Discrete Structuren / Algebra

Deeltentamen, code 2IT25 / 2IJ26, January 12, 2009, 14.00 - 17.00

This examination consists of 5 problems each having the same weight.

Solutions may be given in English or Dutch.

In your solutions all theorems from the course may be used.

Problem 1.

Let $U = \{\bot, \top, a, b\}$ be the lattice in which $\bot$ is the minimum of the lattice, $\top$ is the maximum of the lattice, and a, b are incomparable. Let $f : U \to U$ be defined by $f(\bot) = a$ and $f(x) = x$ for $x \neq \bot$. Does f have a least fixpoint? Prove your answer.

Problem 2.

Let $(U, \sqsubseteq)$ be a complete lattice and $a \in U$. The function $f : U \to U$ is defined by $f(x) = a \sqcup x$ for $x \in U$. Prove that f is continuous.

Problem 3.

Let $(U, \sqsubseteq)$ be a complete lattice. Let A, B be subsets of U such that $a \sqsubseteq b$ for all $a \in A$ and all $b \in B$. Prove that

$$\sup A \sqsubseteq \inf B.$$

Problem 4.

Let (V, E) be a finite acyclic directed graph. Prove that there is a function f from V to the natural numbers such that $f(v) < f(u)$ for all $(u, v) \in E$.

Problem 5.

Let $(V, *, I)$ be a group, let A be a finite subset of V and let $v \in V$. Define

$$B = \{a * v \mid a \in A\}.$$

Prove that $\#A = \#B$.