

A FREE BOUNDARY PROBLEM ARISING IN ORDER BOOK DYNAMICS

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We consider a system of two boundary value problems for the Poisson equation in a domain $D_\gamma \subset \mathbb{R}_+^2$ bounded by the coordinate axes and the free boundary curve γ , which is symmetric about the ray $\{(x, x), x \geq 0\}$. The problems in the system are coupled by an additional requirement that their solutions coincide on γ . This free boundary problem was introduced by Roşu (2009), who used it to approximate utilities of buyers and sellers in a model of order book dynamics with small “granularity” (i.e., the ratio of the interest rate and the customer arrival rates). We establish existence and uniqueness of a classical solution to the above problem and analyticity of the corresponding free boundary. A stochastic interpretation of this problem is also discussed.