Algorithms for inverting transforms

There is a rich literature on the inversion of transform functions, like Fourier transforms, Laplace transforms and z-transforms, in probability theory better known as probability generating functions (PGFs). Abate and Whitt develop in their paper Numerical inversion of probability generating functions (1992, Operations Research Letters 12: 245-251) a numerical algorithm for inverting PGFs.

In this assignment your are asked to implement this algorithm, and to apply the algorithm to the following cases:

1. Consider a PGF for which you know the underlying probability distribution, and compare the numerical values of the algorithm against the true probabilities.
2. Consider the PGF

$$F(z) = \mathbb{E}(z^Q) = \frac{(s - A'(1))(z-1)}{z^s - A(z)} \prod_{k=1}^{s-1} \frac{z - z_k}{1 - z_k}, \tag{1}$$

with $A(z) = \exp(\lambda(z-1))$ and $A'(1) = \lambda/s < 1$ and $z_1, \ldots, z_{s-1}$ the roots of $z^s = A(z)$ in $|z| < 1$. Determine the distribution of Q. You can choose for instance $s = 10$ and $\lambda = 9$, but you may also want to consider different parameter values.

In investigating the performance of the algorithm, take the following aspects into account:

- Abate and Whitt present an error bound, that depends on the choice of parameters. Take this error bounds into account when applying the algorithm and reporting the numerical results.
- The algorithm is expected to perform worse for tail probabilities (rare event probabilities). Apply the technique of dominant singularity analysis to obtain estimates for the tail probabilities, and compare these estimates to the values found by the algorithm.
- Investigate how the algorithm performs in heavy-traffic. Also make a comparison with heavy-traffic approximations that can be obtained in the QED regime **(this will be discussed in class before Christmas)**.

The assignment will count for $1/3$ of the final grade of the course. The assignment can be made in groups of two. Each group should hand in a well-written yet compact report including codes of the numerical program. Email your report to j.s.h.v.leeuwaarden@tue.nl no later than 31 January 2015.