PHILIPS

Initial values for on-line response time calculations

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Initial values for on-line response time calculations for FPPS

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Outline

- Context and motivation
- Schedulability tests
- Example
- Initial values and number of iterations
- Quantitative analysis
- Conclusions

Context and motivation

- A trend of real-time systems
  - Dynamics increases: many different modes that are not predictable or not known statically
  - High utilization required
- Requires on-line and exact schedulability tests
- Special class:
  - Media processing in software in consumer products; see keynote ECRTS 2002
- Goal: reduce cost of exact schedulability test

Schedulability tests (1)

- Discrete model
  - Set Γ of n independent, periodic tasks $\tau_1, \ldots, \tau_n$
  - with periods T_i, deadlines D_i, worst-case computation times C_i, and phasing φ_i
  - where $D_i \leq T_i$, and $\{T_i, D_i, C_i, \varphi_i\} \in Z^+$.
- Necessary (but insufficient) test for Γ:
  - *Liu & Layland*: $U_n = \sum_{j \leq n} C_j/T_j \leq 1$
- Sufficient (but unnecessary) test for Γ for RM:
  - *Liu & Layland* ($LL(n)$): $U_n \leq n(2^{1/n} - 1)$
  - *Bini & Buttazzo*[2] ($HB(n)$): $\prod_{j \leq n} (C_j/T_j + 1) \leq 2$

Schedulability tests (2)

- Exact schedulability test for Γ:
  - $R_i(C_i) \leq D_i$ for $1 \leq i \leq n$.
- Exact analysis
  - Based on notion *critical instant*
  - $R_i(C_i)$ is the smallest x satisfying

$$x = C_i + \sum_{j<i} \lceil x/T_j \rceil C_j$$

  - iterative procedure:
    - $r_i^0 = R_{i-1} + C_i$
    - $r_i^{l+1} = C_i + \sum_{j<i} \lceil r_i^l/T_j \rceil C_j$
  - stop criterion: $r_i^l = r_i^{l+1}$ or $r_i^{l+1} > D_i$

Example

	$T_i = D_i$	C_i	R_i
τ_1	10	3	
τ_2	19	11	

LL(2) fails: $U_2 = 0.88 > 0.83$

HB(2) fails: $\prod_{j\leq 2} (C_j/T_j + 1) = 2.054 > 2$

Both also fail for an arbitrary 3rd task.

Example

	$T_i = D_i$	C_i	R_i
τ_1	10	3	3
τ_2	19	11	17

Exact analysis: succeeds.

