Process Algebra (2IMF10) — Assignment 1

Deadline: Sunday May 14, 2017

This is the first assignment for the course on *Process Algebra* (2IMF10).

You may either do the assignment on your own or together with **at most one** fellow student (the latter is strongly preferred by the lecturer!). If you work together with a fellow student, then the two of you together hand in one set of solutions (clearly list the names and student numbers of both contributors).

Please submit your solutions via Canvas. The only accepted format for your document is PDF.

You are encouraged to discuss the assignment with me before handing it in. In particular, I will give feedback on operational rules and axioms if you show them to me.

Theory of Sequential Processes with Disrupt in Simulation Semantics

The starting point of this assignment is the process theory $\mathrm{MPT_S}(A)$, which has the same signature as $\mathrm{MPT}(A)$ and the following axioms:

$$
\begin{array}{lll}
x + y & = y + x & \text{A1} \\
(x + y) + z & = x + (y + z) & \text{A2} \\
x + x & = x & \text{A3} \\
x + 0 & = x & \text{A6} \\
a.(x + y) & = a.(x + y) + a.y & \text{S}
\end{array}
$$

The axiom S is not valid in bisimulation semantics, but it is valid in *simulation semantics.*

Definition 1 (Simulation semantics). *A binary relation R on the set of states S of a transition-system space is a* simulation relation *if the following condition holds:*

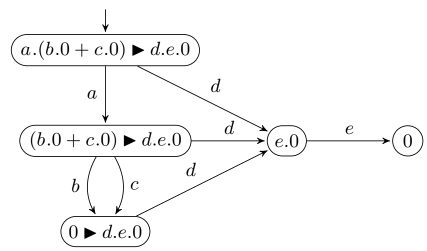
(i) for all states $s, t, s' \in S$, whenever $(s, t) \in R$ and $s \xrightarrow{a} s'$ for some a in the set of labels L of the transition-system space, then there is a state $t' \in S$ such that $t \xrightarrow{a} t'$ and $(s', t') \in R$.

Two states $s, t \in S$ are simulation equivalent *(notation: $s \leftrightarrows t$) if there exist simulation relations R_1 and R_2 such that sR_1t and tR_2s.*

The term deduction system for $\mathrm{MPT_S}(A)$ is the same as the term deduction system for $\mathrm{MPT}(A)$ (see Table 4.2 of [1]). Two closed $\mathrm{MPT_S}(A)$-terms p and q are simulation equivalent if they are simulation equivalent in the transition-system space associated to $\mathrm{MPT_S}(A)$ by the term deduction for $\mathrm{MPT_S}(A)$. In your solution to the assignment you may use the following result:

Theorem 2. *The process theory $\mathrm{MPT_S}(A)$ is a sound and ground-complete axiomatisation of the algebra of closed $\mathrm{MPT_S}(A)$-terms modulo simulation equivalence.*

The assignment is about extending the theory $\mathrm{MPT_S}(A)$ with a binary *disrupt* operator $\blacktriangleright$. The process $p \blacktriangleright q$ executes p, but q may disrupt the execution of p by starting its own execution. Consider the following transition system, associated with the term $a.(b.0 + c.0) \blacktriangleright d.e.0$, which illustrates the behaviour of disrupt:



1. Extend the term deduction system for $\mathrm{MPT_S}(A)$ with operational rules for the disrupt operator to get a term deduction system for $\mathrm{MPT_S{+}D}(A)$.

2. Prove that simulation equivalence is a congruence for $\mathrm{MPT_S{+}D}(A)$.

3. Give a suitable set of axioms for the disrupt operator, and prove that your axioms are valid in the algebra of closed $\mathrm{MPT_S{+}D}(A)$-terms modulo simulation equivalence. Conclude that $\mathrm{MPT_S{+}D}(A)$ is sound for the algebra of closed $\mathrm{MPT_S{+}D}(A)$-terms modulo simulation equivalence.

4. Prove that $\mathrm{MPT_S{+}D}(A)$ is a ground-complete axiomatization for the algebra of closed $\mathrm{MPT_S{+}D}(A)$-terms modulo simulation equivalence.

References

[1] J. C. M. Baeten, T. Basten, and M. A. Reniers. *Process Algebra: Equational Theories of Communicating Processes*. Number 50 in Cambridge Tracts in Theoretical Computer Science. Cambridge University Press, 2010.