Logic and Set Theory
Introduction and Motivation

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A Tennis Tournament

Consider the following:

- a tennis club with 1025 registered players
- organises a tournament to determine the **champion**
- matches are played in rounds
- in each round all players still in the tournament are paired off according to a drawing of lots
- the winner of a match continues to the next round
- the loser of a match is out of the tournament
- the **champion** is the last player remaining in the tournament

Problem: determine how many matches need to be played.

Naive Solution: simple counting

Compute how many matches still need to be played, and add up the numbers:

$$512 + 256 + 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 + 1 = 1024 \ .$$

Inefficient solution: requires 10 additions!

The Power of Logical Reasoning

Reason as follows:

1. each match has a winner and a loser
2. a loser cannot participate in any later rounds, so every player except the champion loses one match
3. therefore: exactly as many matches as there are losers
4. hence, the number of matches is exactly one less than the number of players

We can generalise the argument:

For every n: a tournament with n players has of $n - 1$ matches.

Logic and Set Theory ...

... offers a structured approach to logical reasoning

Some goals:

- formalise statements in unambiguous logical language
- understand principles underlying correct logical reasoning
- apply logic to construct convincing proofs
- reasoning about sets, relations, functions, orderings
- understand and apply mathematical induction (on numbers)
- ...

A major part of the course is about how to present arguments

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What is logic?

Logic

is the formal systematic study
of the principles of

valid inference
and
correct reasoning.

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Correct reasoning?

All *witches* are *things that can burn*
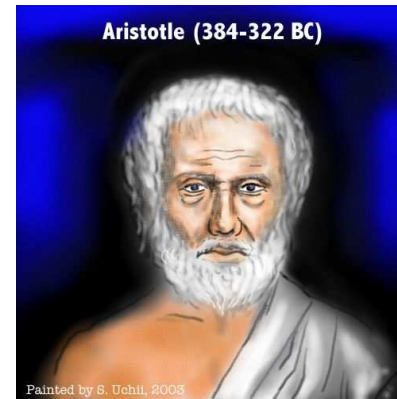All *things that can burn* are *made of wood*
All *witches* are *made of wood*

The reasoning is correct: indeed, the conclusion follows logically from the premisses. Since the second premisse is obviously false this should not, however, be considered a convincing argument for the conclusion.

All *ducks* are *things that float*
All *things made of wood* are *things that float*
All *ducks* are *made of wood*

The reasoning is incorrect!

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General reasoning rules

Aristotle (384-322 BC)
Painted by S. Uchii, 2003

One of Aristotle's 19 syllogisms:

All Ks are Ls
All Ls are Ms
All Ks are Ms

An example of an incorrect rule:

All Ks are Ms
All Ls are Ms
All Ks are Ls

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