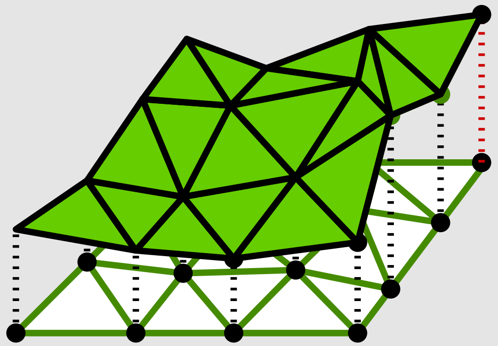
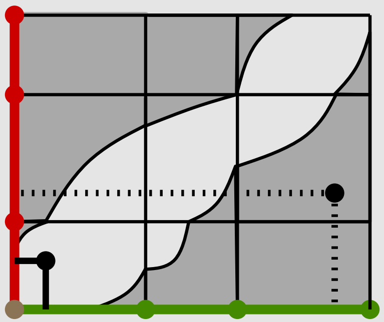
Computational Geometry
(Two) Selected Topics

Mark de Berg

TU Eindhoven





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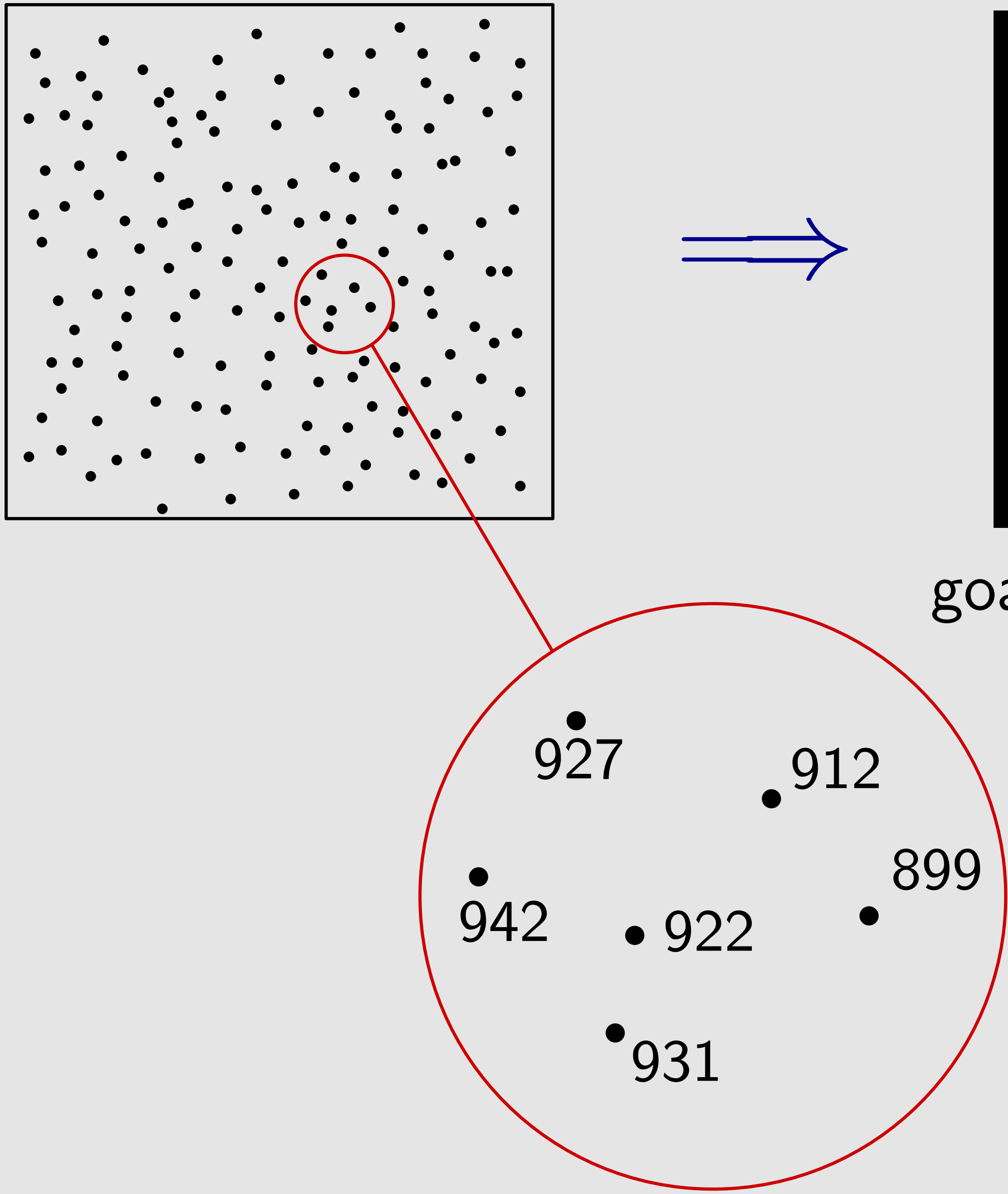
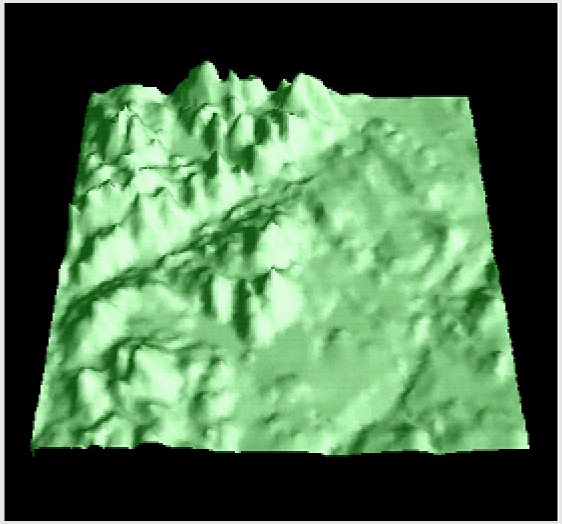
Reconstructing Terrains from Elevantion Data

given: terrain elevation at sample points

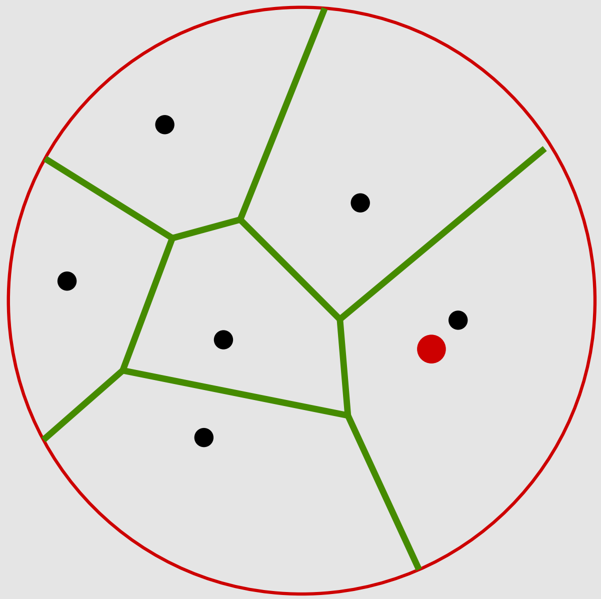




goal: construct terrain surface


927
912
899
942
922
elevation?
931

Idea: use elevation of nearest sample point



Voronoi diagram

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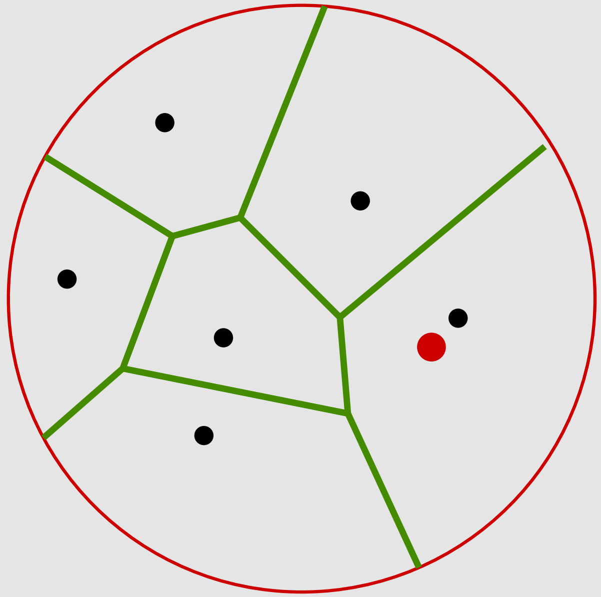
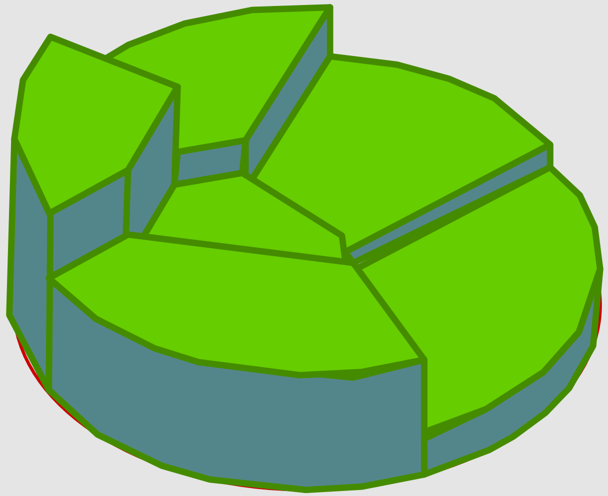
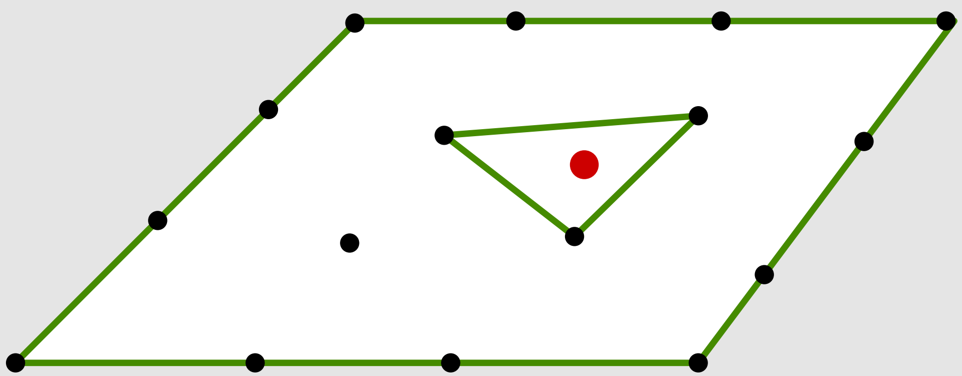


Voronoi diagram

Does not give good result: surface not continuous



Better: determine elevation using interpolation



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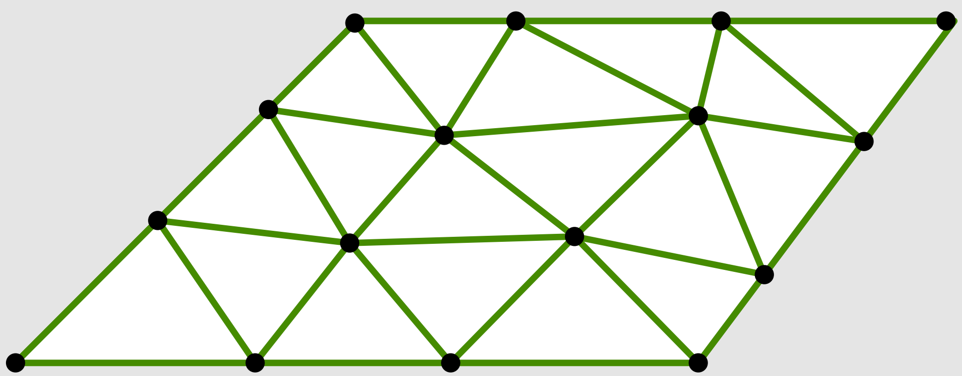

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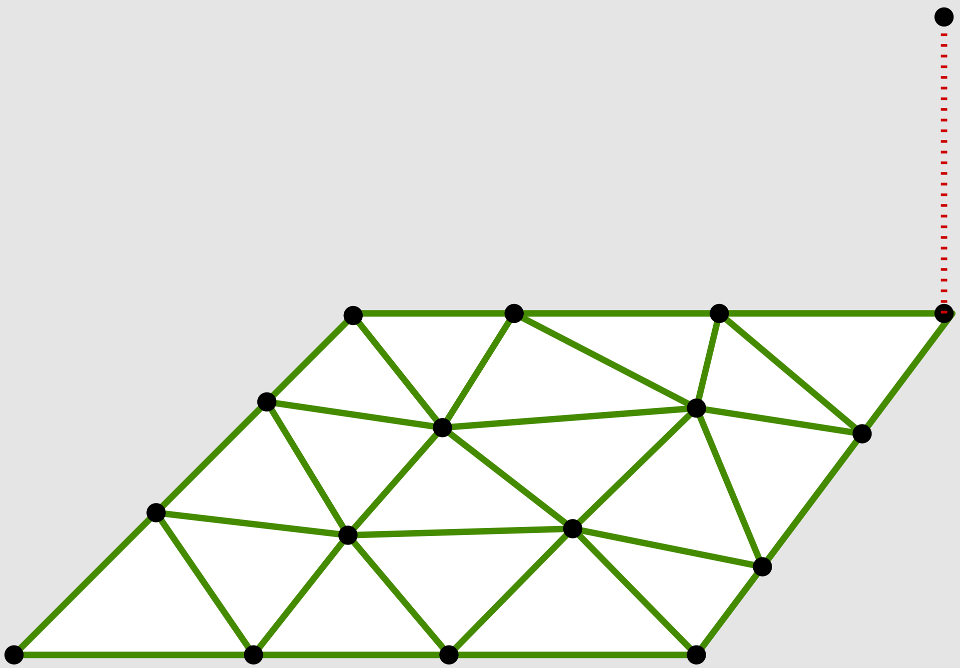

