# DEGREE-DEGREE DEPENDENCIES IN RANDOM GRAPHS WITH HEAVY-TAILED DEGREES

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Mixing patterns in large self-organizing networks, such as the Internet, the World Wide Web, social and biological networks are often characterized by degree-degree dependencies between neighbouring nodes. One of the problems with the commonly used assortativity coefficient is that in disassortative networks its magnitude decreases with the network size. This makes it impossible to compare mixing patterns, for example, in two web crawls of different size. As an alternative, we have recently suggested to use rank correlation measures, such as Spearman's rho. Numerical experiments have confirmed that Spearman's rho produces consistent values in graphs of different sizes but similar structure, and it is able to reveal strong (positive or negative) dependencies in large graphs. In particular, applied to Web crawls, Spearman's rho has revealed much stronger negative degree-degree dependencies in Web graphs than was previously thought. We analytically investigate degree-degree dependencies for scale-free graph sequences, and provide mathematical proofs for the previously obtained numerical results. We start with a simple model of two heavy-tailed highly correlated random variable $X$ and $Y$, and show that the sample correlation coefficient converges in distribution either to a proper random variable on $[-1, 1]$, or to zero, and if $X, Y \geq 0$ then the limit is non-negative. We next adapt these results to the assortativity in networks as described by the degree-degree correlation coefficient, and show that it is non-negative in the large graph limit when the degree distribution has an infinite third moment. Then we consider the alternative degree-degree dependency measure, based on the Spearman's rho, and prove that this statistical estimator converges to an appropriate limit under very general conditions. We verify that these conditions hold in common network models, such as configuration model and Preferential Attachment model. We conclude that rank correlations provide a suitable and informative method for uncovering network mixing patterns.