Example

	$T_i = D_i$	C_i	R_i
τ_1	10	3	3
τ_2	19	11	17

Hyperperiod $H_{i-1} = \mathrm{lcm}(T_1,\ldots,T_{i-1})$; $H_2 = 190$

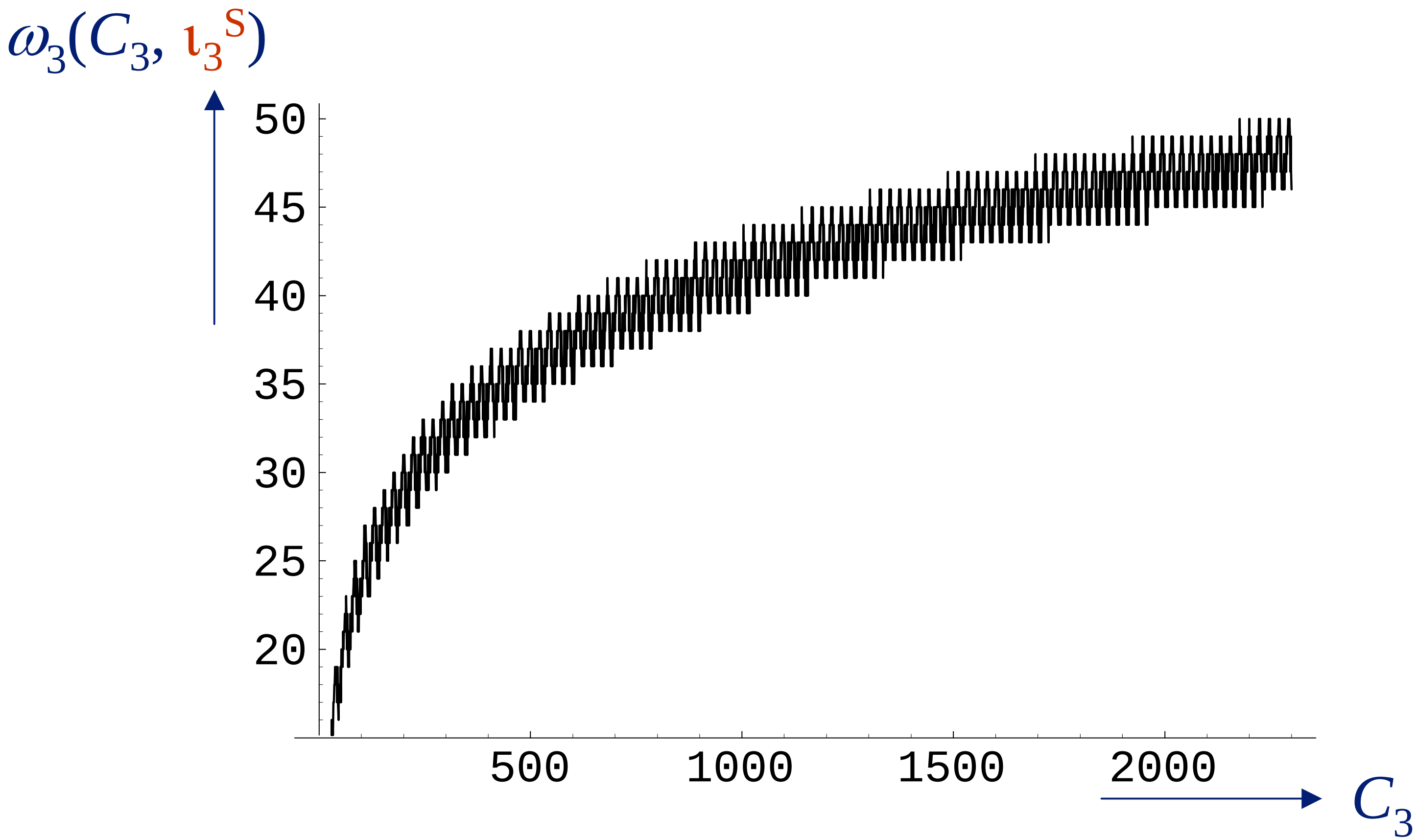
Slack S_{i-1} in H_{i-1}: $S_{i-1} = H_{i-1}(1 - U_{i-1})$; $S_2 = 23$

S_2 available for executions of τ_3.

