

Practical Assignment

Automated Reasoning 2IMF25

prof dr H. Zantema
MF room 6.078
email: h.zantema@tue.nl
November, 2016

The programs to be used

Programs to be used:

- Yices: <http://yices.csl.sri.com/>.

- Z3: <http://z3.codeplex.com/>.

Both Yices and Z3 are programs for satisfiability modulo theories (SMT). They accept standard SMT format, in particular boolean SAT format.

- NuSMV: <http://nusmv.fbk.eu/>.

This is a symbolic model checker based on BDDs

- Prover9 and Mace4: <https://www.cs.unm.edu/~mccune/mace4/>

These are tools for predicate and equational logic: Prover9 for giving proofs based on resolution, and Mace4 for finding counterexamples.

Each of the problems should be solved using one of these tools. The tools should do the job: manual modifications of the problems should be avoided.

For further information about the course we refer to
<http://www.win.tue.nl/~hzantema/ar.html>

The assignment

The practical assignment has to be executed by one or two persons. It consists of two parts.

The results of the assignment have to be described in two reports that should be handed in on paper (printed two-sided, preferably less than 20 pages). For the report on the first part the deadline is **December 12, 2016**, for the report on the second part the deadline is **January 12, 2017**.

For all used formulas an extensive documentation is required, explaining the approach and the overall structure. A generic approach is preferred, since this may result in clearer descriptions, increasing the confidence in the correctness of the results. Formulas of more

than half a page should not be contained in the report, instead the structure of the formula should be explained. From the output of the programs relevant parts should be contained in the report, and observations on computation time should be reported. The answers on the problems should be motivated. Every report should contain name, student number and email address of each of the authors. In case of two authors each of them is considered to be responsible for the full text and all results.

Guidelines for grading:

- Clear and generic descriptions are appreciated, both of the formulas themselves and the way they were designed. An example of appreciated style is given at <http://www.win.tue.nl/~hzantema/prvb.pdf>.
- For both parts at least 3 out of the 4 solutions should be correct to obtain a 7.
- Not giving a solution at all for one problem is preferred over giving a wrong solution.
- Reasons for obtaining higher than a 7 may be:
 - all problems correctly solved,
 - remarkably clear and structured descriptions,
 - approaches allowing generalizations,
 - original approaches and solutions.

The problems for the first part

For the first part (deadline December 12, 2016) you have to find and describe solutions of the following 4 problems using the indicated programs.

1. Eight trucks have to deliver pallets of obscure building blocks to a magic factory. Every truck has a capacity of 8000 kg and can carry at most eight pallets. In total, the following has to be delivered:
 - Four pallets of nuzzles, each of weight 700 kg.
 - A number of pallets of prittles, each of weight 400 kg.
 - Eight pallets of skipples, each of weight 1000 kg.
 - Ten pallets of crottles, each of weight 2500 kg.
 - Twenty pallets of dupples, each of weight 200 kg.

Skipples need to be cooled; only three of the eight trucks have facility for cooling skipples.

Nuzzles are very valuable: to distribute the risk of loss no two pallets of nuzzles may be in the same truck.

- (a) Investigate what is the maximum number of pallets of prittles that can be delivered, and show how for that number all pallets may be divided over the eight trucks.
- (b) Do the same, with the extra information that prittles and crottles are an explosive combination: they are not allowed to be put in the same truck.

2. Give a chip design containing two power components and ten regular components satisfying the following constraints:

- Both the width and the height of the chip is 30.
- The power components have width 4 and height 3.
- The sizes of the ten regular components are 4×5 , 4×6 , 5×20 , 6×9 , 6×10 , 6×11 , 7×8 , 7×12 , 10×10 , 10×20 , respectively.
- All components may be turned 90° , but may not overlap.
- In order to get power, all regular components should directly be connected to a power component, that is, an edge of the component should have at least one point in common with an edge of the power component.
- Due to limits on heat production the power components should be not too close: their centres should differ at least 17 in either the x direction or the y direction (or both).