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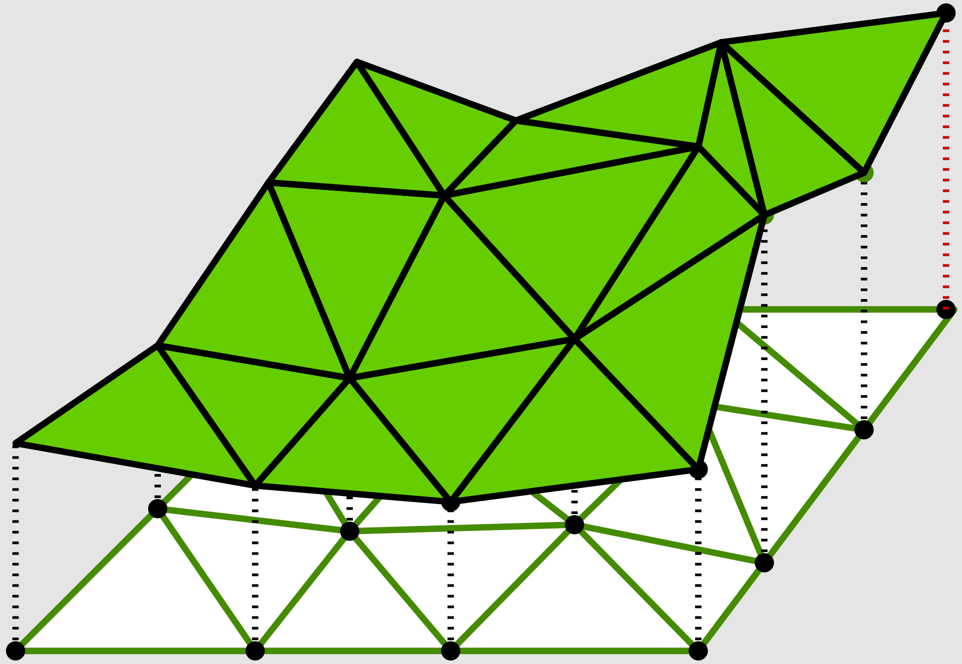
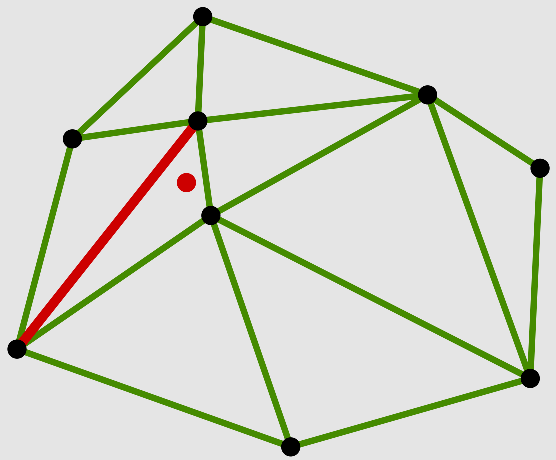
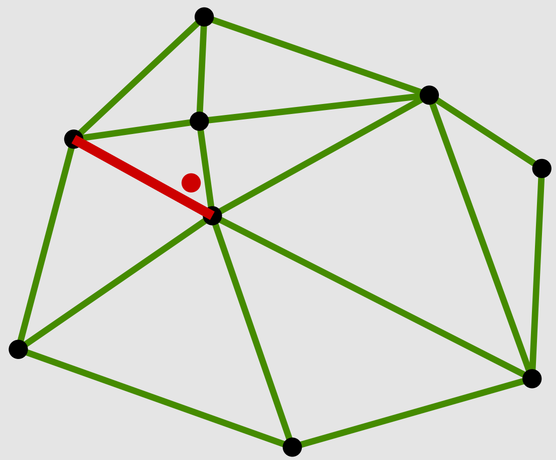


gives continuous surface

What is a good triangulation?



or



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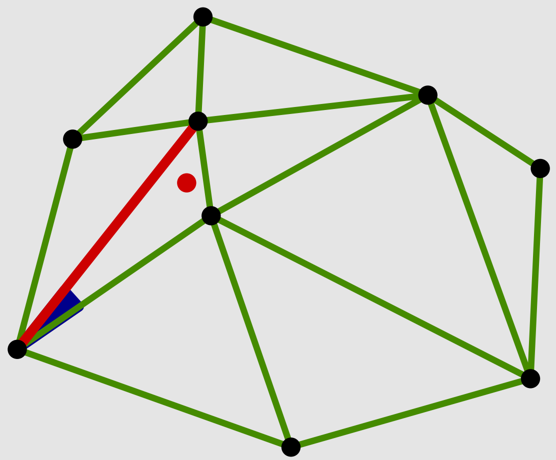

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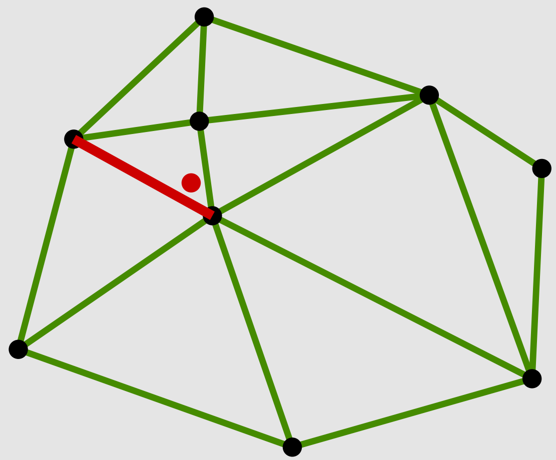

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long and thin triangles are bad $\Longrightarrow$ try to avoid small angles

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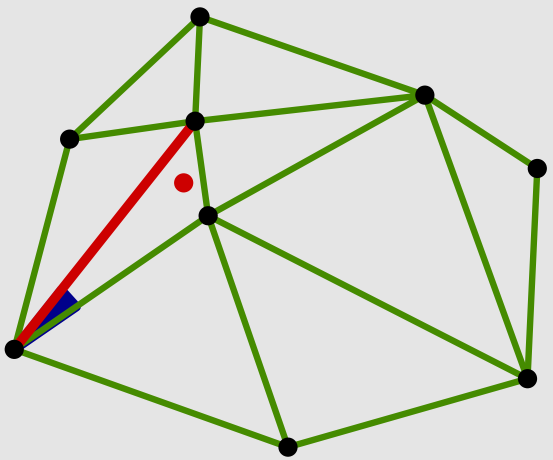

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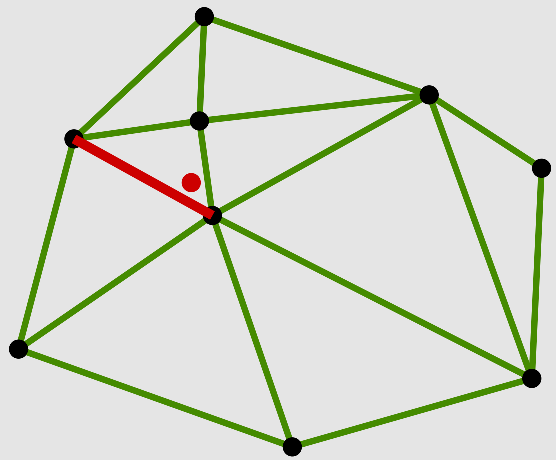

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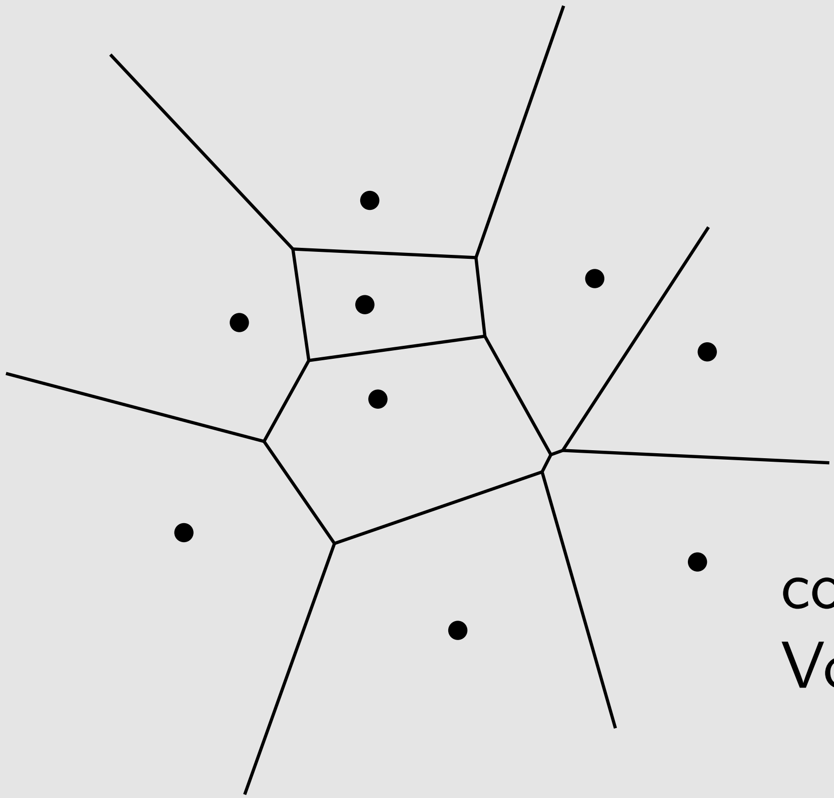
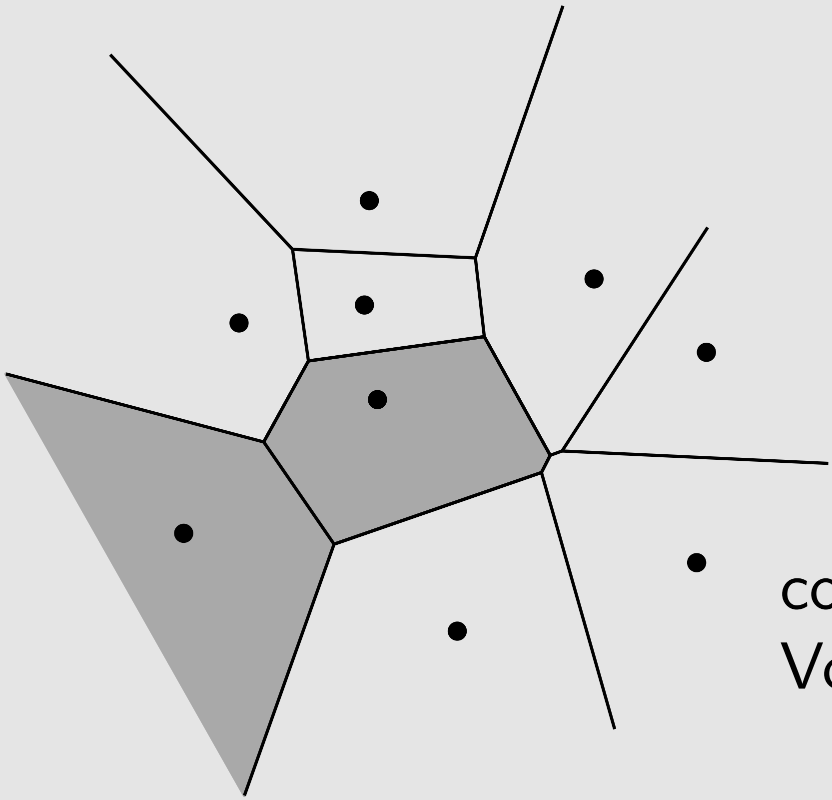
Algorithmic problem: how can we quickly compute a triangulation that maximizes the angles?

Voronoi diagram



connect points whose
Voronoi cells are neighbors

Voronoi diagram



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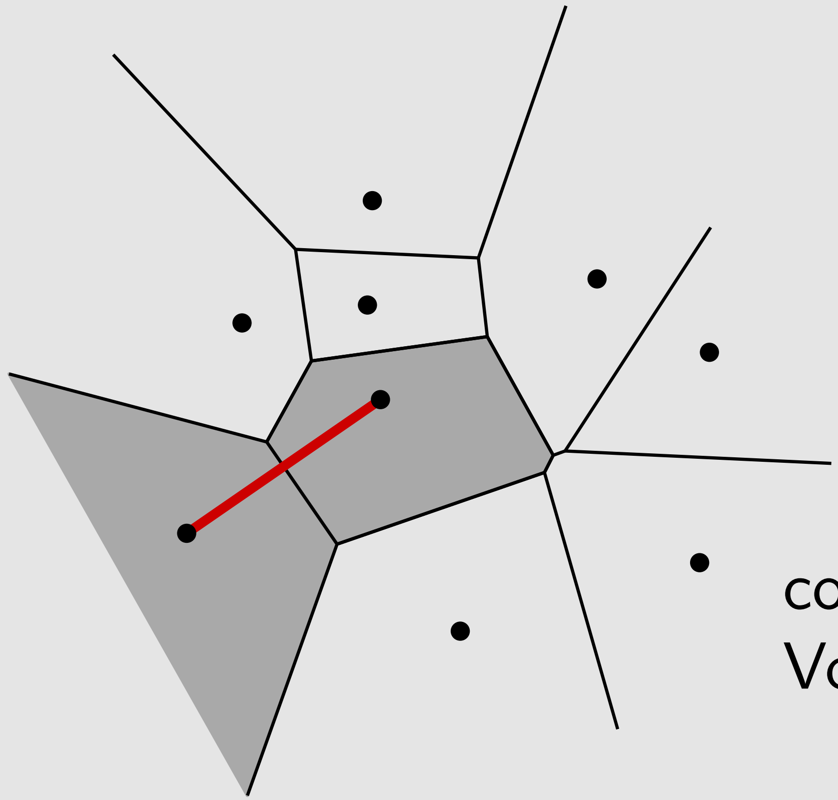

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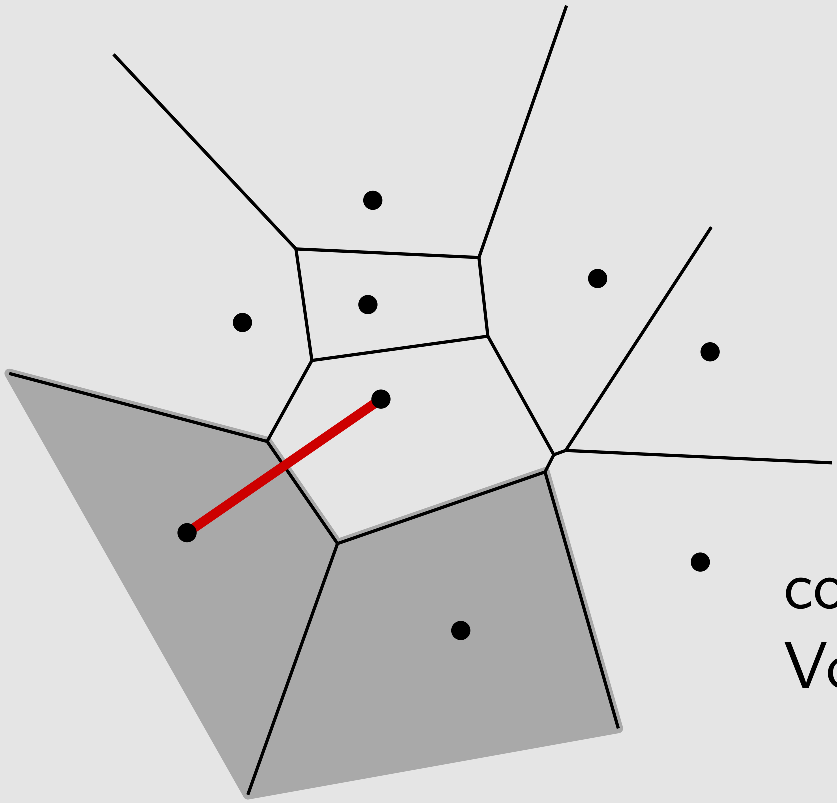

