#### Bas Luttik s.p.luttik@tue.nl HG 6.85

FSA seminar (22 September 2011)



$$x + y = y + x$$

$$(x + y) + z = x + (y + z)$$

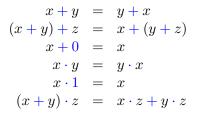
$$x + 0 = x$$

$$x \cdot y = y \cdot x$$

$$x \cdot 1 = x$$

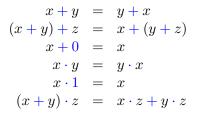
$$(x + y) \cdot z = x \cdot z + y \cdot z$$





This collection of axioms is sound (and complete?) for  $(\mathbb{N}, +, \cdot, 0, 1)$ .

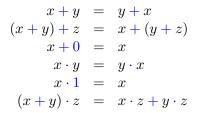




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Let  $\mathbb{P}$  be a set of processes,  $0 \in \mathbb{P}$  is the **deadlocked process**,  $1 \in \mathbb{P}$  is the **(successfully) terminated** process, + is **nondeterministic choice** and  $\cdot$  is **sequential composition**.



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#### Answer

Most likely not, but first tell me a bit more about  $(\mathbb{P}, +, \cdot, 0, 1)!_{I}$ 

**1. syntax:**  $P ::= 0 | 1 | a \cdot P | P \cdot P | P + P$ .



## Recipe for a concrete process algebra (1)

- **1. syntax:**  $P ::= 0 | 1 | a \cdot P | P \cdot P | P + P$ .
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3. **behavioural equivalence**:  $\Delta$  is the largest equivalence relation on the syntax such that, whenever  $P \Delta Q$ ,

• if 
$$P \xrightarrow{a} P'$$
, then  $Q \xrightarrow{a} Q'$  and  $P' \leftrightarrow Q'$ ;

• if  $P \downarrow$ , then  $Q \downarrow$ .



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  - if  $P \xrightarrow{a} P'$ , then  $Q \xrightarrow{a} Q'$  and  $P' \leftrightarrow Q'$ ;
  - if  $P \downarrow$ , then  $Q \downarrow$ .
- 4. Define  $\mathbb{P}$  as the set of  $\pm$ -classes of expressions.



### Recipe for a concrete process algebra (2)

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Suggested definition of  $+, \cdot, 0$  and 1:

$$\begin{split} [P] + [Q] &= [P + Q] \\ [P] \cdot [Q] &= [P \cdot Q] \end{split} \qquad \begin{array}{l} 0 &= [0] \\ 1 &= [1] \end{array}$$

Recall that  $\mathbb{P}$  consists of equivalence classes of expressions. Convenient notation:  $[P] = \{Q \mid Q \leftrightarrow P\}.$ 

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[P] + [Q] = [P + Q]	0 = [0]
$[P] \cdot [Q] = [P \cdot Q]$	1 = [1]

5. Check whether the behavioural equivalence is compatible with the syntax, i.e.,

 $\begin{array}{l} \text{if } P_1 \stackrel{\mbox{$\stackrel{$\leftrightarrow$}$}}{\to} Q_1 \text{ and } P_2 \stackrel{\mbox{$\hookrightarrow$}}{\to} Q_2, \\ \\ \text{then } P_1 + P_2 \stackrel{\mbox{$\leftarrow$}$}{\to} Q_1 + Q_2 \text{ and } P_1 \cdot P_2 \stackrel{\mbox{$\leftarrow$}$}{\to} Q_1 \cdot Q_2. \end{array}$ 

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6. Define  $+, \cdot, 0$  and 1 as suggested above.



Prove that *rooted divergence-preserving branching bisimilarity* is a congruence for a syntax with  $+, \cdot, \ldots$ 

Possible follow-up graduation projects:

- find a collection of axioms and prove that it is sound and complete;
- find SOS format ensuring that rooted divergence-preserving branching bisimilarity is comptabile with all syntactic constructions (i.e., is a congruence).



### Back to the axioms

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#### Theorem

This collection of axioms is sound and complete for  $(\mathbb{P}, +, \cdot, 0, 1)$ .

There are many variations possible with respect to:

- the behavioural equivalence used to define  $\mathbb{P}$ ;
- ► the operations defined on P.

Assignment: extend one of the results in [CFLN08] with sequential composition.