Initial values and number of iterations

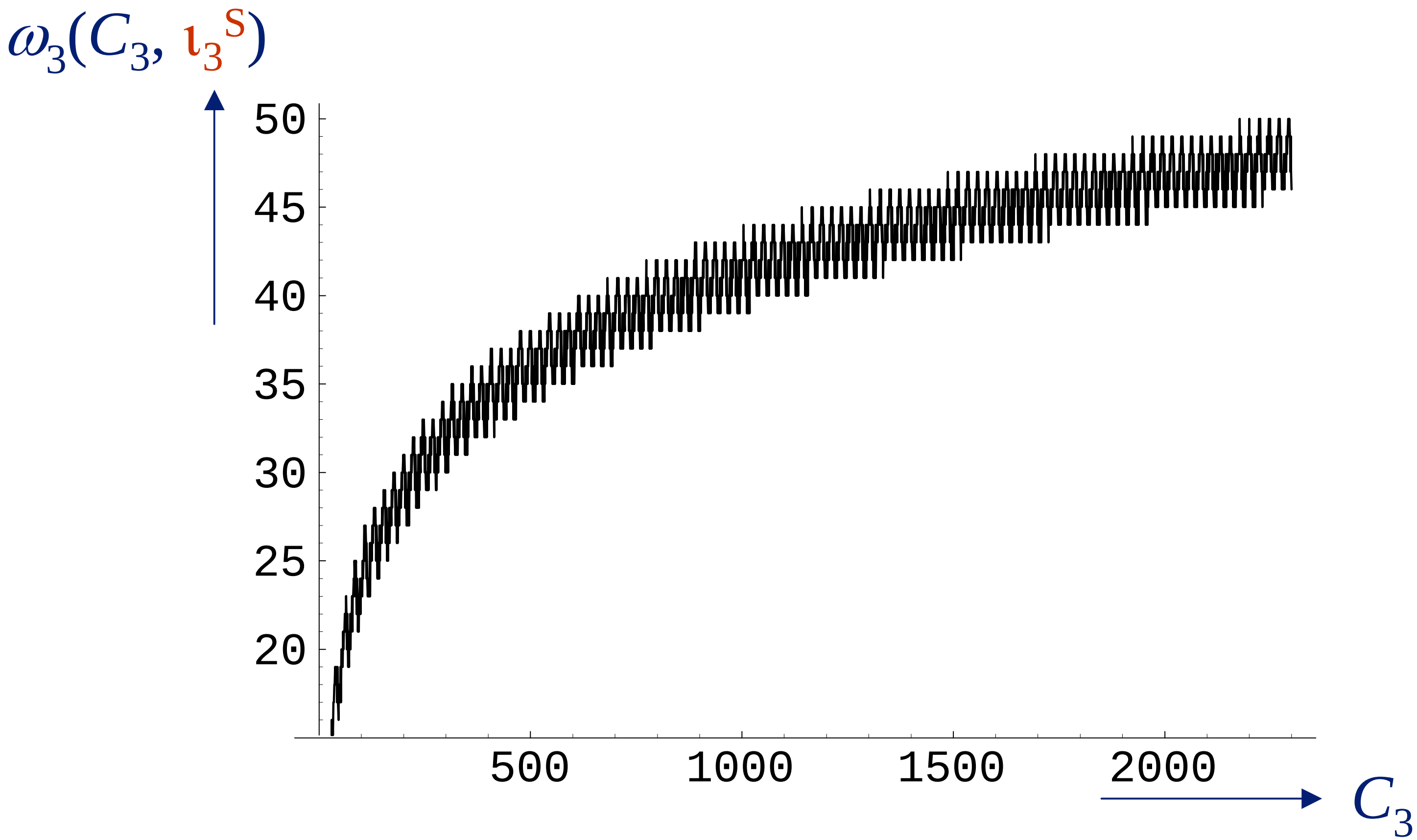
- Initial value:
  lower bound for R_i
- Standard initial value ι_i^S
  $\iota_i^S = R_{i-1} + C_i$;
- New initial value ι_i^N
  $\iota_i^N = C_i/(1 - U_{i-1})$.