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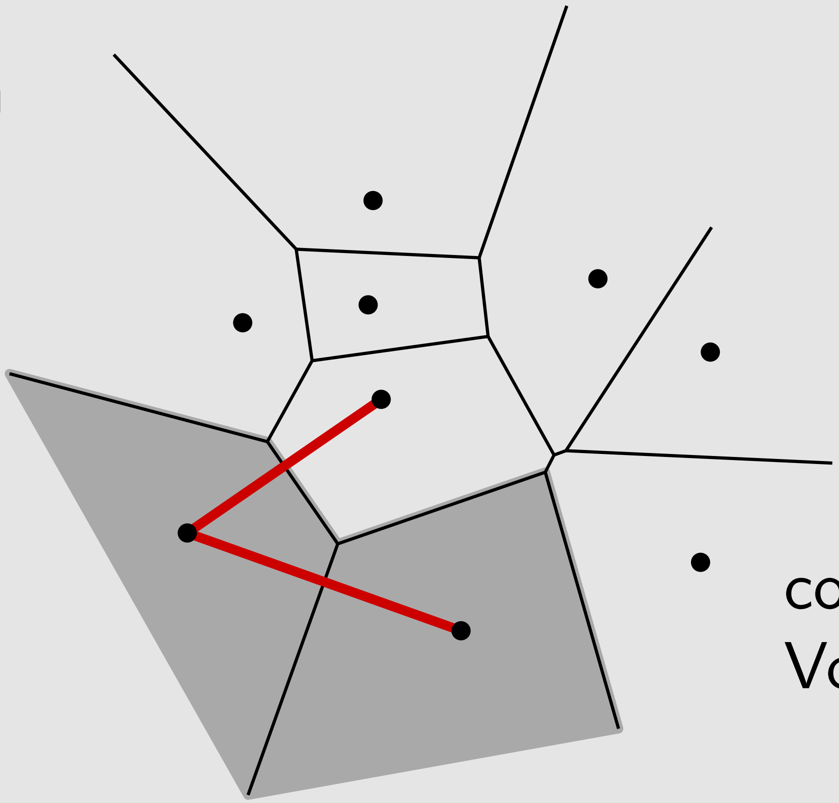

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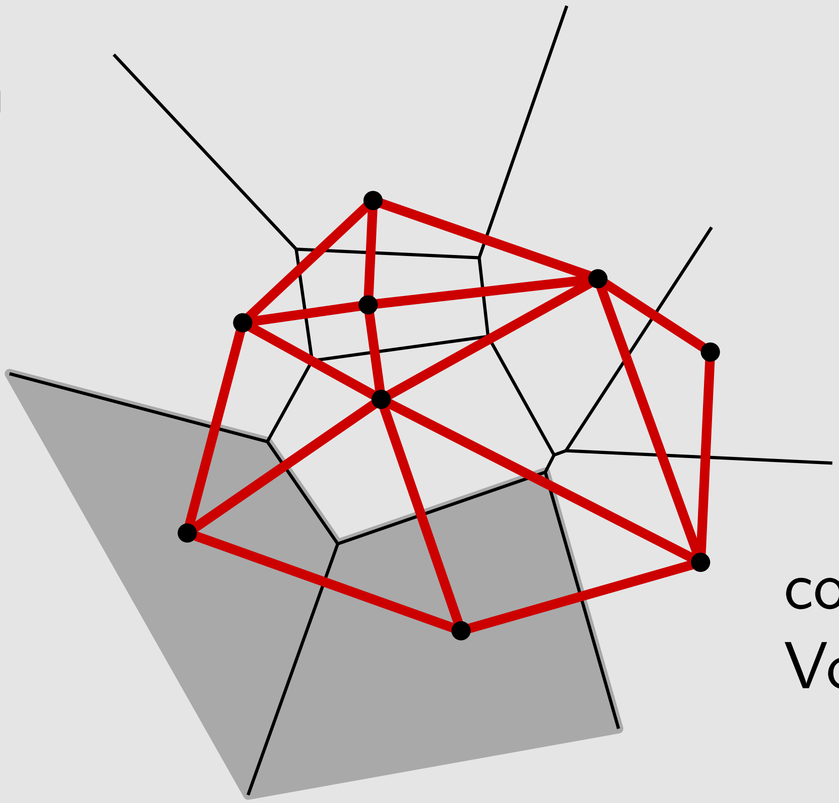

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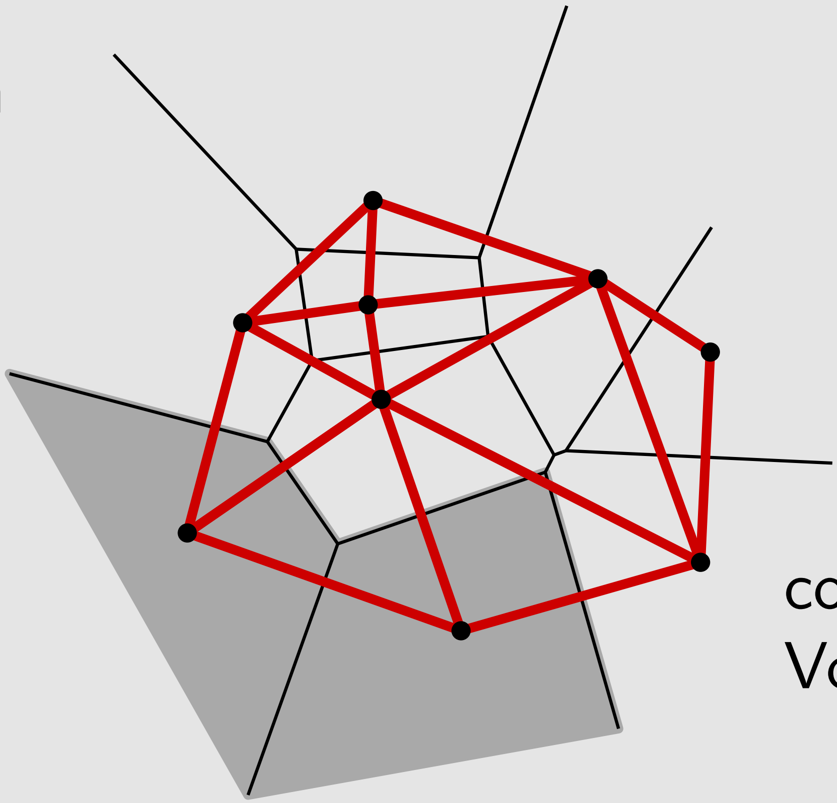

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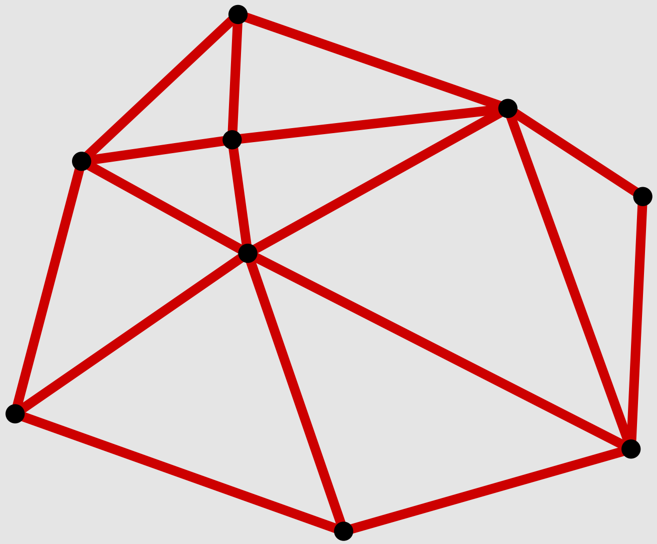
Delaunay triangulation:

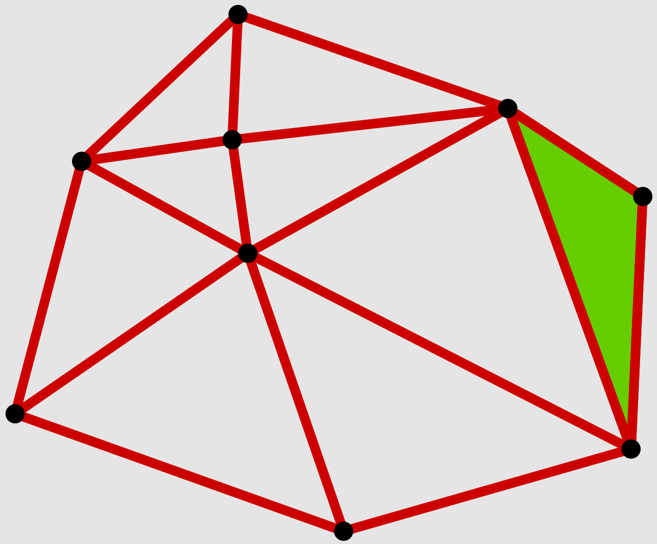
triangulation that maximizes
the minimum angle!



$\Delta(p, q, r)$ is triangle in Delaunay triangulation

$\Longleftrightarrow$

Circle(p, q, r) does not contain any other point



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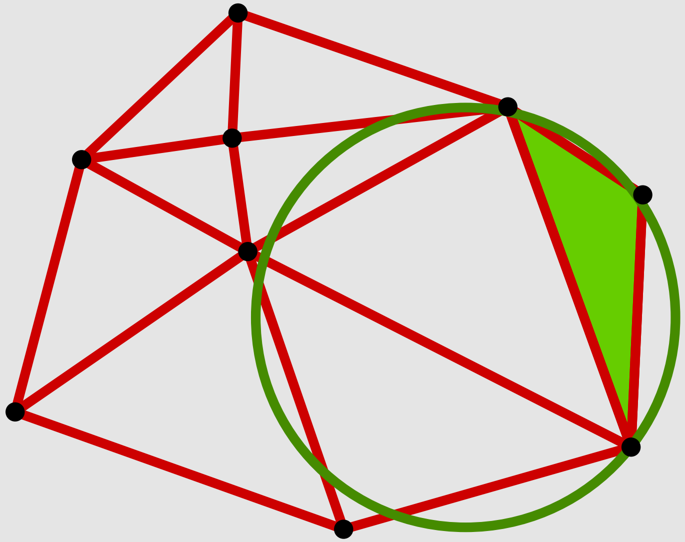

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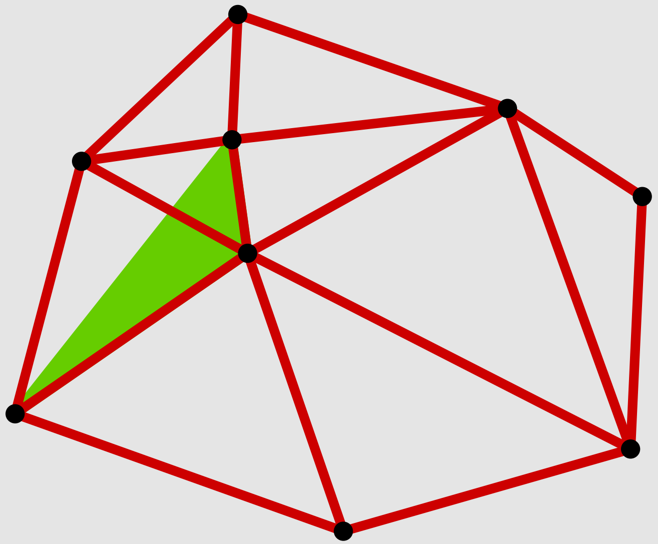

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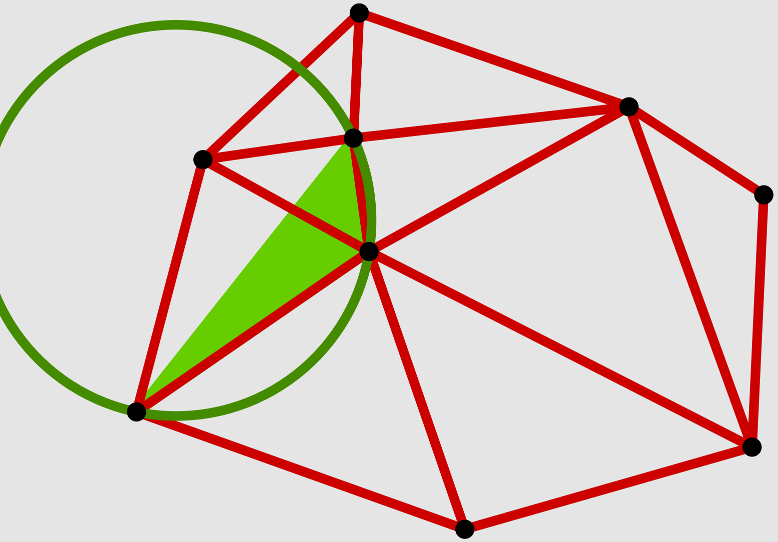

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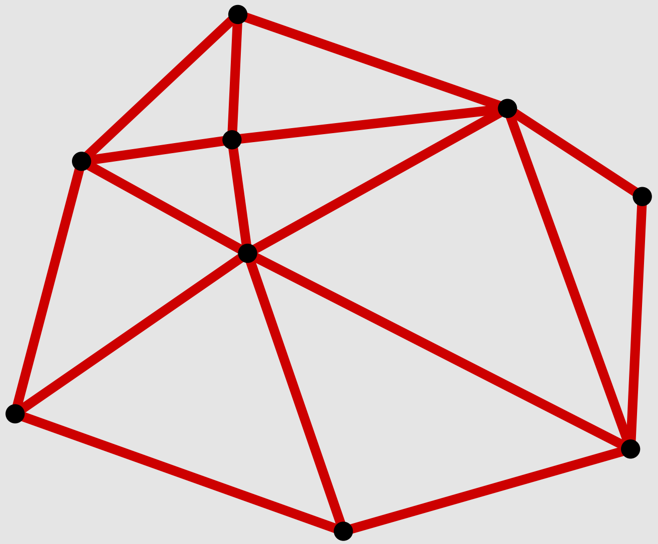

