Model exam Requirement Analysis, Design and Verification RADV, 2IW20, January 2004

It is not allowed to use the study material nor a computer. The axioms formulated in the book are given as appendix to this exam The answers to the questions can be formulated in either English or Dutch. This exam consists of 4 questions. Good luck!

1. (a) Assume the existence of a data type **Bool** with constructors T (true) and F (false). Specify the natural numbers Nat, for instance with constructors 0 and S (successor) with addition and multiplication.

   (b) Describe a process that iteratively reads a natural number n via a port a and nondeterministically delivers $2n$ or n^2 via a port b.

   (c) Describe a similar process as above, which via a control port c requests whether the $2n$ or n^2 must be delivered.

   (d) Draw a graph that represents the behaviour of this process, where suggestive dots can be used where the graph is infinite.

2. (a) Consider the specification of natural numbers of exercise 1 (a). Prove that 0 is not equal to $S(0)$. If this is not possible, extend the specification such that you can prove $0 \neq S(0)$.

   (b) Prove using the axioms that if $x + y = \delta$, then $x = \delta$.

   (c) Prove that the process defined by the equation

   $$X(n{:}Nat) = \sum_{m:Nat} a(m)\, X(n) \lhd eq(m,n) \rhd \delta$$

   equals the process equation

   $$X(n{:}Nat) = a(n)\, X(n).$$

   You may assume that $eq(m,n)$ faithfully reflects equality. I.e. $eq(m,n)$ iff $m = n$.

   (d) Given processes $X(n{:}Nat, m{:}Nat) = a(even(n))\, X(S(S(n)), m)$ and $Y = a(\mathsf{T})\, Y$. Prove that $X(0,0) = Y$. The function $even$ is defined by the equations $even(0) = \mathsf{T}$, $even(S(0)) = \mathsf{F}$ and $even(S(S(n)) = even(n)$. Note that S is the successor.

3. Consider the process $X = a \cdot b \cdot c \cdot X$.

   (a) Give a linear process with the same behaviour as X.

   (b) If X is put 3 times in parallel with itself, how large is the resulting state space?

   (c) Is $\tau_{\{b\}}(X)$ τ-confluent? Is $\tau_{\{b\}}$ τ-convergent? If so, does the application of τ-priorisation have any effect on $\tau_{\{b\}}(X)$? Explain all your answers; a simple yes or no does not suffice.

4. Consider the following datatypes.

sort D, *List*
func $[] :\to List$
$in : D \times List \to List$
map $append : D \times List \to List$
$top, toe : List \to D$
$tail, untoe : List \to List$
$nonempty : List \to Bool$
$length : List \to Nat$
$++ : List \times List \to List$
var $d : D,\ l, l' : List$
rew $append(d, []) = in(d, [])$
$append(d, in(d', l)) = in(d', append(d, l))$
$top(in(d, l)) = d$
$toe(in(d, [])) = d$
$toe(in(d, in(d', l))) = toe(in(d', l))$
$untoe(in(d, [])) = []$
$untoe(in(d, in(d', l))) = in(d, untoe(in(d', l)))$
$nonempty([]) = F$
$nonempty(in(d, l)) = T$
$[]{++}l = l$
$in(d, l){++}l' = in(d, (l{++}l'))$

For indices i, j the specification of a queue that reads data at port r_i and delivers it at port s_j is given by

$$\begin{aligned} Q_{i,j}(q : List) &= \textstyle\sum_{d:D} r_i(d)\, Q_{i,j}(in(d, q)) + \\ &\quad s_j(toe(q))\, Q_{i,j}(untoe(q)) \lhd nonempty(q) \rhd \delta \end{aligned}$$

Prove, for instance with the cones and foci theorem, that

$$\tau_{\{c_2\}}\partial_{\{s_2,r_2\}}(Q_{1,2}(q_1) \parallel Q_{2,3}(q_2))$$

is branching bisimilar to $Q_{1,3}(q_1{++}q_2)$ where $\gamma(r_2, s_2) = c_2$.

END

Score: $(10 + n)/10$ where n is the cumulative judgement given by the following table:

question	(a)	(b)	(c)	(d)
1	5	5	6	5
2	7	5	7	6
3	6	5	9	
4	25			

A1	$x + y = y + x$
A2	$x + (y + z) = (x + y) + z$
A3	$x + x = x$
A4	$(x + y)\cdot z = x\cdot z + y\cdot z$
A5	$(x\cdot y)\cdot z = x\cdot(y\cdot z)$

