Extra exercises Complexiteit IBC028

February, 2018

Exercise 1.

Let $f = \text{fib}$ be defined by $f(i) = i$ for $i = 0, 1$ and $f(i) = f(i-1) + f(i-2)$ for $i > 1$. Prove by induction on n that

$$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^n = \begin{pmatrix} f(n-1) & f(n) \\ f(n) & f(n+1) \end{pmatrix}$$

for all $n \geq 1$.

Exercise 2.

Let $f = \text{fib}$ be defined by $f(i) = i$ for $i = 0, 1$ and $f(i) = f(i-1) + f(i-2)$ for $i > 1$. Prove by induction on n that

$$f(n-1)f(n+1) = f(n)^2 + 1$$

if n is even, and

$$f(n-1)f(n+1) = f(n)^2 - 1$$

if n is odd, for all $n \geq 1$.

Exercise 3.

For each function on the left, $p(n)$, write the letter of a function on the right, $q(n)$, such that $p(n) \in \Theta(q(n))$. If no such function $q(n)$ is listed, then choose (l).

$$f(n) = \sum_{i=1}^{n} (4i - 4)$$ __________

$$g(n) = \sum_{i=1}^{n} \sum_{j=1}^{i} i$$ __________

$$h(n) = \sum_{i=1}^{\lfloor \log n \rfloor} n$$ __________

$$k(n) = \sum_{i=0}^{n} \frac{4}{2^i}$$ __________

(a) 1	(g) $\log n$
(b) n	(h) $n \log n$
(c) $n(\log n)^2$	(i) n^2
(d) $n^2 \log n$	(j) n^3
(e) 2^n	(k) 2^{2n}
(f) n^n	(l) no match

Exercise 4.

Rank the following functions in n by order of growth from low to high; some may be of the same order.

$$n\sqrt{n} \qquad \sum_{i=0}^{n} \log n \qquad n^n \qquad \log\sqrt{n} \quad \log(n^2) \quad (\log n)^2 \qquad 2^n \qquad 3^n$$

$$\sum_{i=0}^{\lfloor \log n \rfloor} i \qquad \sum_{i=0}^{n} i^2 \qquad n^{0.001} \qquad 17n^3 \qquad 17^{\log 89} \qquad n^2 \qquad 100n \quad 1$$

Exercise 5.

Let g be defined by $g(i) = 1$ for $i = 0, 1, 2$ and $g(i) = g(i-2) + g(i-3)$ for $i > 2$. Prove by induction on n that $g(n) > 0.5 * (1.2)^n$ for all $n \geq 0$.

Exercise 6.

Let $T(n) = n + T(n/2) + 2T(n/5)$. Prove that $T(n) = \Theta(n)$.

Exercise 7.

Let $T(n) = n^2 + T(n-1)$. Prove that $T(n) = \Theta(n^3)$.