ON STOCHASTIC APPROXIMATIONS, QUASI-STATIONARY DISTRIBUTIONS, AND DETECTION OF MOVING TARGETS

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Consider a suitably regular Markov process, such as a Langevin diffusion, and consider the problem of computing the limiting distribution obtained by conditioning the process on staying inside, say, a compact set for an arbitrary long time. Such limiting distribution is known as the quasi-stationary distribution (QSD). It arises, as we shall see, in many applications ranging from Biology to one that is of particular interest to us, namely, national security. Since the QSD is obtained by conditioning on an event that is extremely rare, it might come as a surprise that one can design efficient Monte Carlo methods for such estimation problem. Traditional Monte Carlo procedures for estimating a QSD are based on the so-called Fleming-Viot interacting particle system, which provides an estimator that is asymptotically biased. We consider a simple estimator based on the theory of stochastic approximations. Interestingly, we note a phase transition occurring for the validity of the corresponding Central Limit Theorem related to the spectral gap of the associated Markov generator.