Standard initial value: $\iota_i^S = R_{i-1} + C_i$



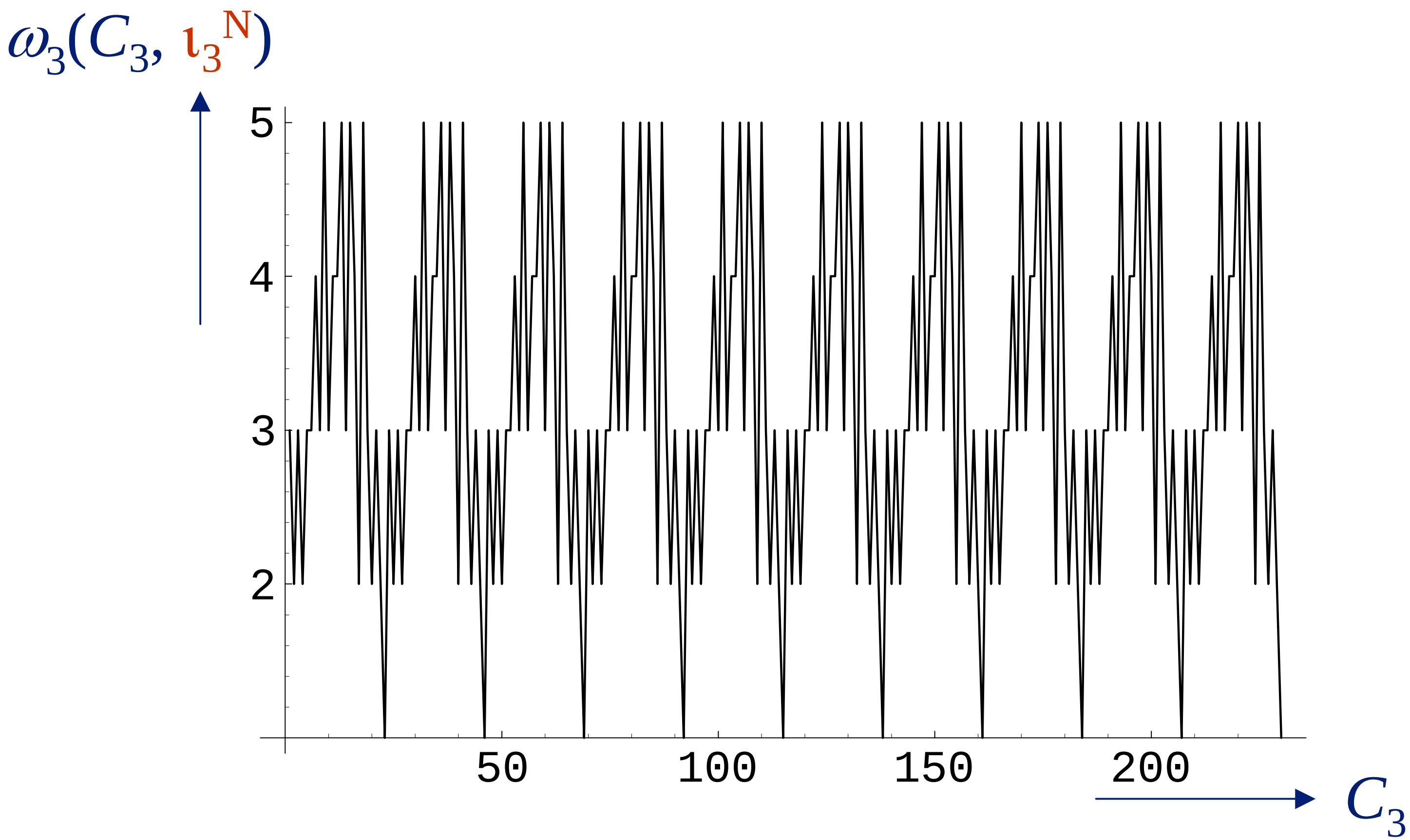
Number of iterations $\omega_3(C_3, \iota_3^S)$ to determine $R_3(C_3)$ for $C_3 = 1, \ldots, 2300$ (i.e. 100 S_2)

