

Exam Process Algebra (2IF45)

20 June 2008, 14.00 –17.00

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This is an open book exam: the syllabus “Process Algebra (equational theories of communicating processes)” of J.C.M. Baeten, T. Basten and M.A. Reniers (2IF45, December 2007) may be used. Extra inscriptions and annotations in the syllabus other than corrections of errors are not permitted. Other documents are also not permitted.

This exam consists of five exercises, each worth maximally 20 points. The final grade is determined by dividing the total number of points obtained by ten and rounding off. All proofs should be given without omitting essential steps.

Opgave 1 Let $a, b \in A$ be different atomic actions. It is given that $\gamma(a, b) = b$ and $\gamma(a, a) = b$.

1. Prove that $\gamma(b, b) = b$.
2. Give a normal form (i.e. a closed BSP-term) for the following closed TCP-terms (with projection and skipping (Section 6.7))
 - (a) $a.1 \parallel b.1 \parallel b.1$;
 - (b) $\partial_{\{a\}}(a.b.1 \parallel a.b.1)$;
 - (c) $\varepsilon_{\{a\}}(a.b.1 \parallel a.b.1)$;
 - (d) $\pi_1(a.1 \parallel b.1)$.

- Opgave 2**
1. Draw the transition systems generated by the operational rules for the processes $a.(\tau.b.1 + \tau.(b.1 + c.1))$ and $a.(b.\tau.1 + \tau.(b.1 + c.1))$.
 2. Are the transition systems drawn rooted branching bisimilar or not? If so, construct an rb-bisimulation; if not, explain why not. Also, reduce the obtained transition systems by removing inert τ -steps.

Opgave 3 1. Derive from the axioms of TCP^{drt*} the equation $1 \mid 1 = 1$.

2. Give two closed TCP^{drt} -terms p, q such that the process $\sigma^*p \mid \sigma^*q$ is not the same as process $\sigma^*(p \mid q)$.

Opgave 4 In this exercise, we work in the theory $\text{TSP}(\{a, b\})$. Consider the following recursive specification E over variables X_n (n a natural number):

$$\begin{aligned} X_0 &= 1 \\ X_{n+1} &= a.X_{n+2} + b.X_n \quad (n \in \mathbb{N}). \end{aligned}$$

Consider also the recursive specification F over variable Y :

$$Y = b.1 + a.Y \cdot Y.$$

Prove that $Y = X_1$, using RSP and RDP.

Opgave 5 The lamp on my desk L is only lit if the plug P is in the socket and the switch S is in the correct position. When I enter, the plug is in the socket and the switch is off: this is the initial state. The set of messages D consists of the following:

- *on*: the lamp is switched on;
- *off*: the lamp is switched off;
- *cf, cn, click*: something happens, but the lamp is not switched on or off.

Besides the standard communications $\gamma(s(\textit{on}), r(\textit{on})) = c(\textit{on}), \gamma(s(\textit{off}), r(\textit{off})) = c(\textit{off})$, there are the extra communications $\gamma(s(\textit{cf}), r(\textit{off})) = c(\textit{click})$ and $\gamma(s(\textit{cn}), r(\textit{on})) = c(\textit{click})$ defined.

There are the following specifications:

$$\begin{aligned} P &= r(\textit{off}).r(\textit{on}).P \\ S &= r(\textit{on}).r(\textit{off}).S \\ L &= s(\textit{on}).s(\textit{off}).L + s(\textit{cf}).s(\textit{cn}).L \end{aligned}$$

The encapsulation set H contains all $s(d), r(d)$ for $d \in D$, the abstraction set I contains *only* the $c(\textit{click})$ communications.

1. Find a linear recursive specification for the process $\partial_H(L \parallel P \parallel S)$, and draw the transition system.
2. In the transition system obtained, rename all $c(\textit{click})$ steps into τ . Find a transition system without divergence, and with no more than three states and four transitions, that is rooted branching bisimilar to this system. (No divergence means no infinite path of τ 's.)
3. Prove $\tau.\tau_I(\partial_H(L \parallel P \parallel S)) = \tau.(c(\textit{on}).c(\textit{off}).1)^*0$. You may use RDP, RSP and a fair abstraction rule.