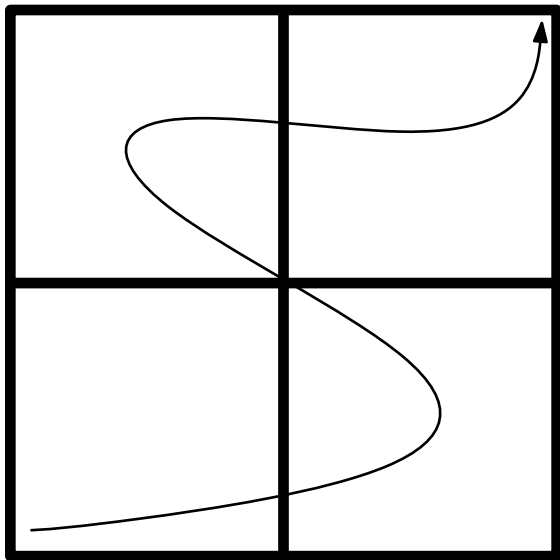
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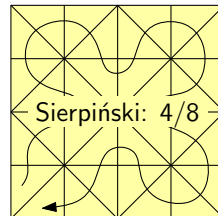
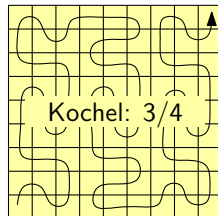
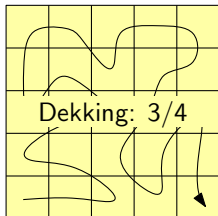
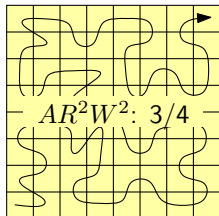
Definition for recursive tilings:

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Definition for space-filling curves (scanning orders of recursive tilings):

**Arrwwid number** = smallest  $a$  such that  $\exists$  constant  $c$  such that any disk  $Q$  is covered by  $\leq a$  **sets of consecutive** tiles of total area  $\leq c \cdot \text{area}(Q)$

Some curves have smaller Arrwwid nr. than the underlying tiling:



# The Arrwwid number of a space-filling curve

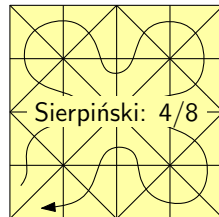
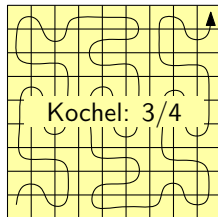
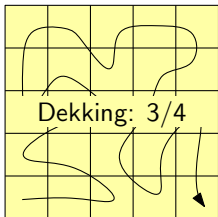
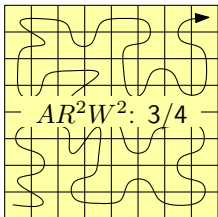
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Some curves have smaller Arrwwid nr. than the underlying tiling:



Some tilings have Arrwwid nr. 3.

**Some curves have Arrwwid nr. <3?**

*Asano et al.*: Not if tiling divides squares into four squares  
*Yours truly*: **Not ever** (assuming tiles are simply connected)

## Results

Herman Haverkort: *Recursive tilings and space-filling curves with little fragmentation*.  
arXiv:1002.1843 [cs.CG], 2010

best Arrwwid nrs. for tilings                      2D

---

uniform squares                                      4 [ARRWW]

best nrs. for space-filling curves              2D

---

uniform squares                                      3 [ARRWW] ← most known curves have 4

---

**Arrwwid number** = smallest  $a$  such that  $\exists$  constant  $c$  such that  
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uniform squares                                      4 [ARRWW]

uniform rectangles                                      3

uniform tiles of any shape                                      3

best nrs. for space-filling curves                      2D

---

uniform squares                                      3 [ARRWW] ← most known curves have 4

uniform rectangles                                      3

uniform tiles of any shape                                      3

lower bound holds for simple tiles

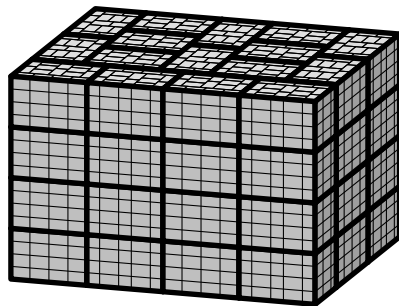
---

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best Arrwwid nrs. for tilings	2D	3D	$d$ -D
uniform hypercubes	4 [ARRWW]	8	$2^d$
uniform hyperboxes	3	6 (lwbd 4)	$\frac{3}{4} \cdot 2^d$ (lwbd:?)
uniform tiles of any shape	3		
<hr/>			
best nrs. for space-filling curves	2D		
uniform hypercubes	3 [ARRWW]		
uniform hyperboxes	3		
uniform tiles of any shape	↪ 3		
lower bound holds for simple tiles			



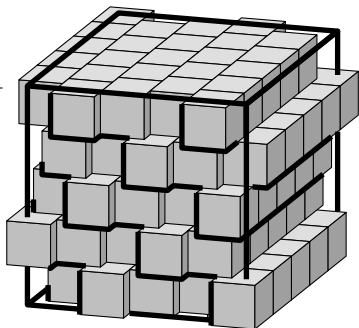
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lower bound holds for simple tiles	



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uniform hypercubes	3 [ARRWW]	8 (lwbd 7)	$2^d$ (lwbd $2^d - 1$ )
uniform hyperboxes	3	6 (lwbd 4)	$\frac{3}{4} \cdot 2^d$ (lwbd:?)
uniform tiles of any shape	3	4	$d + 1$
	lower bound holds for simple tiles	...for convex tiles	(lwbd:?)

**Arrwwid number** = smallest  $a$  such that  $\exists$  constant  $c$  such that any disk  $Q$  is covered by  $\leq a$  (sets of consecutive) tiles of total area  $\leq c \cdot \text{area}(Q)$

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lower bound holds for simple tiles ...for convex tiles

exponential gap between cubes / general shapes (but maybe also in  $c$ )

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<b>THAT'S ALL FOLKS</b>			
best nrs. for space-filling curves	2D	3D	$d$ -D

uniform hypercubes	3 [ARRWW]	8 (lwbd 7)	$2^d$ (lwbd $2^d - 1$ )
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