


Expected time for searching
in a Hash table
(Using Open Addressing)

<https://youtu.be/0VhbvrU9mg0>

Reminder: Expected value/mean

Example: $X \sim$ result of 

$$\text{Mean } E[X] = \frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 2 + \dots + \frac{1}{6} \cdot 6$$

$$= \frac{21}{6} = 3.5 \quad \sum_{i=1}^6 i \cdot \Pr(X=i)$$

$$\sum_{i=1}^6 \Pr(X \geq i) = 1 + \frac{5}{6} + \frac{4}{6} + \dots + \frac{1}{6} = \frac{21}{6} = 3.5$$

$E[\# \text{ probes}]$ (for unsuccessful search)

$$= \sum_{i=1}^n \Pr(\# \text{ probes} \geq i)$$

$$\Pr(\# \text{ probes} \geq i) = \left(\frac{n}{m}\right) \cdot \left(\frac{n-1}{m-1}\right) \cdot \left(\frac{n-2}{m-2}\right) \cdots \left(\frac{n-i+2}{m-i+2}\right)$$

$$\leq \left(\frac{n}{m}\right)^{i-1} = \alpha^{i-1}$$

$$E[\# \text{ probes}] \leq \sum_{i=1}^n \alpha^{i-1} = \sum_{i=1}^{\infty} \alpha^{i-1} = \frac{1}{1-\alpha}$$