EXPLICIT SOLUTIONS FOR THE STATIONARY DISTRIBUTION OF A GENERAL CLASS OF MARKOV PROCESSES

M.S. Squillante, IBM Thomas J. Watson Research Center, New York, USA, mss@us.ibm.com
J.S.H. van Leeuwaarden, Eindhoven University of Technology, Eindhoven, The Netherlands, j.s.h.v.leeuwaarden@tue.nl
E.M.M. Winands, Rabobank and University of Amsterdam, Amsterdam, The Netherlands, e.m.m.winands@uva.nl

We consider a general class of two-dimensional Markov processes defined over the nonnegative quarter-plane, potentially infinite in both dimensions, and obtain exact solutions for the stationary distribution. More specifically, we derive an explicit solution for the rate matrix $\mathbf{R}$ of the invariant probability vector, as well as for the associated matrix $\mathbf{G}$. The probabilistic interpretations of these fundamental solution matrices allow us to describe their elements in terms of paths on the two-dimensional lattice. Then determining explicit expressions for the matrices becomes equivalent to solving a lattice path counting problem, the solution of which is derived using path decomposition, Bernoulli excursions, and hypergeometric functions. Time permitting, we consider a few applications and related large-deviation decay rates.