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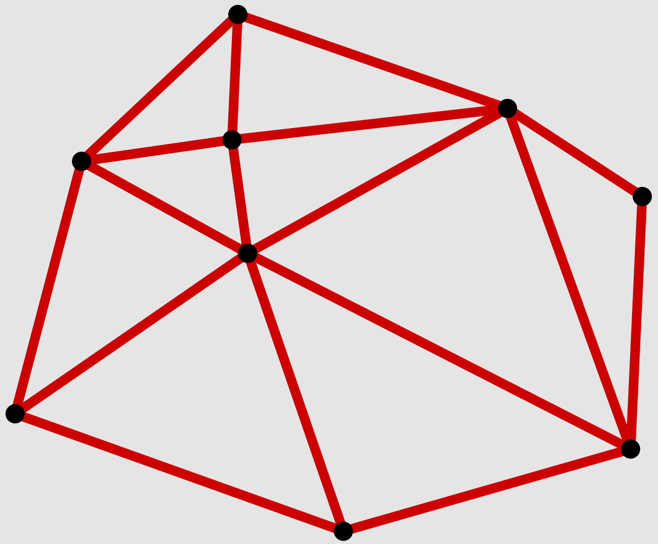

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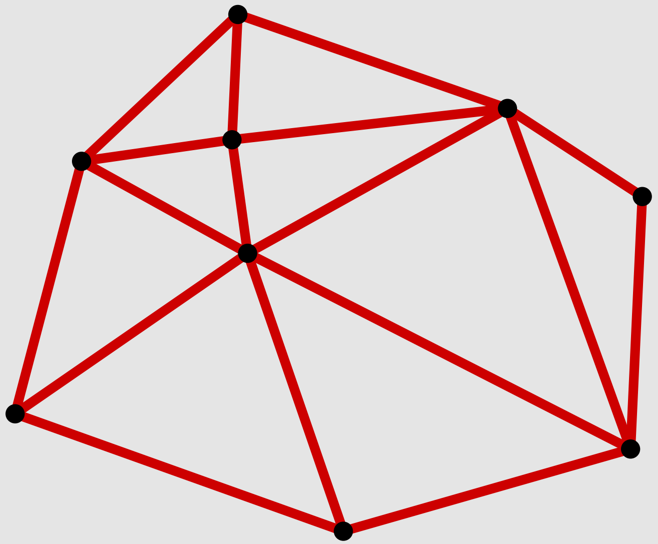
Algorithm for computing Delaunay triangulations:

1. **for** every triple of points
2. **do if** all other points lie outside Circle(p, q, r)
3. **then** Add triangle $\Delta(p, q, r)$ to DT



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- running time $O(n^4)$: very slow!
- can we do better? yes: there are algorithms running in $O(n \log n)$ time

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Analyzing Trajectories of Moving Objects

Moving objects (spatio-temporal data): now widely available

- GPS data, RFID tags, surveillance cameras, simulations

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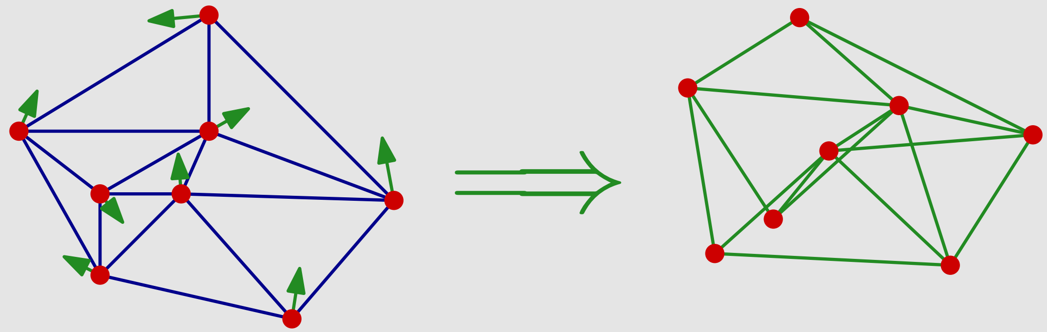

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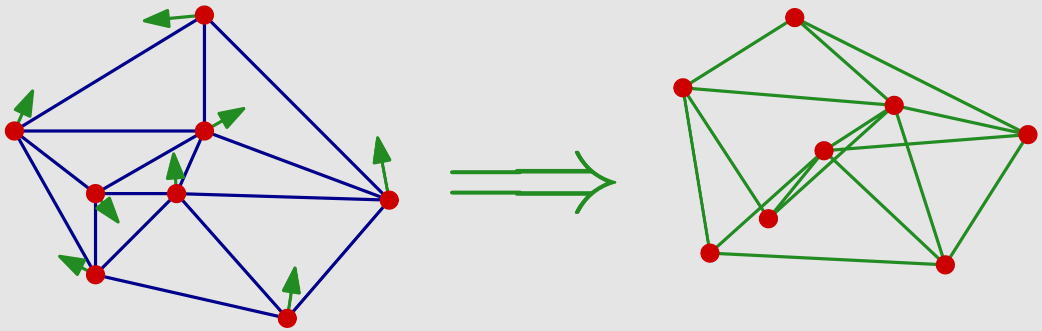
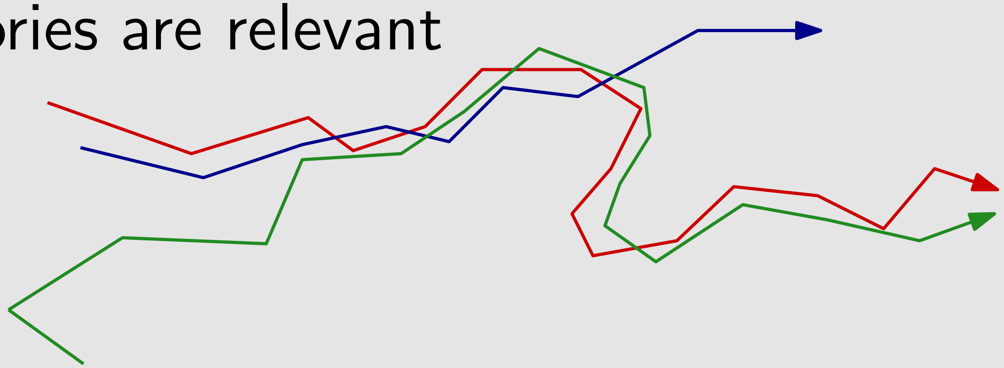
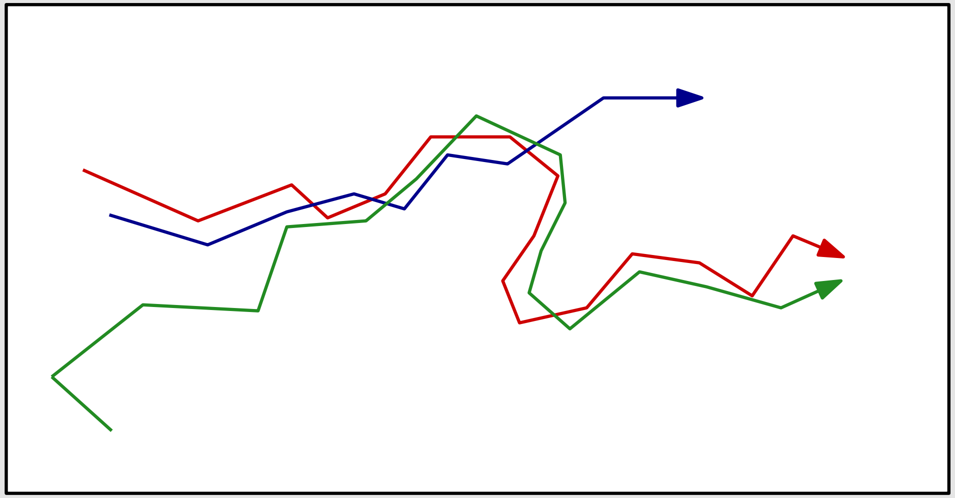


- Trajectories are relevant



Analyzing trajectories: applications



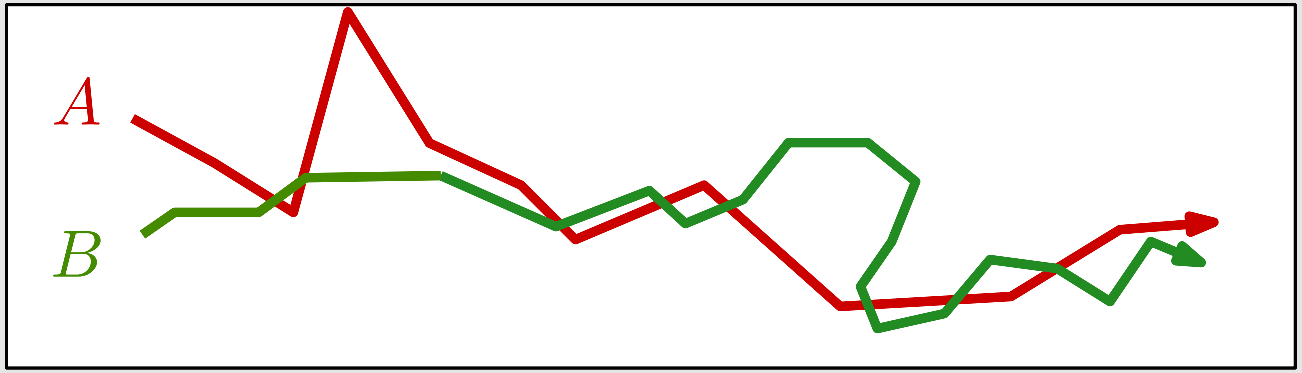
- behavorial studies on animals
- analysis of sports games
- analyzing traffic

Analyzing trajectories: examples of interesting questions

- given two trajectories, how similar are they?
- given a collection of trajectories, report all subtrajectories similar to a query trajectory
- is there a certain motion pattern? (leader following, flocking, . . .)

Basic question: how to measure similarity of two trajectories

We need a distance function *dist*$(\cdot,\cdot)$ on the trajectories:
the smaller *dist*(A, B), the more similar A and B are



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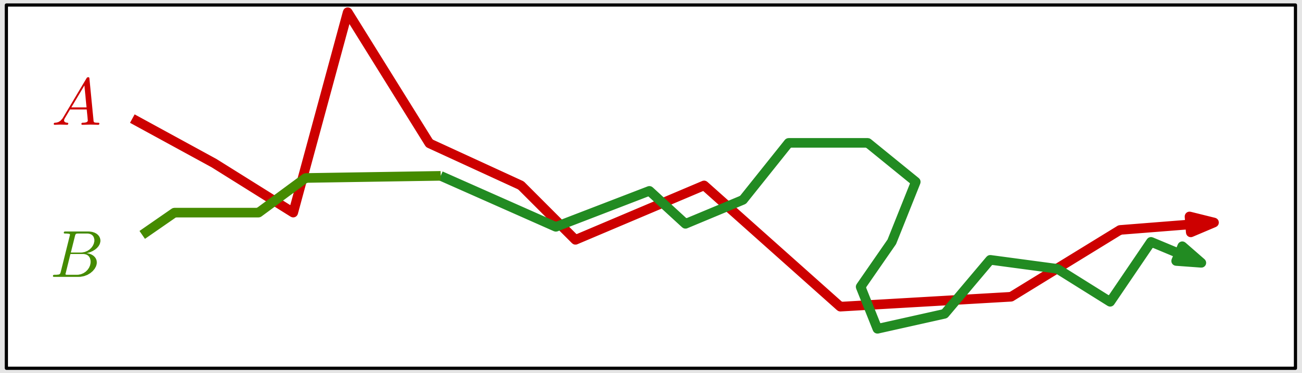

First attempt: Hausdorff distance

$$\mathit{dist}(A,B) \;=\; \max\left(\max_{a\in A}\min_{b\in B} d(a,b),\; \max_{b\in B}\min_{a\in a} d(b,a)\right)$$

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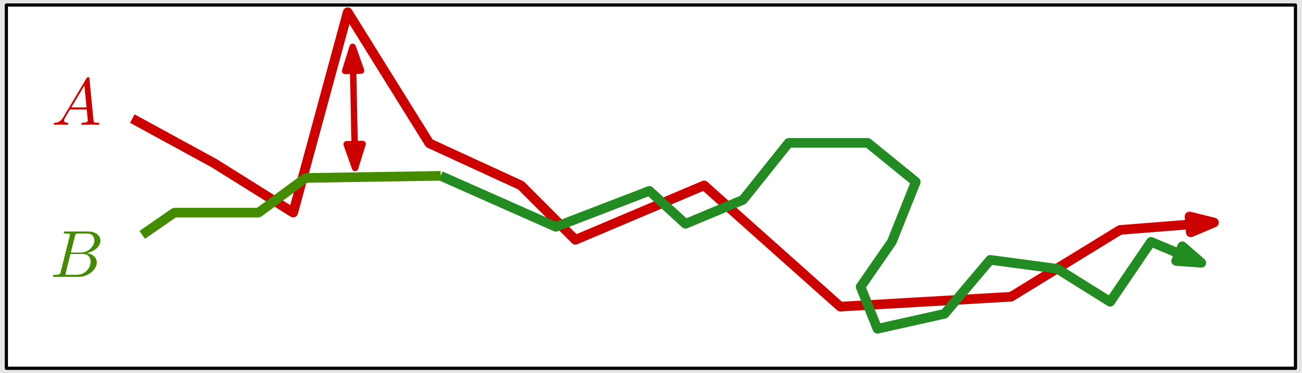

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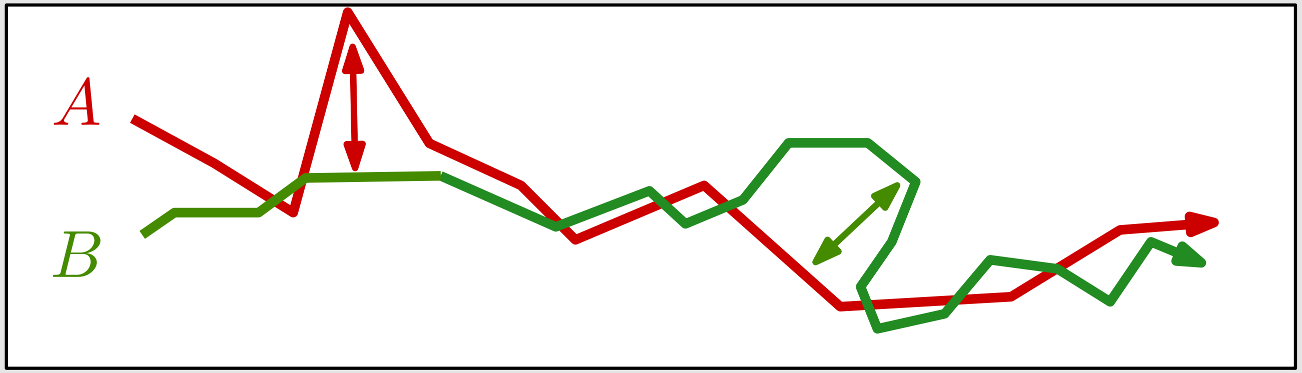

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Is the Hausdorff distance a good measure for trajectory similarity?

