A BAYESIAN APPROACH TO STOCHASTIC ROOT FINDING

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We consider the problem of finding the root of a monotone function on the one-dimensional interval $[0, 1]$ when only noisy function evaluations are available. The probabilistic bisection algorithm (PBA) begins with a prior distribution on the location of the root, and successively updates this prior to reflect one's belief about the location of the root. In doing so, it needs to know the probability that the sign of the noisy function evaluation is correct. In practice, this information is not available. We explore a method for circumventing this problem, and develop convergence results. If a certain conjecture holds, then the rate of convergence of the PBA in terms of the number of simulation replications is arbitrarily close to, but slightly slower than, the asymptotic rate of convergence of stochastic approximation. The PBA has the advantages that it performs very stably for finite sample sizes, and can return confidence intervals for the location of the root.