TU/e Technische Universiteit **Eindhoven** University of Technology

Student name:

Student number:

Examination cover sheet

(to be completed by the examiner)

Course name: Process Algebra — Course code: 2IMF10

Date: June 27, 2016

Start time: 13:30 — End time : 16:30

Number of pages: 2

Number of questions: 4

Maximum number of points/distribution of points over questions:20 (The number between parentheses in front of a problem indicates how many points you score with a correct answer to it.)

Method of determining final grade: Your grade for this examination will be determined by dividing the total number of scored points by 2, and it will contribute for 70% to your final grade for the course. Your average grade for the three homework assignments will contribute for 30% to your final grade for the course. To pass the course your grade for this examination should be at least 5.0.

Answering style: open questions

Exam inspection: Make an appointment with the lecturer.

Other remarks: Special exam for Olav Bunte

You are allowed to use, without proof, results stated in the book, provided that you carefully refer to them in your solutions.

A partially correct answer is sometimes awarded with a fraction of the points.

Instructions for students and invigilators

Permitted examination aids (to be supplied by students):

- ☐ Notebook
- ☐ Calculator
- ☐ Graphic calculator
- ☑ Lecture notes/book
- ☐ One A4 sheet of annotations
- ☐ Dictionar(y)(ies). If yes, please specify:
- ☑ Other: Slides

Important:

- examinees are only permitted to visit the toilets under supervision
- it is not permitted to leave the examination room within 15 minutes of the start and within the final 15 minutes of the examination, unless stated otherwise
- examination scripts (fully completed examination paper, stating name, student number, etc.) must always be handed in
- the house rules must be observed during the examination
- the instructions of examiners and invigilators must be followed
- no pencil cases are permitted on desks
- examinees are not permitted to share examination aids or lend them to each other

During written examinations, the following actions will in any case be deemed to constitute fraud or attempted fraud:

- using another person's proof of identity/campus card (student identity card)
- having a mobile telephone or any other type of media-carrying device on your desk or in your clothes
- using, or attempting to use, unauthorized resources and aids, such as the internet, a mobile telephone, etc.
- using a clicker that does not belong to you
- having any paper at hand other than that provided by TU/e, unless stated otherwise
- visiting the toilet (or going outside) without permission or supervision

(1) 1. (a) Prove that $\mathrm{TCP}_\tau(A, \gamma) \vdash x \parallel y = y \parallel x$.

(2) (b) Prove that $\mathrm{TCP}_\tau(A, \gamma) \vdash a.(\tau.x \parallel \tau.y) = a.(x \parallel y)$.

2. A word $\sigma \in A^*$ is a *trace* of a state s in a transition-system space over the set of labels A if, and only if, there exists a state s' such that $s \xrightarrow{\sigma}{}^* s'$. We denote by $tr(s)$ the set of all traces of s. Two states are *trace equivalent* (notation: $s \sim_T t$) if they have the same sets of traces, i.e., if $tr(s) = tr(t)$.

(2) (a) Prove that trace equivalence is a congruence on $\mathrm{MPT}(A)$.

(2) (b) Give an equation that is valid in $\mathbb{P}(\mathrm{MPT}(A))/_{\sim_T}$, but not in $\mathbb{P}(\mathrm{MPT}(A))/_{\underline{\leftrightarrow}}$. Motivate your answer with two proofs: one showing that your equation is indeed valid in $\mathbb{P}(\mathrm{MPT}(A))/_{\sim_T}$ and the other showing that it is not valid in $\mathbb{P}(\mathrm{MPT}(A))/_{\underline{\leftrightarrow}}$.

(2) (c) Is the process theory $\mathrm{MPT}(A)$ a ground-complete axiomatisation of the model $\mathbb{P}(\mathrm{MPT}(A))/_{\sim_T}$? (Motivate your answer.)

3. Consider the recursive specification E over $\mathrm{TSP}(A)$ with the following equations

$$X = a.X \cdot Y + 1$$
$$Y = b.1 + 1$$

(2) (a) Formally derive, within the term deduction system for $\mathrm{TSP}_{\mathrm{rec}}(A)$, three transitions that each have the closed $\mathrm{TSP}_{\mathrm{rec}}(A)$-term

$$((\mu X.E) \cdot (\mu Y.E)) \cdot (\mu Y.E)$$

as source. (You may write X instead of $\mu X.E$ and Y instead of $\mu Y.E$.)

(2) (b) Sketch the transition system associated with $\mu X.E$ by the term deduction system for $\mathrm{TSP}_{\mathrm{rec}}(A)$.

(3) (c) Prove that the process in $\mathbb{P}(\mathrm{TSP}_{\mathrm{rec}}(A))/_{\underline{\leftrightarrow}}$ denoted by $\mu X.E$ is *not* finitely definable over $\mathrm{BSP}(A)$.

(See next page)

4. Let $D = \{0, 1\}$, and let $D_\perp = D \cup \{\perp\}$. We presuppose that the set of actions A is defined by $A = \{i?d, o!d \mid d \in D\} \cup \{\ell?d, \ell!d, \ell\text{‽}d \mid d \in D_\perp\}$, that the communication function γ satisfies $\gamma(\ell?d, \ell!d) = \gamma(\ell!d, \ell?d) = \ell\text{‽}d$ for all $d \in D_\perp$ and is undefined otherwise, and that $H, I \subseteq A$ are defined by $H = \{\ell?d, \ell!d \mid d \in D_\perp\}$ and $I = \{\ell\text{‽}d \mid d \in D_\perp\}$.

   We consider the recursive specification E consisting of following equations:

$$
\begin{aligned}
B &= \sum_{d \in D} i?d.B_d \\
B_d &= \ell!\perp.B_d + \ell!d.B \qquad (d \in D) \\
C &= \ell?\perp.C + \sum_{d \in D} \ell?d.o!d.C \ .
\end{aligned}
$$

(3) (a) Give a recursive specification F over $\mathrm{BSP}(A)$ (i.e., not using parallel composition and encapsulation) including the definition of a recursion variable X such that

$$
\mathrm{BCP}_{\mathrm{rec}}(A, \gamma) + \mathrm{RSP} \vdash \mu X.F = \partial_H(\mu B.E \parallel \mu C.E) \ . \qquad (1)
$$

   Motivate your answer by proving Equation (1); in particular, explain how RSP is used.

(1) (b) Let G be the recursive specification consisting of the following recursive equations

$$
\begin{aligned}
B &= \sum_{d \in D} i?d.B'_d \\
B'_d &= o!d.B + \sum_{e \in D} i?e.o!d.B'_e \qquad (d \in D) \ .
\end{aligned}
$$

   Prove that the transition systems associated with the closed $\mathrm{TCP}_{\tau,\mathrm{rec}}(A, \gamma)$-terms $\tau_I(\partial_H(\mu B.E \parallel \mu C.E))$ and $\mu B.G$ are rooted branching bisimilar.

(<u>End of Examination</u>)