Hausdorff distance:

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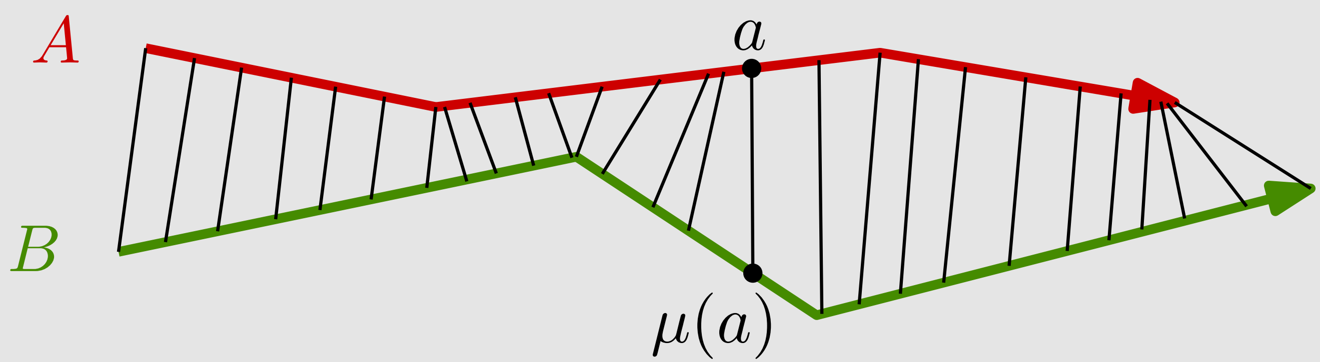
Hausdorff distance:

Hausdorff distance can be small, even when trajectories are very different

Hausdorff distance does not take continuity of trajectories into account

A better measure: Fréchet distance

- take continuous mapping $\mu : A \rightarrow B$
- take as distance $\max_{a \in A} d(a, \mu(a))$



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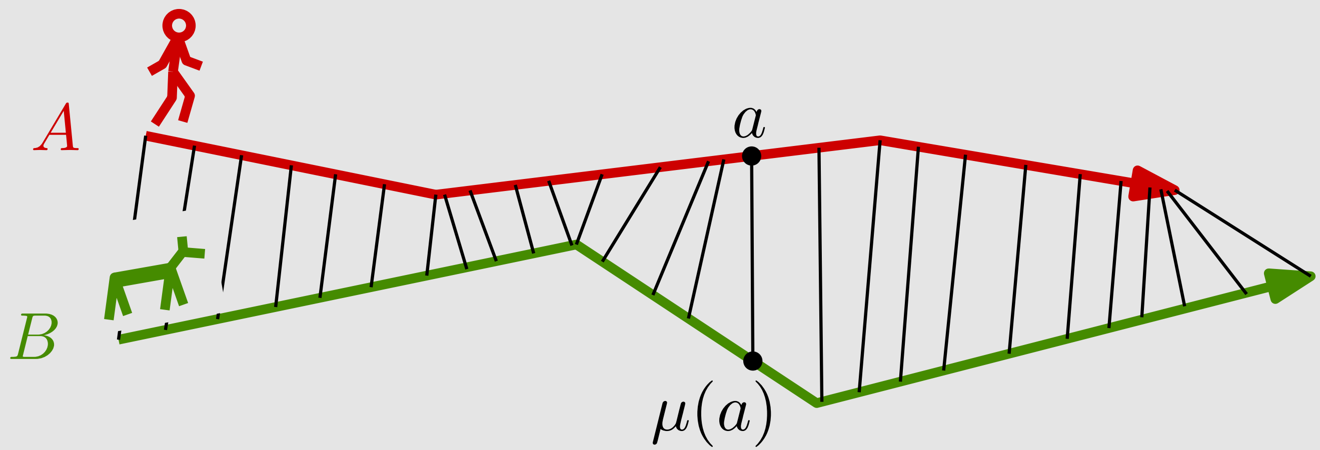

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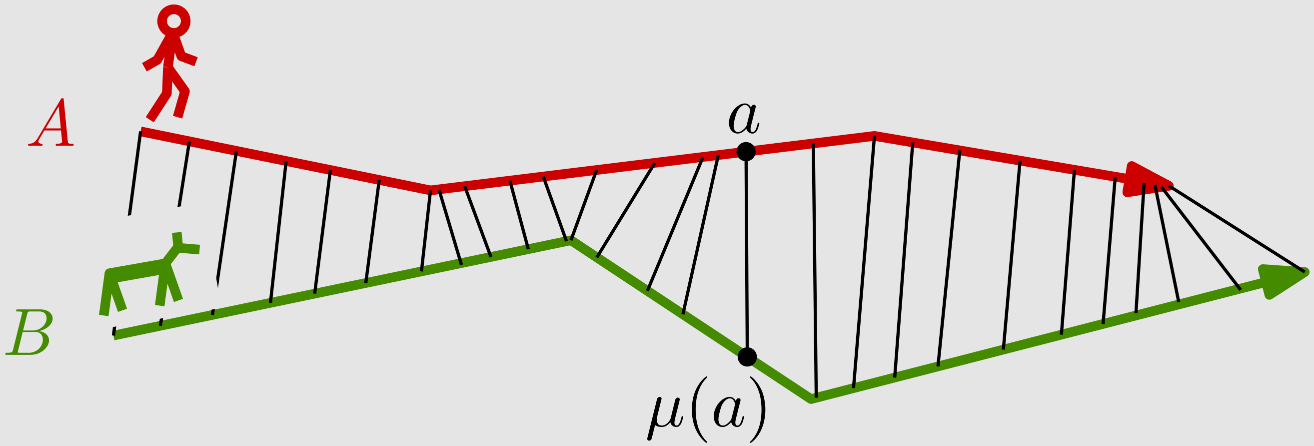
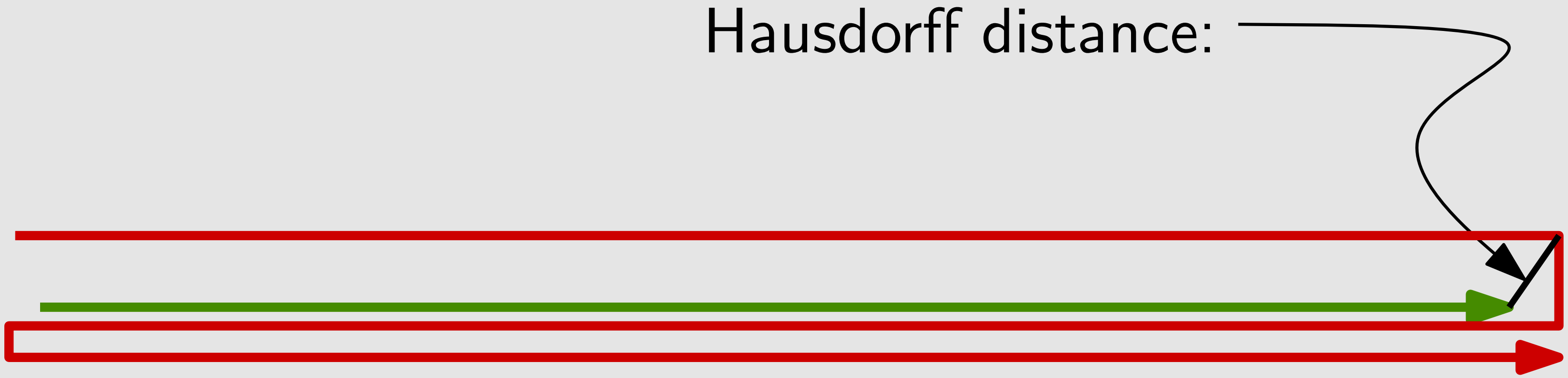


Which mapping? Take the best one:

$$\text{Fréchet-dist}(A, B) = \min_{\mu} \max_{a \in A} d(a, \mu(a)),$$

where min is taken over all continuous 1-to-1 mappings

Hausdorff distance:


Hausdorff distance:

Fréchet distance:

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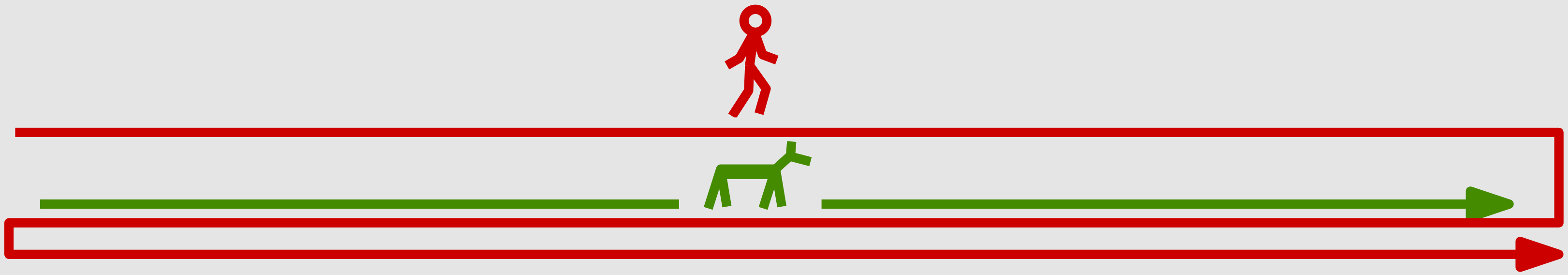

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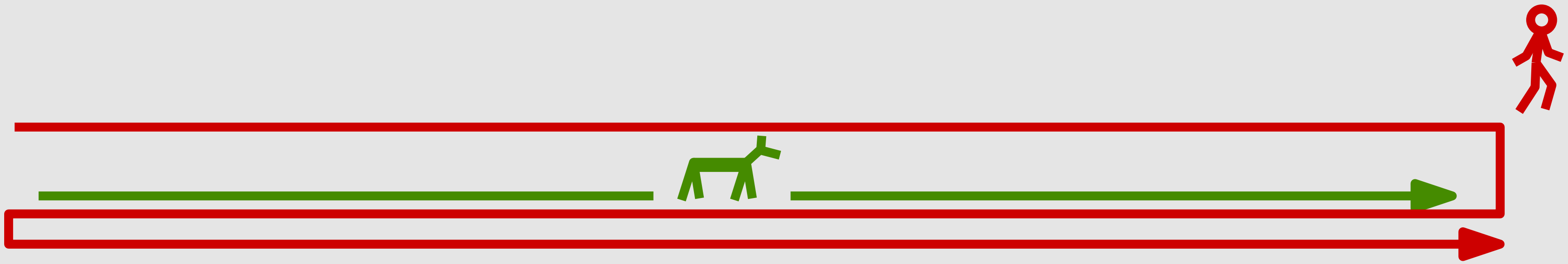

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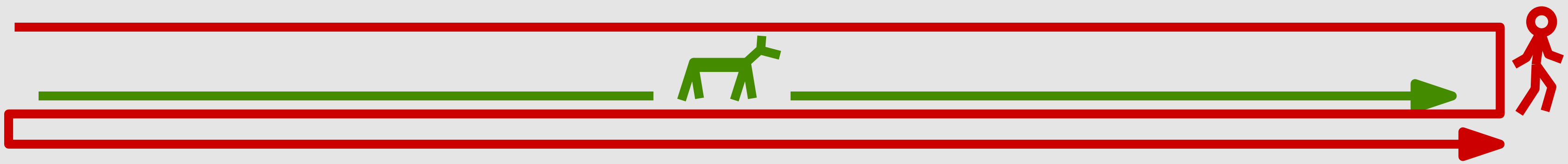

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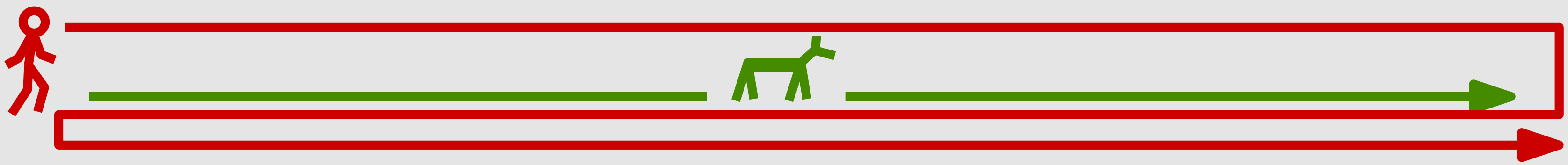

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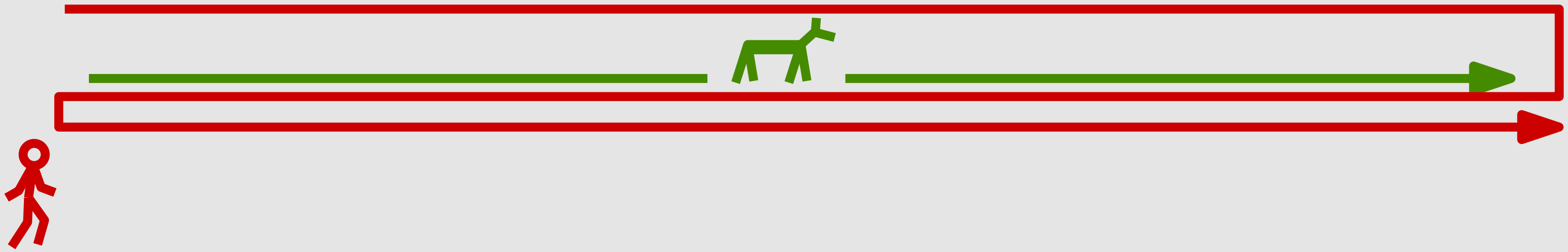

