2IL76 Algorithms for Geographic Data

Spring 2015

Lecture 4: Schematization
Schematic maps
Ancient schematic maps
Ancient schematic maps
Ancient schematic maps
Not just for transport ...

Water Pollution: This map shows the anticipated geographic water stress levels by 2020.
More schematic maps
Schematic?
Schematization

What is schematization?

- a stylized, abstract representation
  - usually simpler than input (relation to simplification)
  - iconic: few directions of lines, specific curves, …
  - might preserve topology
  - some visual resemblance to input

Most commonly schematized

- subdivisions
- networks

vertex-restricted or non-vertex-restricted
area preserving, topology preserving, using curves or straight lines …
Network Schematization
Network schematization
Common criteria

- many stations aligned horizontally, vertically or diagonally
- sufficient spacing between different lines
- connections have at most two bends
- stations are not displaced too much

*maximum displacement can be different for different stations*
Algorithmic solutions

- many stations aligned horizontally, vertically or diagonally
- sufficient spacing between different lines
- connections have at most two bends
- stations are not displaced too much
  *maximum displacement can be different for different stations*

Many iterative approaches
  *solution quality and convergence cannot be guaranteed*
Algorithmic solutions

- many stations aligned horizontally, vertically or diagonally
- sufficient spacing between different lines
- connections have at most two bends
  - stations are not displaced too much
  - maximum displacement can be different for different stations

[Cabello, de Berg, van Kreveld, 2005]

combinatorial approach:
- replace every connection by one of a schematic type
- define a top-to-bottom placement order on connections
- place each connection in its topmost position that gives no overlap
Algorithmic solutions

[Cabello, de Berg, van Kreveld, 2005]
combinatorial approach:

- replace every connection by one of a schematic type
- define a top-to-bottom placement order on connections
- place each connection in its topmost position without overlap
Formalization

Definitions

A (polygonal) map $M$ is a set of simple polygonal paths $\{c_1, \ldots, c_m\}$ such that two paths do not intersect except at shared endpoints.

A monotone map is a map where all paths are $x$-monotone.
Potential issues

- keep cyclic order of paths around endpoints
- each path of the schematic map is a deformation of the original path, without passing over endpoints
- or, each path in the original map is a deformation of a path in the schematic map, without passing over the endpoints

more formally …
Formalization

Definitions

Two maps $M$ and $M'$ are equivalent if and only if

- they have the same endpoints
- each path of $M$ can be continuously deformed to a path of $M'$ without passing over the endpoints

schematic path: axis-aligned, $x$-monotone, at most 3-links …

Formal problem statement

Given a polygonal map $M$, compute an equivalent schematic map $M'$ whose paths are schematic.

optional: minimum vertical distance, shared (pieces of) paths …
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Combinatorial approach:

- replace every connection by one of a schematic type
- define a top-to-bottom placement order on connections
- place each connection in its topmost position without overlap
Below and above a curve

Definition

Point $p$ is below/above path $c$ if any continuous deformation of $c$, that does not pass over $p$, intersects the vertical upper/lower ray from $p$.

- to decide whether a point is above or below a path, we do not consider other points
Below and above a curve

Definition

Point $p$ is below/above path $c$ if any continuous deformation of $c$, that does not pass over $p$, intersects the vertical upper/lower ray from $p$.

$p$ is both above and below $c$

$p$ and $c$ have no relation

$p$ is below $c$
Order among paths

Definition

Path $c$ is **above** path $c'$ if any endpoint of $c$ is above $c'$ or any endpoint of $c'$ is below $c$.

Lemma

The above-below relation among paths is invariant between equivalent maps.
Order among paths

Lemma

The above-below relation among paths is invariant between equivalent maps.

⇒ the above-below relation is preserved in the schematic map
⇒ if the above-below relation is acyclic, extend to order and use to place schematic connections topmost

Remaining questions

Is there always an order?
If there is an order, can we compute it efficiently?
Does an order imply a schematic map exists?
   At least for certain types of connections?
Order among paths

Is there always an order? No

Lemma

For a monotone map $M$, the above-below relation among paths is acyclic. Furthermore, if $M$ has complexity $n$, a total order extending this relation can be computed in $O(n \log n)$ time.

Definition

Path $c$ is above path $c'$ if any endpoint of $c$ is above $c'$ or any endpoint of $c'$ is below $c$.

For x-monotone paths equivalent to

Path $a$ is above path $b$ (denoted $a \triangleright b$) if and only if there are points $(x, y_a) \in a$ and $(x, y_b) \in b$ with $y_a > y_b$. 
Computing above-below relationships

If there is an order, can we can compute it efficiently?

**Theorem**

For a map $M$ of total complexity $n$, we can decide in $O(n \log n)$ time whether an equivalent, monotone map exists.

