# ESTIMATING WAITING TIMES WITH THE TIME-VARYING LITTLE'S LAW

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When waiting times cannot be observed directly, Little's law can be applied to estimate the average waiting time by the average number in system divided by the average arrival rate, but that simple indirect estimator tends to be biased significantly when the arrival rates are time-varying and the service times are relatively long. Here it is shown that the bias in that indirect estimator can be estimated and reduced by applying the time-varying Little's law (TVLL). If there is appropriate time-varying staffing, then the waiting time distribution may not be time-varying even though the arrival rate is time-varying. Given a fixed waiting time distribution with unknown mean, there is a unique mean consistent with the TVLL for each time $t$. Thus, under that condition, the TVLL provides an estimator for the unknown mean wait, given estimates of the average number in system over a subinterval and the arrival rate function. Useful variants of the TVLL estimator are obtained by fitting a linear or quadratic function to arrival data. When the arrival rate function is approximately linear (quadratic), the mean waiting time satisfies a quadratic (cubic) equation. The new methods are shown to be effective in estimating the bias in the indirect estimator and reducing it, using simulations of multi-server queues and data from a call center.