

Examination Automated Reasoning

Code 2IW15, April 16, 2013, 9.00 - 12.00

This examination consists of 5 problems each having the same weight. The final result for this course will be the average of the result for the practical assignment and the result for this examination, as long as both results are at least 5. Here for the practical assignment the average of both parts is taken.

Problem 1.

- a. Prove that the CNF consisting of the following clauses is unsatisfiable, by using the four rules UnitPropagate, Decide, Fail and Backtrack; as the first decision literal choose p^d . Make clear at every step what is the corresponding list M of literals and which clause was used.

- | | |
|---------------------------------|----------------------------|
| (1) $p \vee q$ | (5) $\neg p \vee \neg r$ |
| (2) $\neg q \vee r \vee \neg s$ | (6) $\neg q \vee r \vee s$ |
| (3) $\neg p \vee \neg r \vee s$ | (7) $p \vee \neg r \vee t$ |
| (4) $p \vee \neg r \vee \neg t$ | (8) $\neg p \vee q \vee r$ |

- b. Show that it is possible to remove one clause from this CNF such that the remaining CNF of seven clauses is still unsatisfiable. Which clause may be removed?

Problem 2.

Compute the ROBDD of

$$(p_1 \wedge p_3) \vee (p_2 \wedge p_4 \wedge p_5) \vee (p_6 \wedge p_7)$$

with respect to the order

$$p_1 < p_2 < p_3 < p_4 < p_5 < p_6 < p_7.$$

Problem 3.

Apply the simplex algorithm to find the minimal value of $2y - x$ for $x, y \geq 0$ satisfying

$$2x + y \leq 10 \wedge 2x - y \geq 0 \wedge x + y \leq 4.$$

Problem 4.

- a. Describe all single resolution steps that are possible starting from the two clauses

$$P(f(x, x), x) \vee Q(x) \quad \text{and}$$

$$\neg P(x, f(y, y)) \vee \neg Q(x),$$

in which x, y are variables.

- b. Apply skolemization to a prenex normal form of

$$(\exists x \forall y P(x, y)) \rightarrow \neg \forall y (R \vee \neg Q(f(y)))$$

Problem 5.

Given the term rewriting system R consisting of the two rules

$$\begin{aligned} f(g(x, y)) &\rightarrow g(h(y), x) \\ h(f(x)) &\rightarrow f(x), \end{aligned}$$

where f and h are unary symbols and g is a binary symbol.

- Is R terminating? Prove your answer.
- Compute all non-trivial critical pairs of R .
- Is R confluent? Prove your answer.