

# Algebra

Examination 2IJ26, January 19, 2011, 14.00 - 17.00

This examination consists of 5 problems each having the same weight.

Solutions may be given in English or Dutch.

Motivate your answers.

## Problem 1.

Let  $n \in \mathbf{N}$  satisfy  $n > 1$ . Let the relation  $R$  on  $X = \{x \in \mathbf{N} \mid 1 \leq x \leq n\}$  be defined by

$$R = \{(n, 1)\} \cup \{(x, x + 1) \mid 1 \leq x \leq n - 1\}.$$

Prove that  $R^+ = X \times X$ .

## Problem 2.

Give an example of two equivalence relations  $R, S$  on a finite set for which  $R; S$  is not an equivalence relation.

## Problem 3.

Let  $(V, E)$  be a finite connected undirected graph. Let  $W$  be a finite set and  $f : W \rightarrow V$  a function. Prove that the undirected graph

$$(V \cup W, E \cup \{(w, f(w)) \mid w \in W\})$$

is connected.

## Problem 4.

Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow X$  be functions for which both  $f \circ g$  and  $g \circ f$  are bijective. Prove that  $f$  is bijective.

## Problem 5.

A poset  $(U, \sqsubseteq)$  is given with two subsets  $X$  and  $Y$  for which  $\sup(X)$  and  $\sup(Y)$  exist, and  $\sup(X) \in Y$ . Prove that  $\sup(Y)$  is an upper bound of  $X$ .