

Examination Automated Reasoning

Code 2IW15, August 24, 2010, 14.00 - 17.00

This examination consists of 5 problems all having the same weight. The final result for this course will be the average of the result for the practical assignment and the result for this examination, as long as both results are at least 5.

Problem 1.

Consider the CNF consisting of the following eight clauses

- | | |
|--------------------------|---------------------------------|
| (1) $p \vee \neg q$ | (5) $r \vee \neg s \vee \neg t$ |
| (2) $\neg q \vee r$ | (6) $s \vee \neg t$ |
| (3) $\neg p \vee t$ | (7) $p \vee \neg r \vee t$ |
| (4) $\neg r \vee \neg s$ | (8) $\neg p \vee q \vee s$ |

Determine whether this CNF is satisfiable by using the four rules Unit-Propagate, Decide, Fail and Back-track; as the first decision literal choose p^d . Make clear at every step what is the corresponding list M of literals and which clause was used.

Problem 2.

Prove by semantic tableaux that

$$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$$

is a tautology.

Problem 3.

Compute the ROBDD of

$$((q \leftrightarrow s) \leftrightarrow p) \wedge (r \rightarrow (p \vee s))$$

with respect to the order

$$p < q < r < s.$$

Problem 4.

a. Consider the two clauses

$$P(f(x, x), x) \vee Q(x)$$

and

$$\neg P(x, f(y, y)) \vee \neg Q(x),$$

where x, y are variables. Describe all single resolution steps that are possible starting from these two clauses.

b. Compute a prenex normal form of

$$(\exists x \forall y P(x, y)) \rightarrow \neg \forall y (R \vee \neg Q(f(y)))$$

and apply Skolemization to it.

Problem 5.

The term rewriting system R is defined to consist of the rules

$$\begin{aligned} f(x, g(y)) &\rightarrow g(f(y, x)), \\ f(g(x), y) &\rightarrow g(f(y, x)). \end{aligned}$$

a. Prove that R is terminating by monotone interpretations.

b. Give all non-trivial critical pairs of R and prove that R is confluent.

c. Investigate whether $f(g(a), g(b))$ is convertible to $f(g(b), g(a))$, that is, whether $f(g(a), g(b)) \leftrightarrow_R^* f(g(b), g(a))$.