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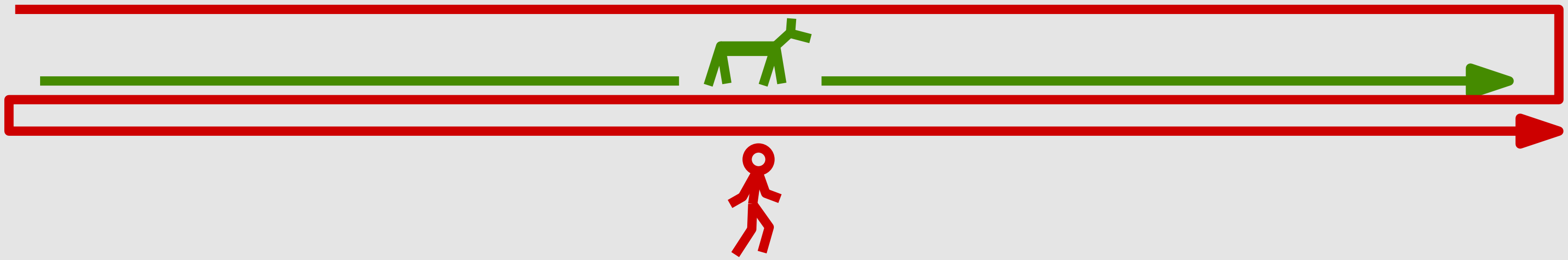

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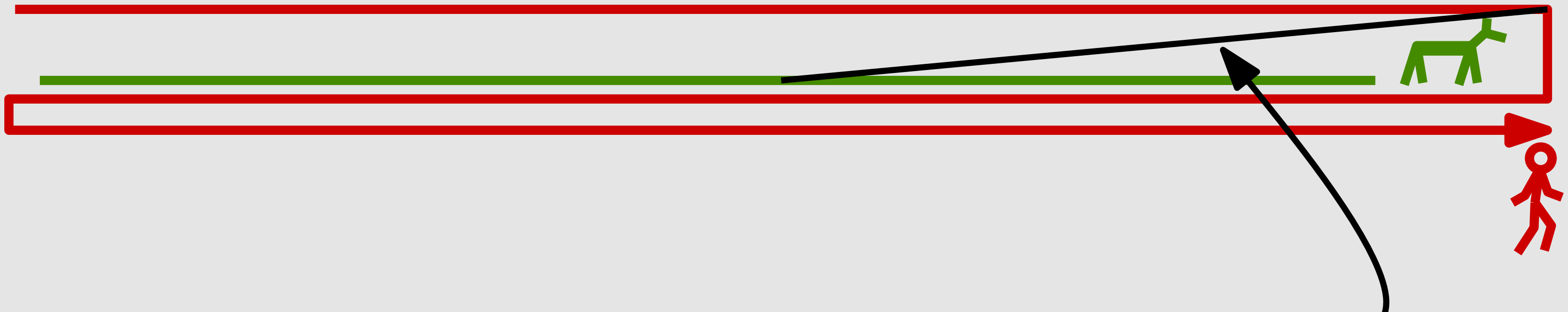
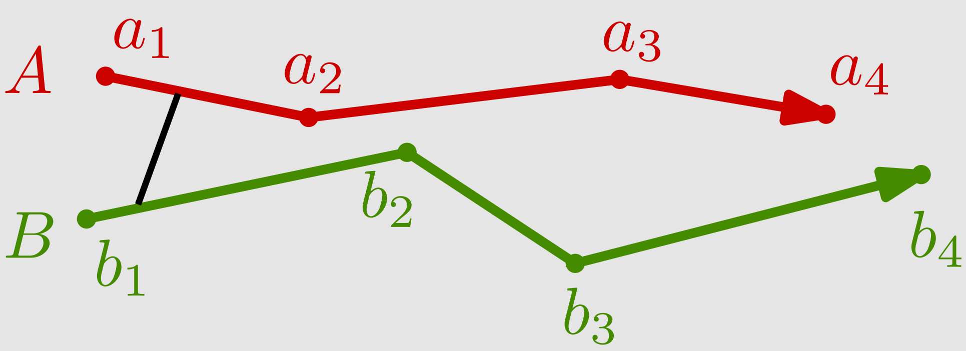
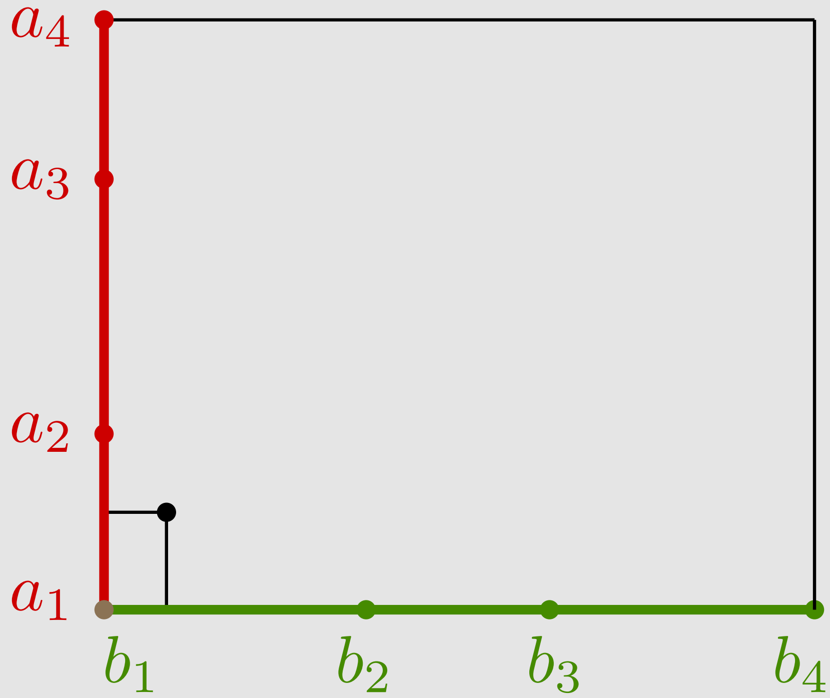


Fréchet distance:

Computing similarity: deciding if Fréchet-dist$(A, B) \leq \varepsilon$



free-space diagram (for given ε)



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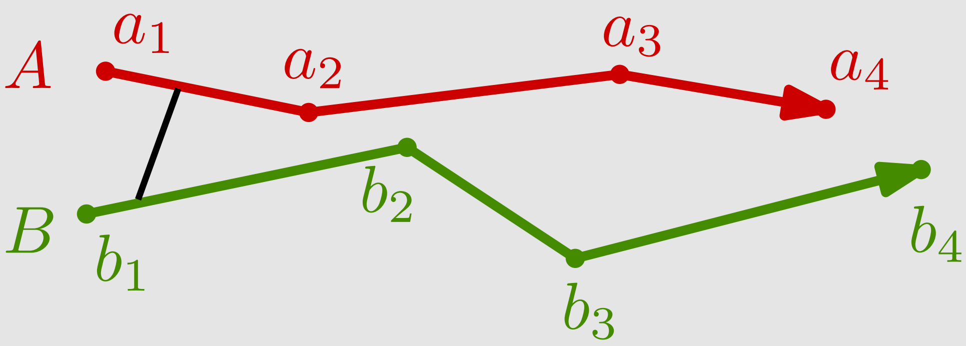

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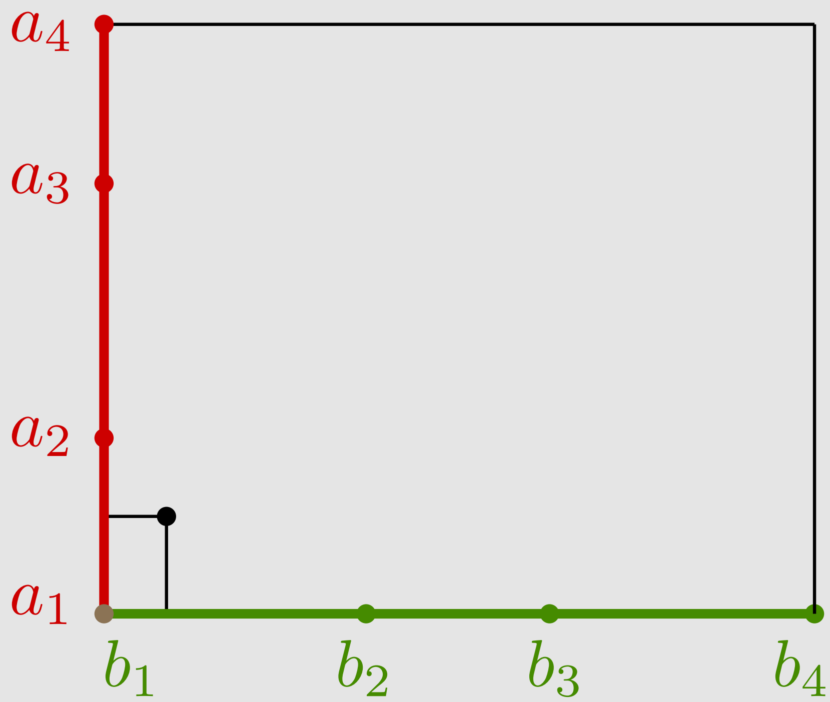

what does continuous 1-to-1 mapping correspond to in diagram?

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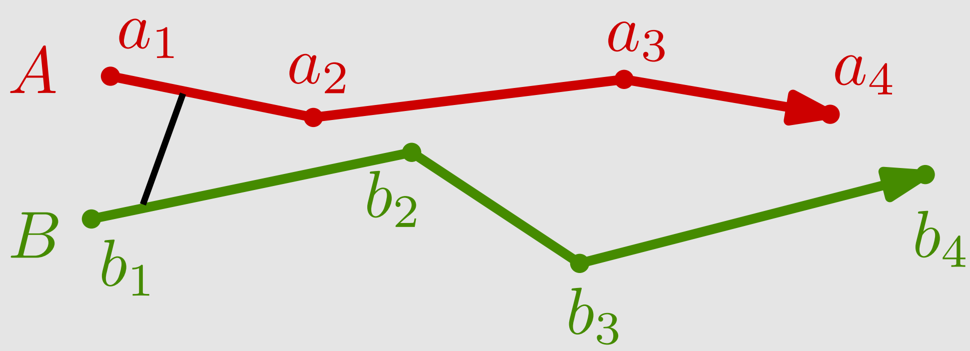

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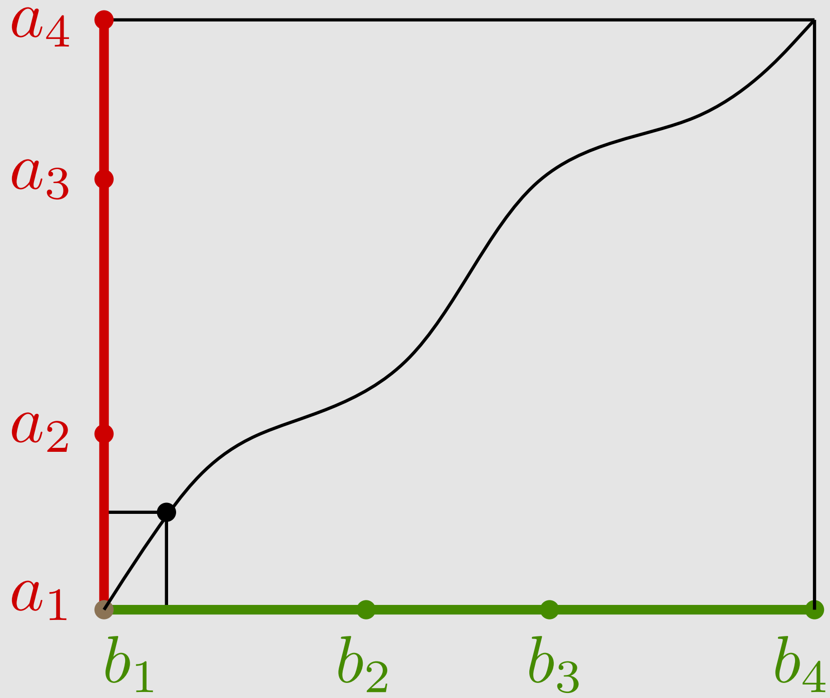

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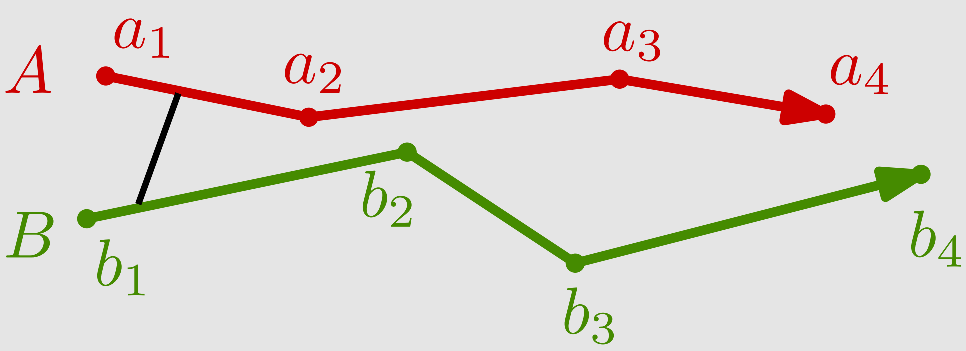
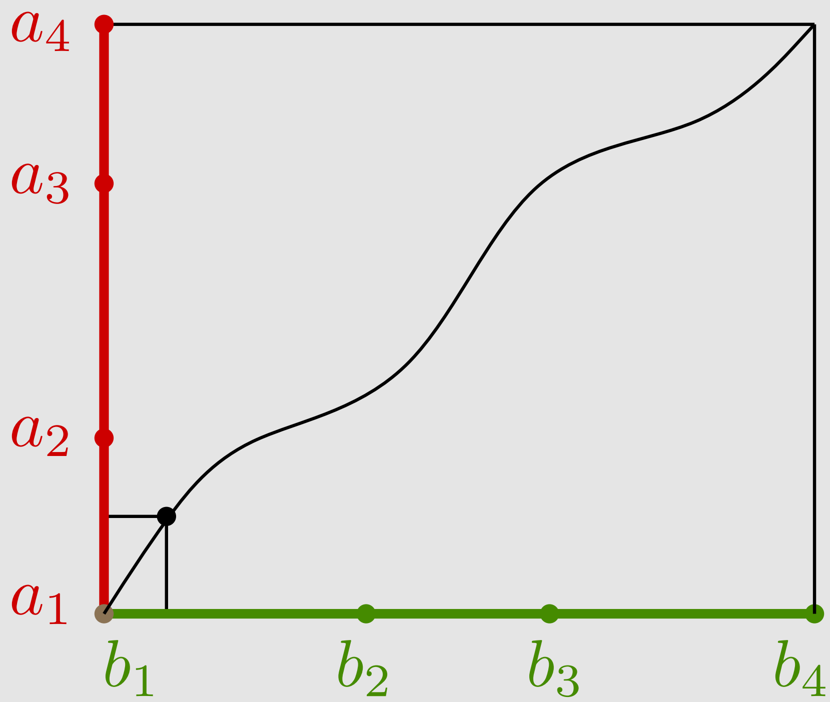
xy-monotone path from lower left to upper right