**Proof**

uses canonical sequences, rectified maps, universal cover …

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**Schematization of networks**

Sergio Cabello, Mark de Berg, and Marc van Kreveld

*Computational Geometry* 30:223–238, 2005

**Testing Homotopy for Paths in the Plane**

Sergio Cabello, Yuanxin Liu, Andrea Mantler, and Jack Snoeyink

*Discrete & Computational Geometry* 31(1):61-81, 2004
Schematic maps

Does an order imply a schematic map exists?  No
At least for certain types of connections?  No

But if we specify the types of schematic connections, we can test …

Intuition
we can only use schematic connections together that have a clear topmost placement …

x-monotone ordered schematic map model
Schematic map models

x-monotone ordered schematic map models
Algorithm

Input: a map $M$ and an $x$-monotone ordered schematic map model
Output: an equivalent schematic map $M'$ or “does not exist”

1. compute above-below relations among paths of $M$
2. if acyclic, complete to order, otherwise return “does not exist”
3. place paths (topmost each) in order
   return “does not exist” if placement is not possible

Running time: $O(n \log n)$ where $n$ is the complexity of $M$

 maintain lower envelop of already-placed connections …
Results

\{ \text{-, -}, |, |, \} \rightarrow \text{Diagram with straight lines and nodes}
Results
Results

\{\ldots, \cdot, \cdot, \cdot, \cdot\}

It is not possible
Territorial Outlines
Area-preserving schematization
Area-preserving schematization
Requirements

- Few orientations
- Few lines
- Preserve “shape”
  - Area
  - Topology

- Single operation
- Complete for polygons

- Minimize distance function?
Distance measures

Symmetric difference

Hausdorff distance

Turning angle distance

CDTW distance

Fréchet distance
The idea

Given a simple subdivision

1. Convert to *rectilinear*
   - area-preserving

2. **Contract configuration**
   - choose greedily
   - until satisfied
Rectilinearization

- Convert simple subdivision into simple rectilinear subdivision
  - Area-preserving
  - Preserve adjacencies
  - Minimize angular deviation
Assign direction to each vertex of each edge

- Minimize angular deviation
- Sharp endpoints
Rectilinearization

- For each edge
  - Create staircase
  - Use evasive behaviour for sharp endpoints
Rectilinearization

- Increase in complexity
  - Depends on distance between non-adjacent edges
  - Depends on angle between adjacent edges
Rectilinearization
S-configuration

- S-configuration
  - 3 consecutive edges
  - 2 different turns
S-contraction

- S-contraction
  - Replace by 1 edge
  - Weighted average

- Feasible
  - Contraction area is empty
Deadlock
C-contractions

- **C-configuration**
  - 3 consecutive edges
  - 2 similar turns
  - Inner & outer C-configuration

- **C-contraction**
  - Inner & outer C-configuration
  - Remove smallest
  - Compensate for area change
Completeness for polygons

Theorem

A rectilinear polygon with at least 6 edges has

- a feasible inner C-configuration
- a feasible S-configuration
  or a feasible outer C-configuration
A contraction is always possible

Generalizes to C-oriented polygons …
Results for polygons
Results for polygons

- Building generalization
C-oriented polygons: edge moves
C-oriented polygons: edge moves
Edge-moves
Combining edge-moves
Completeness

**Theorem.** Every simple non-convex polygon has a non-conflicting pair of complementary feasible contractions.

**Corollary.** Every simple $C$-oriented polygon can be schematized area-preservingly with at most $2|C|$ edges using only edge-moves.
Schematization algorithm

- Convert input to C-oriented subdivision

- Until satisfied
  - Execute pair of edge-moves with smallest contraction area
Experimental results
Experimental results
Experimental results
Experimental results
Experimental results
Experimental results
Still open ...

- Orientation selection
- Edge-move selection
- Adding orientations
- When to stop?
Curved Schematization
Curved networks
Curved networks
Curved networks
Curved networks
Curved networks
Curved networks
Curved outlines

Area preserving circular arcs
Curved outlines

Area preserving circular arcs
Curved outlines

Area preserving circular arcs

Bézier curves
Curved outlines
Extreme Schematization
Chorematic diagrams

- Extreme generalization combined with schematized geometries

**Generalization**

“process by which information is selectively removed from a map in order to simplify pattern without distortion of overall content”

[Heywood et al. ’98]
Chorematic diagrams
Chorematic diagrams
Extreme Schematization
Extreme Schematization
Inspiration

Stenography

The Dog
Stenomaps
Stenomaps
Hurricane path prediction
Hurricane path prediction
Rivers as locational aid

[Guylaine Brun-Trigaud]

[Grosser Atlas zur Weltgeschichte, 1991]
Stenomaps
Solar potential in Europe
Solar potential in Europe
Solar potential in Europe