Table 1: Basic axioms for μCRL

CM1	$x \parallel y = (x \mathbin{\lfloor\!\lfloor} y + y \mathbin{\lfloor\!\lfloor} x) + x \mid y$
CM2	$a(\vec{d}) \mathbin{\lfloor\!\lfloor} x = a(\vec{d})\cdot x$
CM3	$a(\vec{d})\cdot x \mathbin{\lfloor\!\lfloor} y = a(\vec{d})\cdot(x \parallel y)$
CM4	$(x + y) \mathbin{\lfloor\!\lfloor} z = x \mathbin{\lfloor\!\lfloor} z + y \mathbin{\lfloor\!\lfloor} z$
CF	$a(\vec{d}) \mid b(\vec{d}) = c(\vec{d})$ if $a \vert b = c$
CF′	$a(\vec{d}) \mid b(\vec{e}) = \delta$ if $\vec{d} \neq \vec{e}$ or a and b do not communicate
CM5	$a(\vec{d})\cdot x \mid b(\vec{e}) = (a(\vec{d}) \mid b(\vec{e}))\cdot x$
CM6	$a(\vec{d}) \mid b(\vec{e})\cdot x = (a(\vec{d}) \mid b(\vec{e}))\cdot x$
CM7	$a(\vec{d})\cdot x \mid b(\vec{e})\cdot y = (a(\vec{d}) \mid b(\vec{e}))\cdot(x \parallel y)$
CM8	$(x + y) \mid z = x \mid z + y \mid z$
CM9	$x \mid (y + z) = x \mid y + x \mid z$

Table 2: Axioms for parallellism in μCRL

A6	$x + \delta = x$	
A7	$\delta \cdot x = \delta$	
DD	$\partial_H(\delta) = \delta$	
D1	$\partial_H(a(\vec{d})) = a(\vec{d})$	if $a \notin H$
D2	$\partial_H(a(\vec{d})) = \delta$	if $a \in H$
D3	$\partial_H(x + y) = \partial_H(x) + \partial_H(y)$	
D4	$\partial_H(x \cdot y) = \partial_H(x) \cdot \partial_H(y)$	
CD1	$\delta \,\|\!\_\, x = \delta$	
CD2	$\delta \mid x = \delta$	
CD3	$x \mid \delta = \delta$	

Table 3: Axioms for deadlock

C1	$x \lhd \mathtt{T} \rhd y = x$
C2	$x \lhd \mathtt{F} \rhd y = y$

Table 4: Axioms for conditionals

SUM1	$\sum_{d:D} x = x$
SUM3	$\sum_{d:D} X(d) = \sum_{d:D} X(d) + X(d_0) \quad (d_0 \in D)$
SUM4	$\sum_{d:D}(X(d) + Y(d)) = \sum_{d:D} X(d) + \sum_{d:D} Y(d)$
SUM5	$(\sum_{d:D} X(d)) \cdot x = \sum_{d:D}(X(d) \cdot x)$
SUM6	$(\sum_{d:D} X(d)) \,\|\!\_\, x = \sum_{d:D}(X(d) \,\|\!\_\, x)$
SUM7	$(\sum_{d:D} X(d)) \mid x = \sum_{d:D}(X(d) \mid x)$
SUM7′	$x \mid (\sum_{d:D} X(d)) = \sum_{d:D}(x \mid X(d))$
SUM8	$\partial_H(\sum_{d:D} X(d)) = \sum_{d:D} \partial_H(X(d))$
SUM11	$(\forall d_0 \in D \; X(d_0) = Y(d_0)) \;\Rightarrow\; \sum_{d:D} X(d) = \sum_{d:D} Y(d)$

Table 5: Axioms for summation

R1	$\rho_f(\delta) = \delta$
R3	$\rho_f(a(\vec{d})) = f(a)(\vec{d})$
R4	$\rho_f(x + y) = \rho_f(x) + \rho_f(y)$
R5	$\rho_f(x \cdot y) = \rho_f(x) \cdot \rho_f(y)$
SUM9	$\rho_f(\sum_{d:D} X(d)) = \sum_{d:D} \rho_f(X(d))$

Table 6: Axioms for renaming

B1	$x \cdot \tau = x$	
B2	$x \cdot (\tau \cdot (y + z) + y) = x \cdot (y + z)$	
TID	$\tau_I(\delta) = \delta$	
TI1	$\tau_I(a(\vec{d})) = a(\vec{d})$	if $a \notin I$
TI2	$\tau_I(a(\vec{d})) = \tau$	if $a \in I$
TI3	$\tau_I(x + y) = \tau_I(x) + \tau_I(y)$	
TI4	$\tau_I(x \cdot y) = \tau_I(x) \cdot \tau_I(y)$	
SUM10	$\tau_I(\sum_{d:D} X(d)) = \sum_{d:D} \tau_I(X(d))$	
R2	$\rho_f(\tau) = \tau$	

Table 7: Axioms for hidden actions and hiding