

Discrete Structuren

Examination 2IT25, June 18, 2009, 14.00 - 17.00

This examination consists of 5 problems each having the same weight.

Solutions may be given in English or Dutch.

Motivate your answers.

Problem 1.

- a. Give all solutions in the integer numbers of the equation

$$256x + 729y = 23.$$

- b. Give a closed expression for a_n , defined by

$$a_0 = 2, \quad a_1 = 9,$$

$$a_n = 6a_{n-1} - 9a_{n-2} \quad \text{for } n > 1.$$

Problem 2.

The order \sqsubseteq on the set \mathbf{N} of natural numbers (including 0) is defined by

$$x \sqsubseteq y \iff \exists c \in \mathbf{N} : y = cx.$$

- a. Determine how many numbers x of the shape p^2q for distinct prime numbers p, q satisfy $x \sqsubseteq 25!$.
- b. Give the supremum and the infimum of both the set \mathbf{N} and the set $\{2, 10, 14\}$.

Problem 3.

Let

- (U, \sqsubseteq) be a complete lattice,
- $f, g : U \rightarrow U$ be two monotonic functions,
- a be the least fixed point of f , and
- b be the least fixed point of g .

- a. Prove that $f \circ g$ has a least fixed point.
- b. Let c be the least fixed point of $f \circ g$. Prove that if $g(a) = a$, then $c \sqsubseteq a$.

Problem 4.

Let (V, E) be a finite connected undirected graph. Let W be any set and $f : V \rightarrow W$ a surjective function. Prove that the undirected graph $(V \cup W, E \cup \{(v, f(v)) \mid v \in V\})$ is connected.

Problem 5.

Let $(V, *, I)$ be a group. Let $a, b, c \in V$ satisfy $a * b = c$. Prove that $b^{-1} * a^{-1} = c^{-1}$.