Standard initial value: $\iota_i^S = R_{i-1} + C_i$



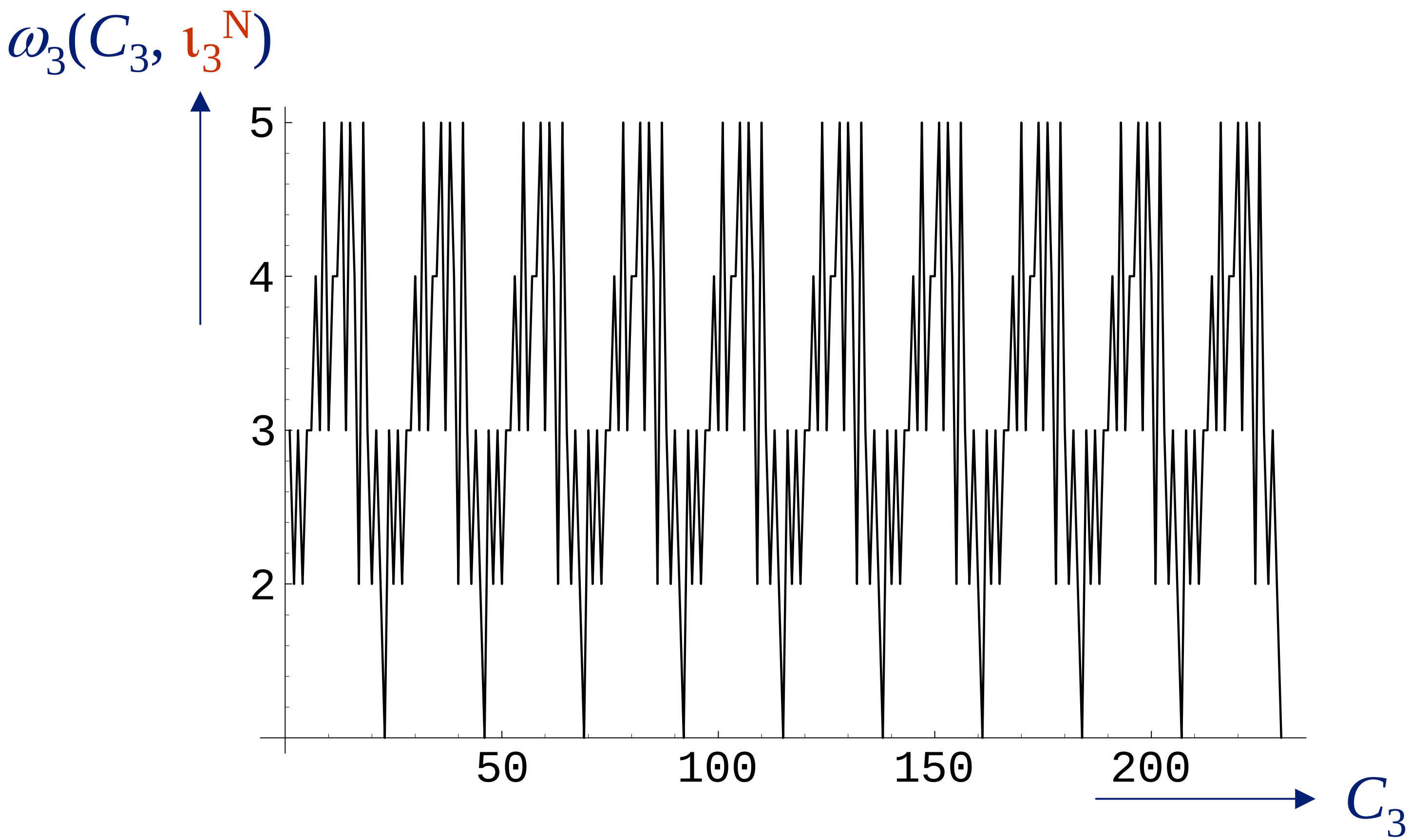
Logarithmic growth of $\omega_3(C_3, \iota_3^S)$!

New initial value: $\iota_i^N = C_i/(1 - U_{i-1})$



Number of iterations $\omega_3(C_3, \iota_3^N)$ to determine $R_3(C_3)$ for $C_3 = 1, \ldots, 230$ (i.e. 10 S_2)

New initial value: $\iota_i^{\mathrm{N}} = C_i/(1 - U_{i-1})$



Bounded and periodic $\omega_3(C_3, \iota_3^{\mathrm{N}})$!

