Due Thursday December 4, 23:59, by e-mail (pdf)

Requirements: Assignments have to be typeset in English. They have to be handed in by each student separately, except for exercises marked with (G), which should be solved and handed in by each (project) group. Please include your name on top of the first sheet.

How to describe algorithms: Whenever you are asked to describe an algorithm, you should present three things: the algorithm, a proof of its correctness, and a derivation of its running time.

A careful geometric proof can be quite cumbersome at times, so you are highly encouraged to use intuition and give illustrations whenever appropriate. However, beware of “proof by picture”: a poorly drawn figure might make you ignore special cases that are not apparent in your drawing. Try to keep your answers concise and to the point. Never give detailed code. Only give sufficient detail to convince us that your method and your running time analysis are complete and correct.

Unless stated otherwise, you can assume that objects are in general position, that is, no three points are collinear and no four points are cocircular. You may also assume that any geometric primitive that involves only a constant number of objects which are each of constant complexity can be computed in \( O(1) \) time.

Note concerning Voronoi diagram computation: We didn’t explicitly cover in the lecture how to compute the Voronoi diagram from the Delaunay triangulation, but you may use in your solutions that this can be done in linear time.

Exercise 1: [5 points]

(a) Show that a set of \( n \) points in the plane has at most \( 2^{\binom{n}{2}} \) different triangulations.

(b) Show that for arbitrarily large \( n \) there is a set of \( n \) points that has at least \( 2^{n-2\sqrt{n}} \) different triangulations.

(c) Show that for arbitrarily large \( n \) there is a set of \( n \) points that has a triangulation whose total edge length is smaller than that of the Delaunay triangulation.

Exercise 2: [5 points] Given \( n \) 1-Euro coins and one 2-Euro coin lying flat on a table. Describe an algorithm that decides whether the 2-Euro coin can be moved (without lifting it) to the edge of the table without moving the 1-Euro coins.

Exercise 3 (G): [4 points] The goal of this exercise is to show that several interesting graphs are contained in the Delaunay triangulation. The Gabriel graph of a set \( P \) of points in the plane is defined as follows: two points \( p \) and \( q \) are connected by an edge of the Gabriel graph if and only if the circle with diameter \( pq \) does not contain any other point of \( P \) in its interior. A Euclidean minimum spanning tree (EMST) of a point set \( P \) in the plane is a tree of minimum total edge length connecting all the points. A nearest neighbor graph of a point set \( P \) in the plane is a directed graph containing for each point \( p \in P \) a directed edge to a nearest neighbor in \( P \setminus \{p\} \), i.e., as we define the graph, it contains exactly one outgoing edge per point.
(a) Briefly argue that the set of edges of a EMST of $P$ contains the (undirected) edges of a nearest neighbor graph.

(b) Prove that the set of edges of the Gabriel graph of $P$ contains an EMST of $P$.

(c) Prove that the Delaunay graph of $P$ contains the Gabriel graph of $P$.

(d) Prove that $p$ and $q$ are adjacent in the Gabriel graph of $P$ if and only if the Delaunay edge between $p$ and $q$ intersects its dual Voronoi edge.