Computing similarity: deciding if Fréchet-dist$(A, B) \leq \varepsilon$



must decide if monotone path exists such that distance is always $\leq \varepsilon$

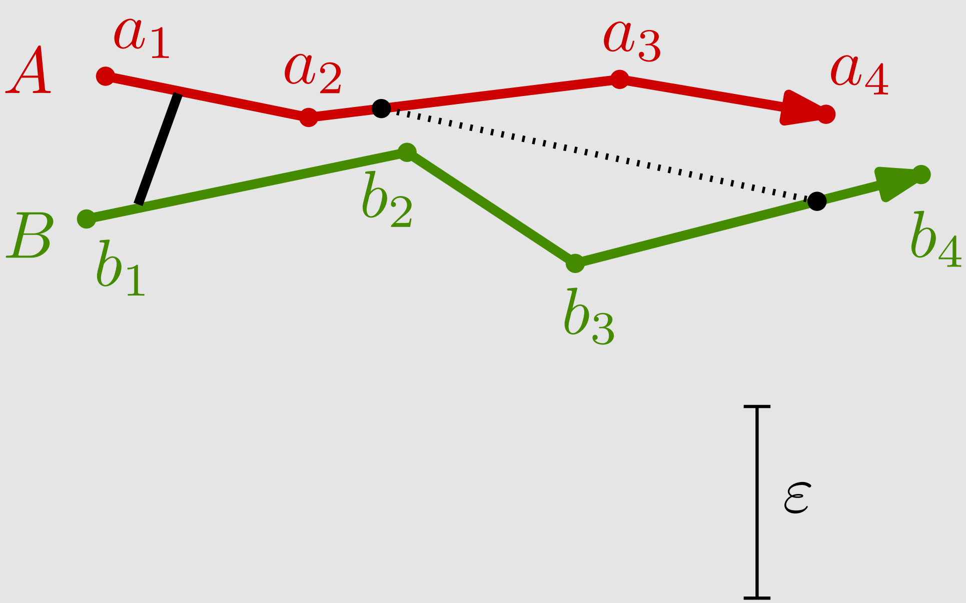
free-space diagram (for given ε)



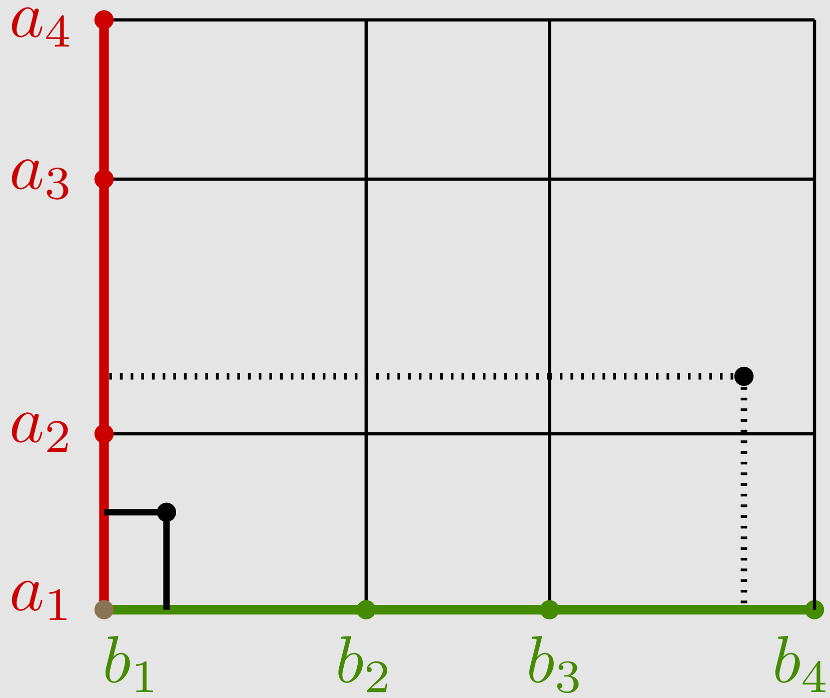
what does continuous 1-to-1 mapping correspond to in diagram?

xy-monotone path from lower left to upper right

Computing similarity: deciding if Fréchet-dist$(A, B) \leq \varepsilon$

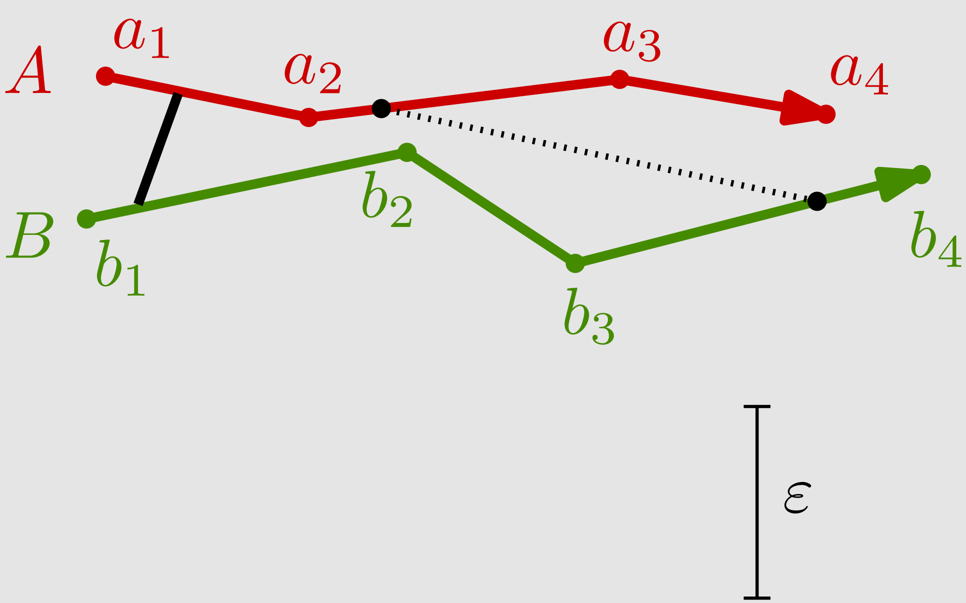


free-space diagram (for given ε)

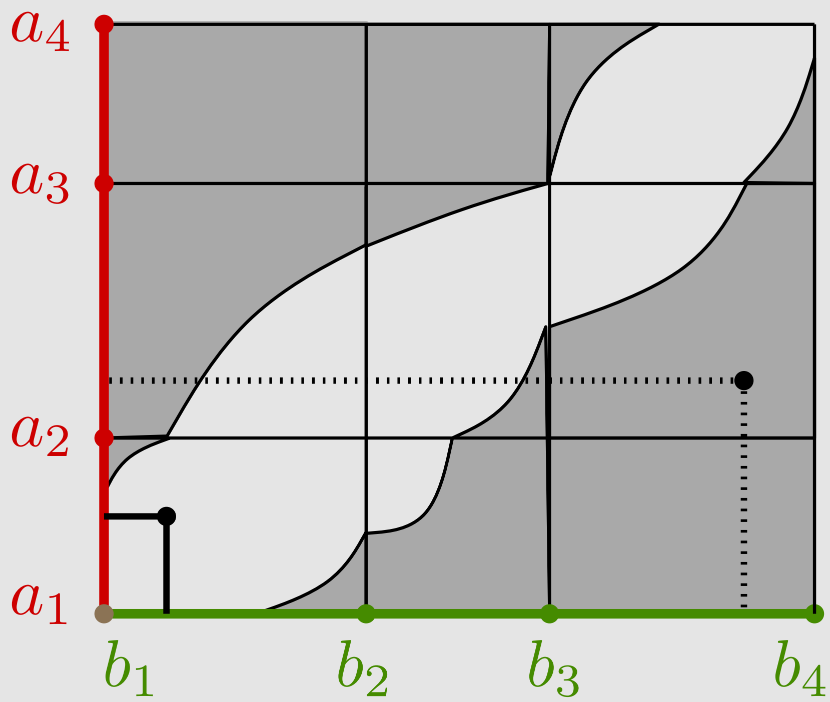


forbidden region in diagram:
(a, b) such that $d(a, b) > \varepsilon$

Computing similarity: deciding if Fréchet-dist$(A, B) \leq \varepsilon$



free-space diagram (for given ε)



forbidden region in diagram:
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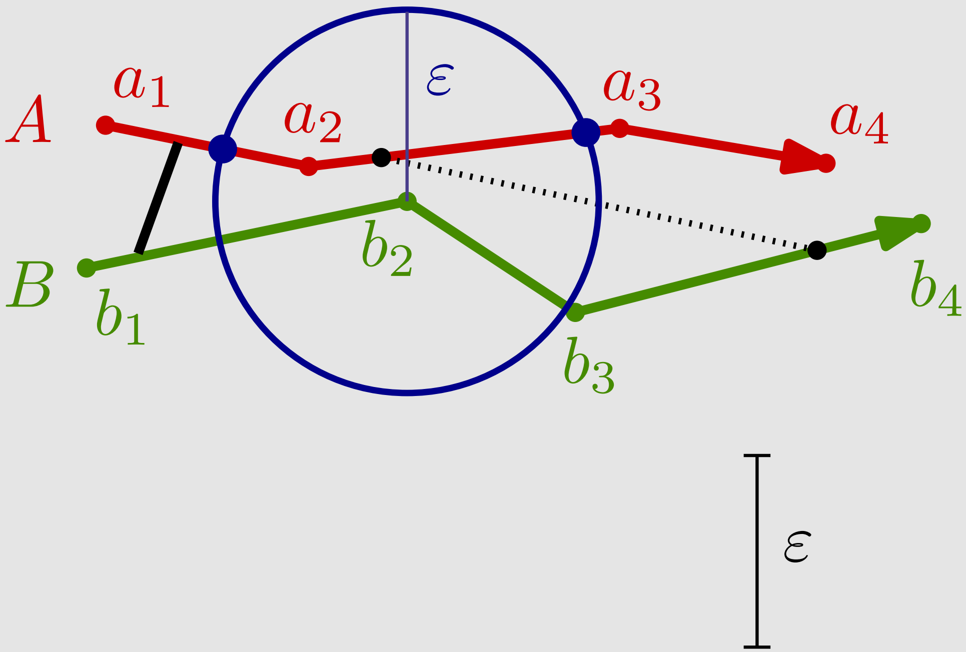
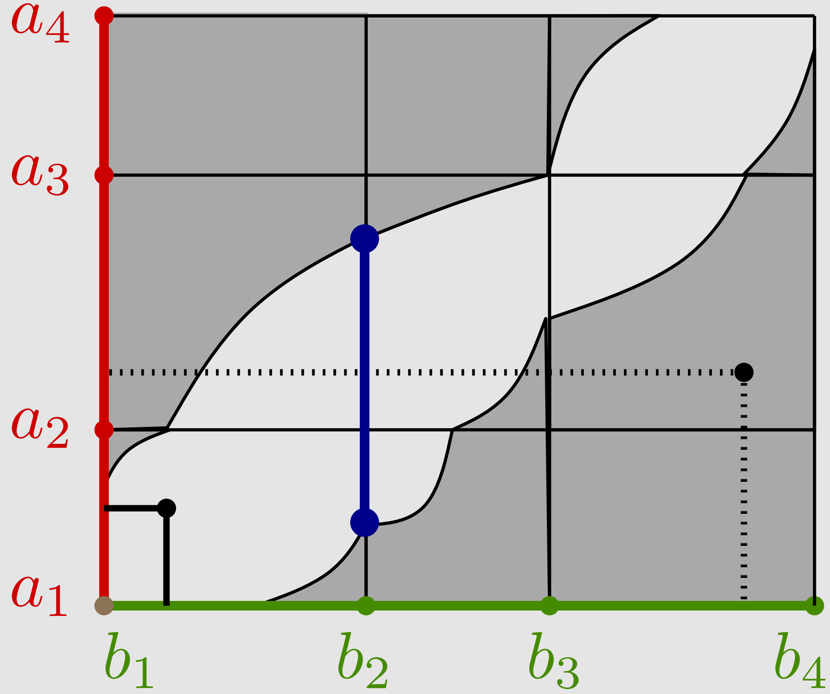


free-space diagram (for given ε)



forbidden region in diagram:
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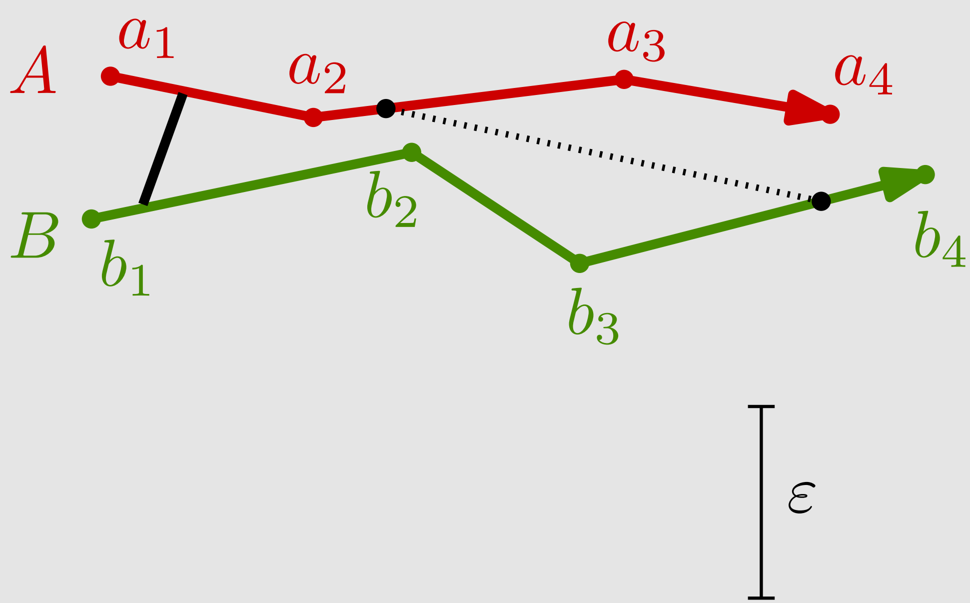
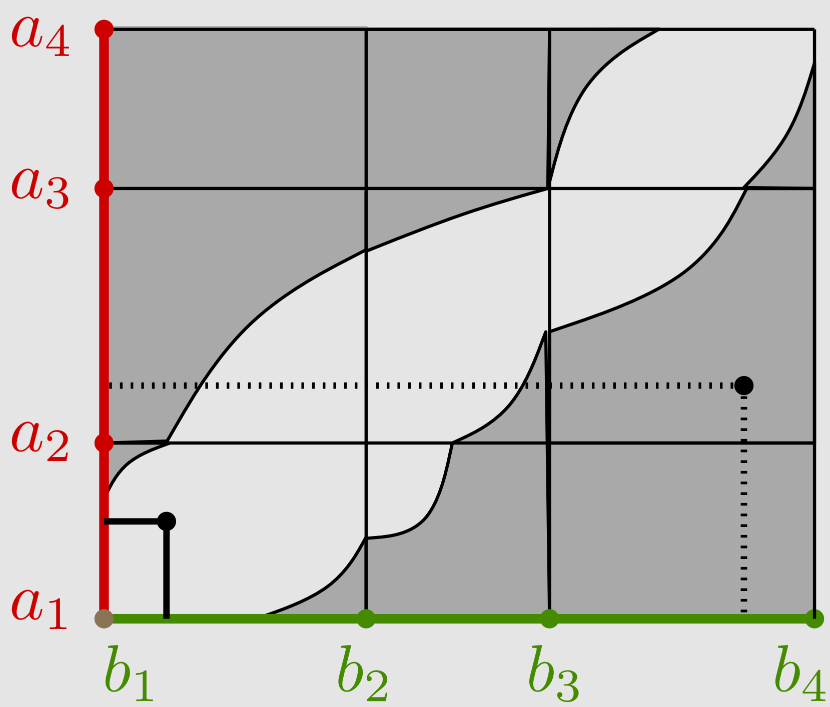