Summary initial values

- Standard initial value $\iota_i^S = R_{i-1} + C_i$:
  - Logarithmic growth of $\omega_i(C_i, \iota_i^S)$.
- New initial value $\iota_i^N = C_i/(1 - U_{i-1})$:
  - $\omega_i(C_i, \iota_i^N)$ is bounded by a constant, and
  - periodic with period S_{i-1}.
- Proposed initial value $\iota_i^W = \max(\iota_i^S, \iota_i^N)$:
  - Best of both worlds…

Quantitative analysis (1)

- Goal: compare cost of using ι^S and ι^W.
- Cost:
  - Based on recursive equation:
    $$x = C_i + \sum_{j<i} \lceil x/T_j \rceil C_j$$
  - Single task τ_i:
    $$\xi_i^\tau(C_i, \iota_i) \approx \omega_i(C_i, \iota_i)\,(i-1)\xi_{\text{sum}}$$
  - Cumulative for $\tau_1, \ldots, \tau_i$:
    $$\xi_i(\iota) = \sum_{j \le i} \xi_i^\tau(C_j, \iota_j)$$
  - Task set Γ:
    $$\xi^\Gamma(\iota) = \xi_m(\iota)$$

  with τ_m 1st task exceeding D_m, or $m = n$.

Quantitative analysis (2)

- Experiment definition:
  - $U_n \in \{0.2, \ldots, 1.0\}$, uniform distribution for U_i^τ
  - *spread* $\sigma_n = \log T_n/T_1$, $\sigma_n \in \{0.1, \ldots, 4.0\}$, uniform distribution for T_i for $1 < i < n$, and T_i sorted (RMA);
  - $n \in \{3, 4, 6, 10, 20\}$;
  - 10,000 samples for every (U_n, σ_n, n).

Quantitative analysis (3)

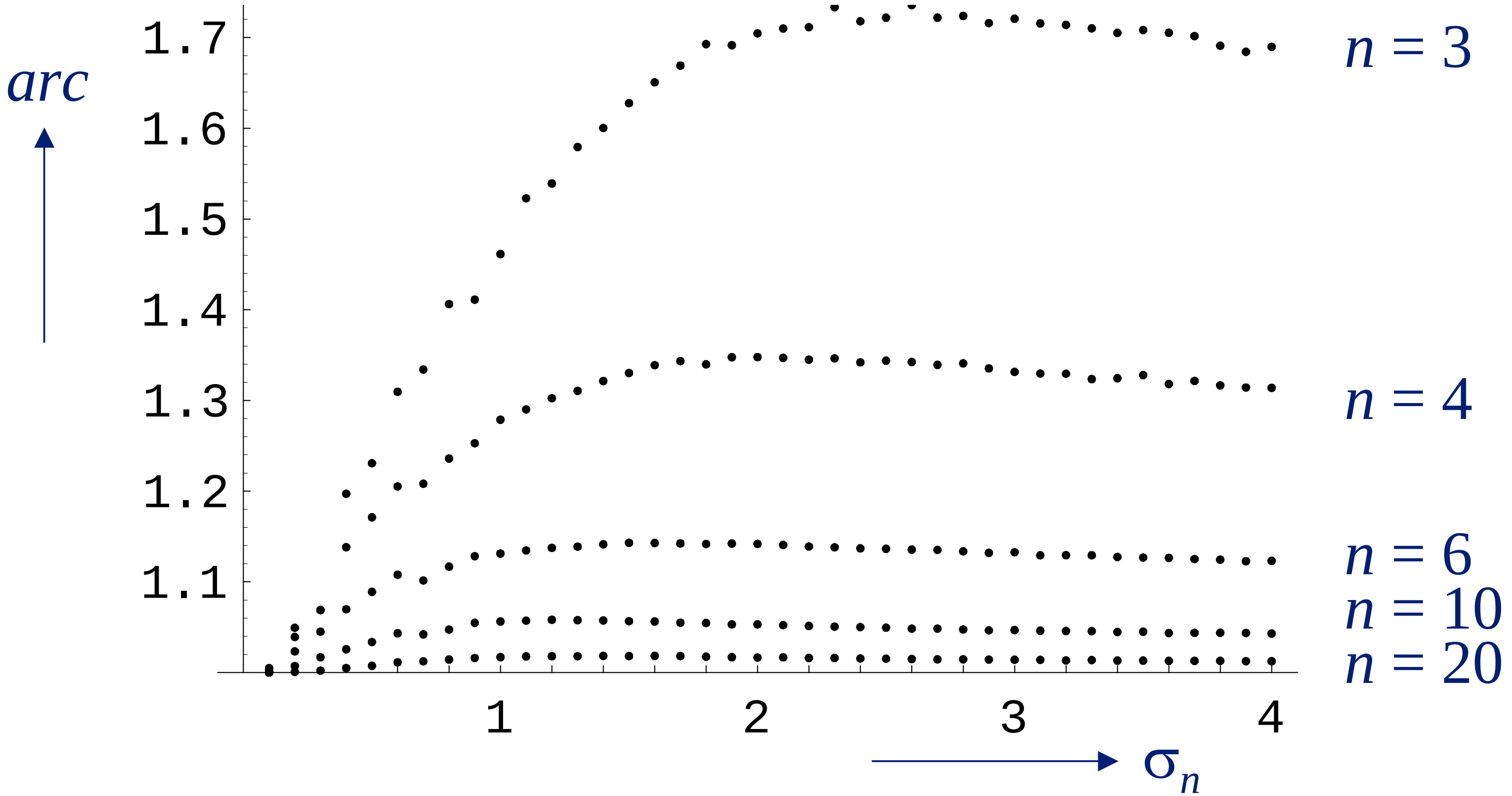
- Experiment definition (cntd):
  - *average relative complexity*:

    $$arc = \sum_e^{10,000} \xi^{\Gamma}(\iota^{S}, e) / \sum_e^{10,000} \xi^{\Gamma}(\iota^{W}, e)$$

  - *maximum relative complexity*:

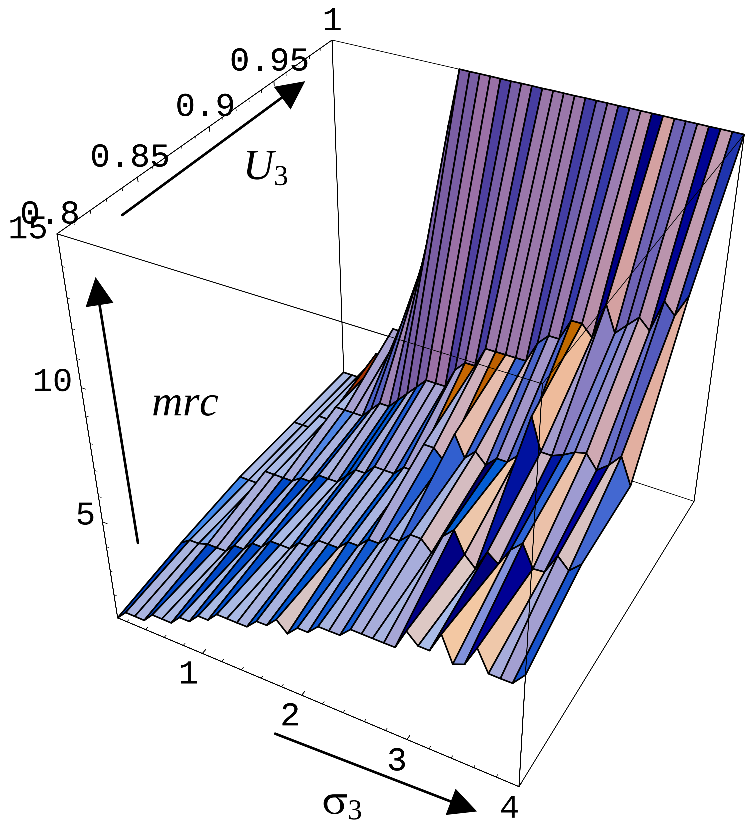
    $$mrc = \max_e^{10,000} \xi^{\Gamma}(\iota^{S}, e) / \xi^{\Gamma}(\iota^{W}, e)$$