3. Twelve jobs numbered from 1 to 12 have to be executed satisfying the following requirements:

- The running time of job i is $i + 5$, for $i = 1, 2, \dots, 12$.
- All jobs run without interrupt.
- Job 3 may only start if jobs 1 and 2 have been finished.
- Job 5 may only start if jobs 3 and 4 have been finished.
- Job 7 may only start if jobs 3, 4 and 6 have been finished.
- Job 8 may not start earlier than job 5.
- Job 9 may only start if jobs 5 and 8 have been finished.
- Job 11 may only start if job 10 has been finished.
- Job 12 may only start if jobs 9 and 11 have been finished.
- Jobs 5, 7 and 10 require a special equipment of which only one copy is available, so no two of these jobs may run at the same time.

(a) Find a solution of this scheduling problem for which the total running time is minimal.

(b) Do the same with the extra requirement that job 6 and job 12 need a resource of limited availability, by which job 6 may only run during the 17 time units in which job 12 is running.

4. Eight integer variables $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$ are given, for which the initial value of a_i is i for $i = 1, \dots, 8$. The following steps are defined: choose i with $1 < i < 8$ and execute

$$a_i := a_{i-1} + a_{i+1},$$

that is, a_i gets the sum of the values of its neighbors and all other values remain unchanged.

(a) Show how it is possible that $a_3 = a_7$ after a number of steps.

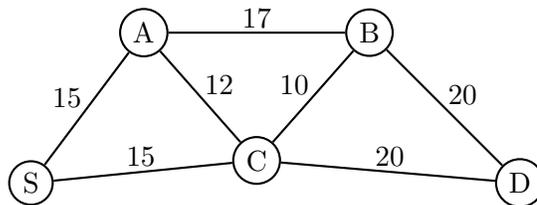
(b) Show how it is possible that $a_3 = a_5 = a_7$ after a number of steps.

For this problem both try to use an SMT solver and NuSMV, and discuss how they perform.

The problems for the second part

For the second part (deadline January 12, 2017) you have to find and describe solutions of the following 4 problems using the indicated programs.

1. Four non-self-supporting villages A, B, C and D in the middle of nowhere consume one food package each per time unit. The required food packages are delivered by a truck, having a capacity of 240 food packages. The truck has to pick up its food packages at location S containing an unbounded supply. The locations of this supply location and the villages and the roads between them are shown in the following picture. Here every number indicates a distance, more precisely, the number of time units the truck needs to travel from one village to another, including loading and delivering. The villages only have a limited capacity to store food packages: for A this capacity is 120, for B and D it is 160, and for C it is 100. Initially, the truck is in S and is fully loaded, and in each of the four villages there are 80 food packages.



(a) Show that it is impossible to deliver food packages in such a way that each of the villages consumes one food package per time unit forever.

(b) Show that this is possible if the capacity of the truck is increased to 260 food packages. (Note that a finite graph contains an infinite path starting in a node v if and only if there is a path from v to a node w for which there is a non-empty path from w to itself.)

2. Let an NFA be defined as in wikipedia

https://en.wikipedia.org/wiki/Nondeterministic_finite_automaton

Find an NFA M over $\Sigma = \{a, b\}$ with the least possible number of states for which the only non-empty words in $L(M)$ of length < 5 are aa , bb , aba , baa , $abab$, $babb$ and $bbba$.

3. The goal of this problem is to exploit the power of the recommended tools rather than elaborating the questions by hand.

(a) For a binary operator a we have the idempotence, commutativity and associativity rule, that is, the three rewrite rules

$$a(x, x) \rightarrow x, \quad a(x, y) \rightarrow a(y, x), \quad a(x, a(y, z)) \rightarrow a(a(x, y), z).$$

Figure out whether $a(p, a(q, a(p, a(q, a(p, a(q, p))))))$ rewrites to $a(p, q)$ in a finite number of steps, for constants p, q .

(b) In mathematics, a *group* is defined to be a set G with an element $I \in G$, a binary operator $*$ and a unary operator inv satisfying

$$x * (y * z) = (x * y) * z, \quad x * I = x \quad \text{and} \quad x * inv(x) = I,$$

for all $x, y, z \in G$. Determine whether in every group each of the four properties

$$I * x = x, \quad inv(inv(x)) = x, \quad inv(x) * x = I \quad \text{and} \quad x * y = y * x$$

holds for all $x, y \in G$. If a property does not hold, determine the size of the smallest finite group for which it does not hold.

4. Give a precise description of a non-trivial problem of your own choice, and encode this and solve it by one of the given programs.

Here ‘trivial problems’ include both minor modifications of earlier problems and single solutions of simply specified puzzles like sudokus. In case of doubt please contact the lecturer.