

# TIMING AND ROUTING GAMES IN TRANSITORY GENERALIZED JACKSON NETWORKS

**H. Honnappa**, University of Southern California, Los Angeles CA, honnappa@usc.edu

**R. Jain**, University of Southern California, Los Angeles CA, rahul.jain@usc.edu

How do customers passing through a network of FIFO queues, such as passengers at an airport, choose a time to arrive and a route through the network to minimize their delay? What is the interplay between these choices? We study a game theoretic model of arrival timing and routing in feedforward generalized Jackson networks that models this scenario. We assume a finite population of customers (hence, *transitory*) who arrive and route themselves such that the individual sojourn times are minimized, at equilibrium. For tractability, we study the system in the fluid regime.

[1] introduced the single server *concert queueing* game and proved that the unique Nash equilibrium arrival profile is a uniform distribution function. Routing games, too, have been studied extensively in the literature mostly assuming separable delay functions. In general, customers' arrival timing decisions will be affected by their routing decisions (and vice versa), and in this work we provide what is, to the best of our knowledge, the first analysis of this complex interplay.

We first extend the arguments from [1] to tandem and trellis network topologies (a trellis network has a number of parallel networks connected in tandem), and prove that the unique equilibrium arrival profile is a uniform distribution function; the Wardrop equilibrium routing profile, too, is unique. Furthermore, the equilibrium is fairly efficient, with a price of anarchy upper-bounded by 2. For general network topologies, we use the fact that the arrival and routing decisions form a multi-stage extensive form game to prove that a sub-game perfect equilibrium exists and it is *essentially* unique. We note that this analysis extends standard multi-commodity flow games to transient settings and non-separable delay functions.

## References

[1] R. Jain, S. Juneja, and N. Shimkin. The Concert Queueing Game: To Wait or To be Late. *Discrete Event Dynamic Systems*, 21(1):103134, 2011.