Quantitative analysis (4)



Average relative complexity *arc* for $U_n = 0.9$

Quantitative analysis (5)

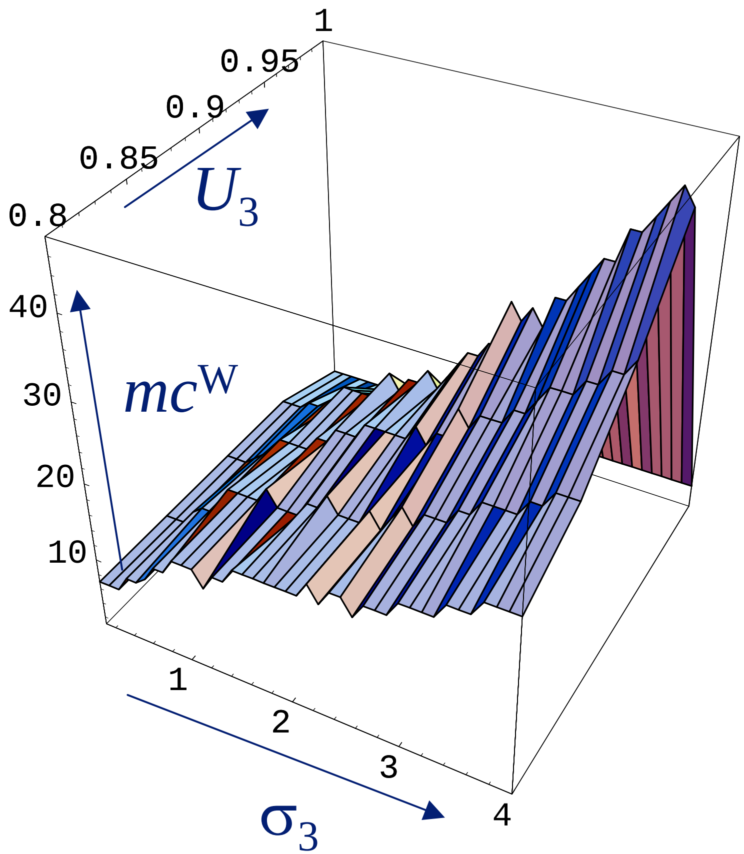


Maximum ($U_3 = 1.0$, $\sigma_3 = 3.6$):

- mrc = 1631
- $\xi^{\Gamma}(\iota^{S}, e) = 4893\xi_{sum}$
- $\xi^{\Gamma}(\iota^{W}, e) = 3\xi_{sum}$

Maximum relative complexity mrc for $n = 3$

Quantitative analysis (6)



Maximum ($U_3 = 0.95$, $\sigma_3 = 3.9$):

- $mc^W = 48\ \xi_{sum}$

Maximum complexity mc^W for ι^W for $n = 3$:

$$mc^W = \max_e{}^{10{,}000}\ \xi^\Gamma(\iota^W, e)$$

Conclusions

- Number of iterations:
  - $\omega_i(C_i, \iota_i^S)$ grows logarithmically;
  - $\omega_i(C_i, \iota_i^N)$ bounded by a constant and periodic.
- Experiment reveals:
  - On average (10,000 samples), minor advantage of ι^W compared to ι^S (e.g. only 45% for n = 3 and U_3 = 0.9)
  - Major advantage for special cases:
    - reduction of *worst-case* cost of schedulability test.
- ι^N also applicable in other contexts
  - Best-case response times, jitter, …