Assignment 2

Let us consider the production system in figure 1.

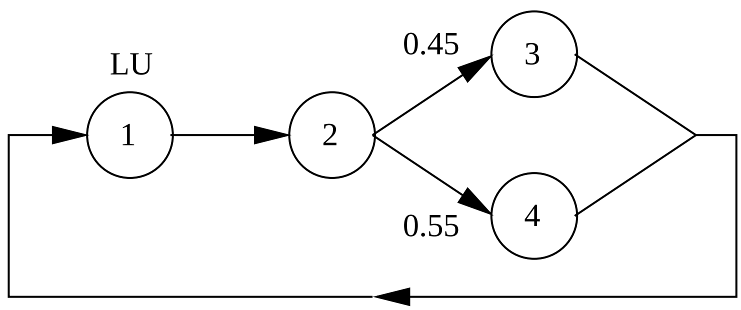

Figure 1: Closed production system with 4 stations and N circulating pallets

Station 1 is the Load/Unload (LU) station and there are N circulating pallets (carrying jobs). In the LU station, finished jobs are removed from the pallet, and a new (raw) job is attached to the pallet and send to one of the two workstations for processing. Station 2 models the transportation time from the LU station to the two workstations; it is an infinite server station. Workstations 3 and 4 can process one job at a time; both stations have ample buffer space. The processing (or transportation) times in station i are exponential with rate μ_i per hour. The processing rates are listed in table 1.

Station i	μ_i
1	8
2	10
3	4
4	5

Table 1: Processing rates

Performance characteristics of interest are:

- Throughput: Number of pallets leaving the LU station per unit of time;
- WIP of station i: Mean number of jobs (or Work In Process) in workstation i.

This production system can be described by a Markov process with states (n_1, n_2, n_3, n_4) where n_i denotes the number of jobs in station i. So $n_i \geq 0$ for all i and $n_1 + \cdots + n_4 = N$. (So what is the total number of states?) The equilibrium distribution is given by the product form

$$p(n_1, n_2, n_3, n_4) = C \left(\frac{1}{\mu_1}\right)^{n_1} \frac{1}{n_2!} \left(\frac{1}{\mu_2}\right)^{n_2} \left(\frac{0.45}{\mu_3}\right)^{n_3} \left(\frac{0.55}{\mu_4}\right)^{n_4},$$

where C is the normalization constant.

a. Determine numerically, for various values of N, the equilibrium distribution by using the iterative method with bounds, as well as its Gauss-Seidel variant, and compare the number of iterations required by both iteration schemes to produce estimates for performance characteristics (WIP and throughput), the error of which is at most 0.1%.

b. Calculate, for $N = 10$, performance characteristics as a function of time (and show the result in a graph). How fast does the system reach equilibrium, assuming that at time 0 all pallets are located at the LU station? Is the rate of convergence affected by the initial state?