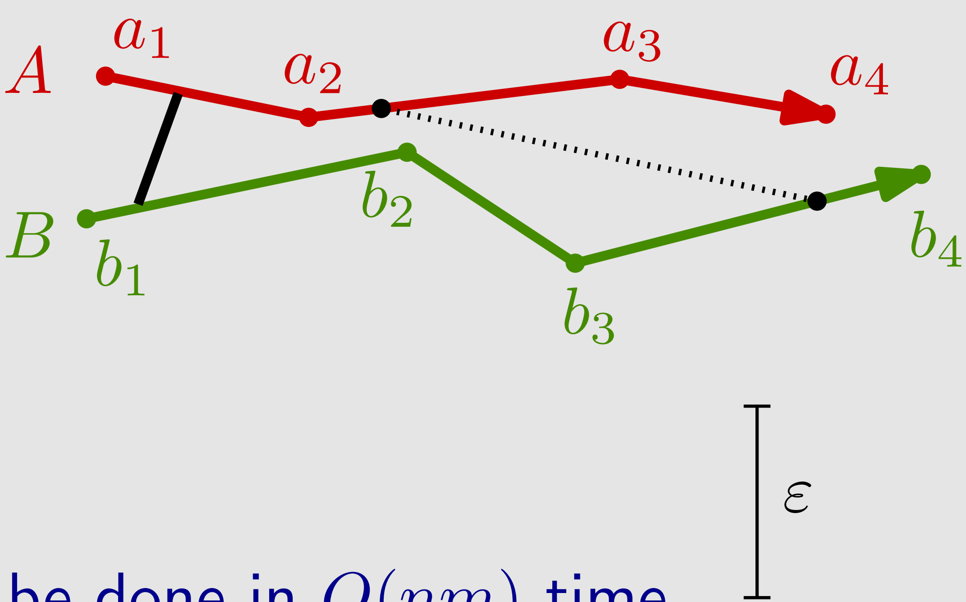
free-space diagram (for given ε)



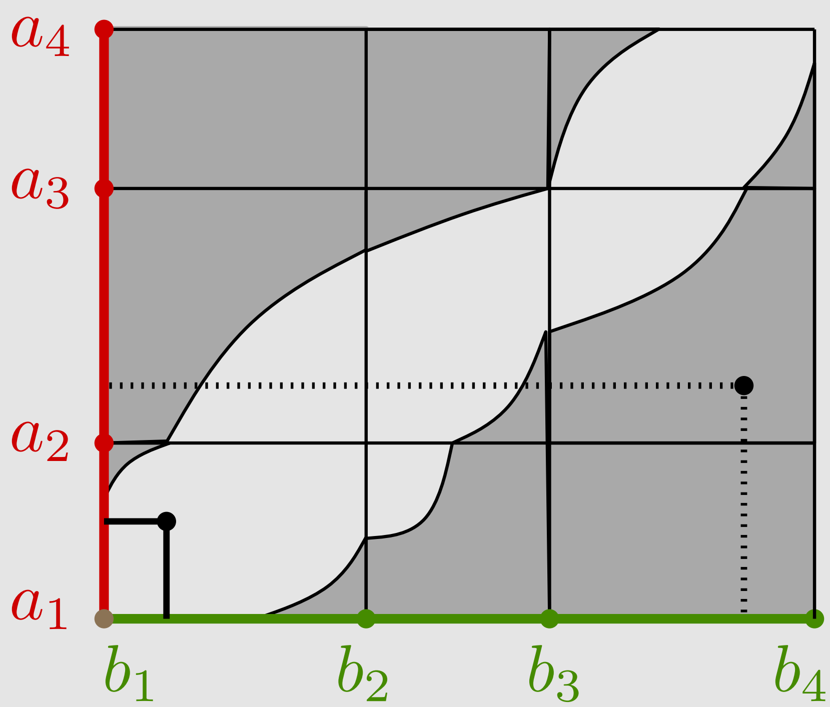
Algorithm:
Compute decomposition into free and forbidden regions, and check if monotone path exists in free region.

forbidden region in diagram:
(a, b) such that $d(a, b) > \varepsilon$

Computing similarity: deciding if Fréchet-dist$(A, B) \leq \varepsilon$



free-space diagram (for given ε)

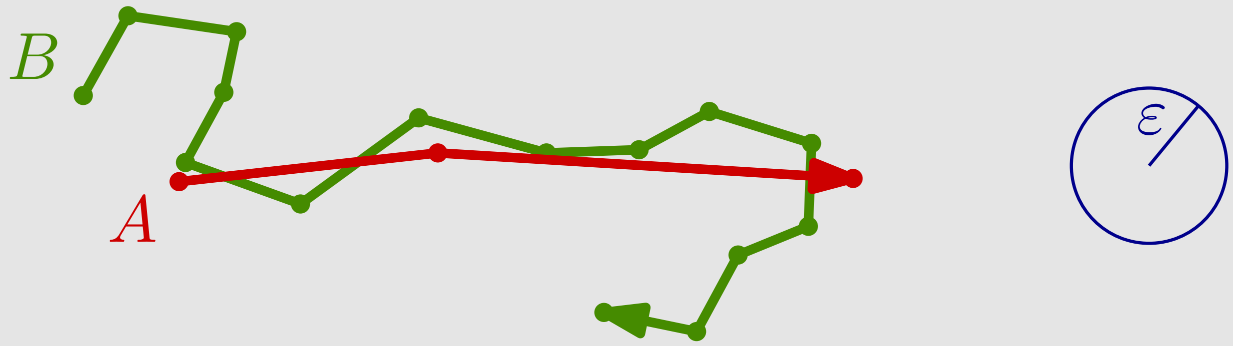


Can be done in $O(nm)$ time

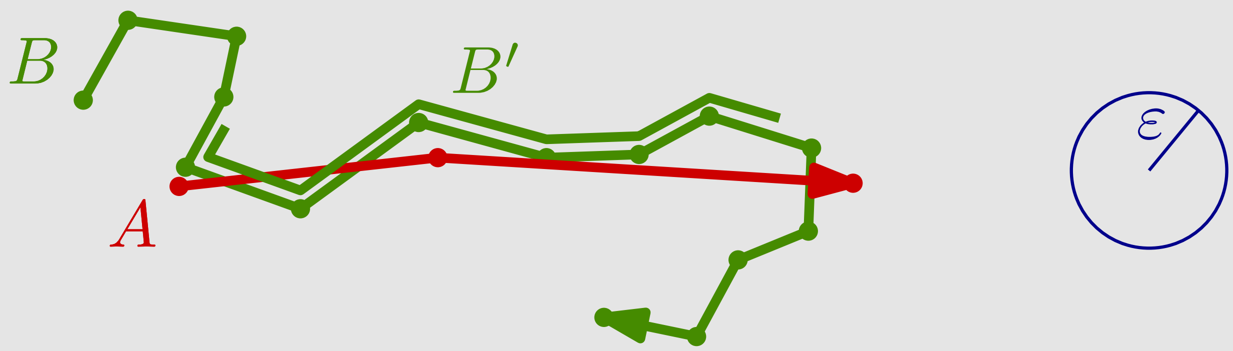
Algorithm:
Compute decomposition into free and forbidden regions, and check if monotone path exists in free region.

forbidden region in diagram:
(a, b) such that $d(a, b) > \varepsilon$

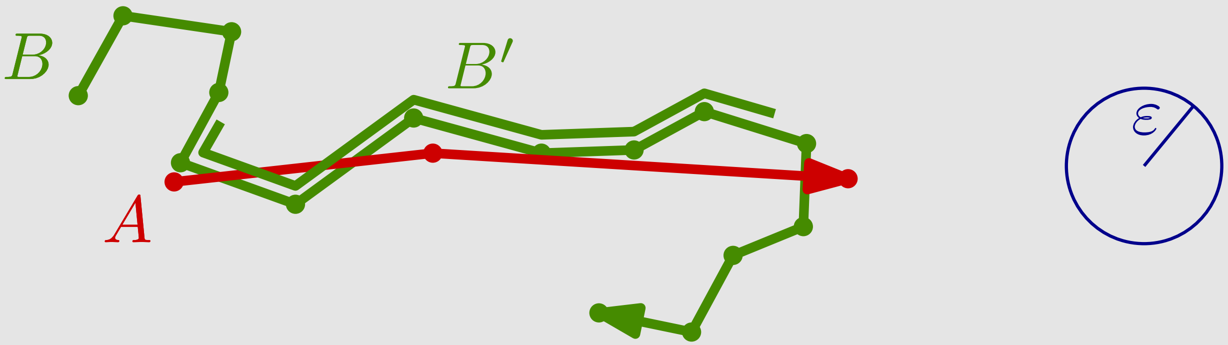
Partial matching: what if we want to decide if there is a connected subcurve of B similar to A?

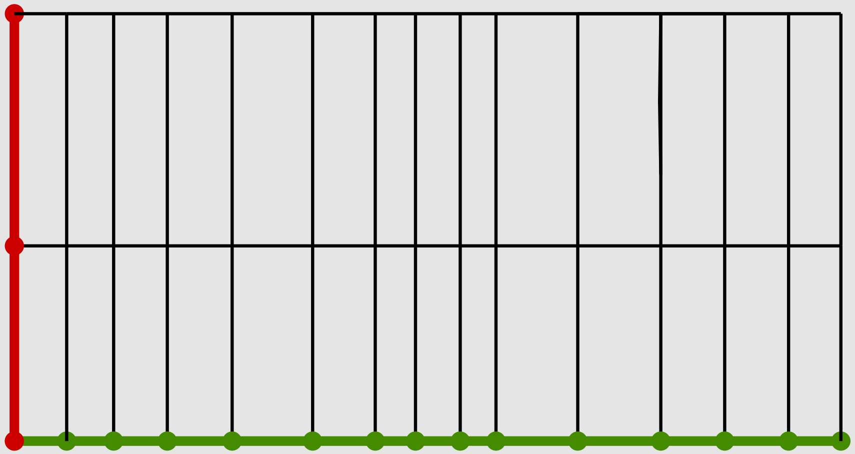


Partial matching: what if we want to decide if there is a connected subcurve of B similar to A?

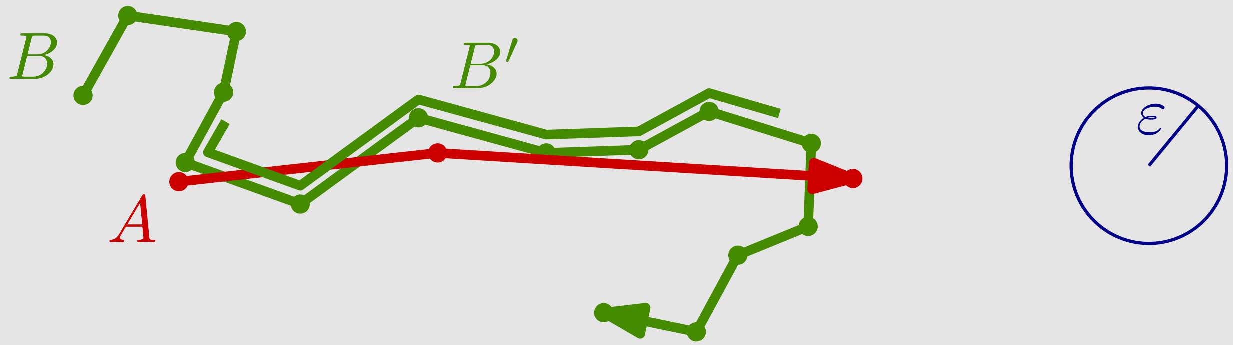


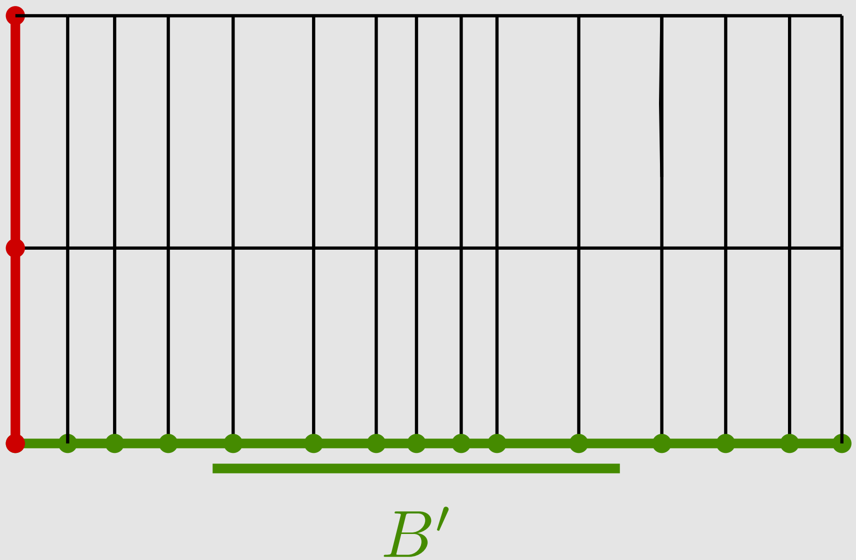
Partial matching: what if we want to decide if there is a connected subcurve of B similar to A?



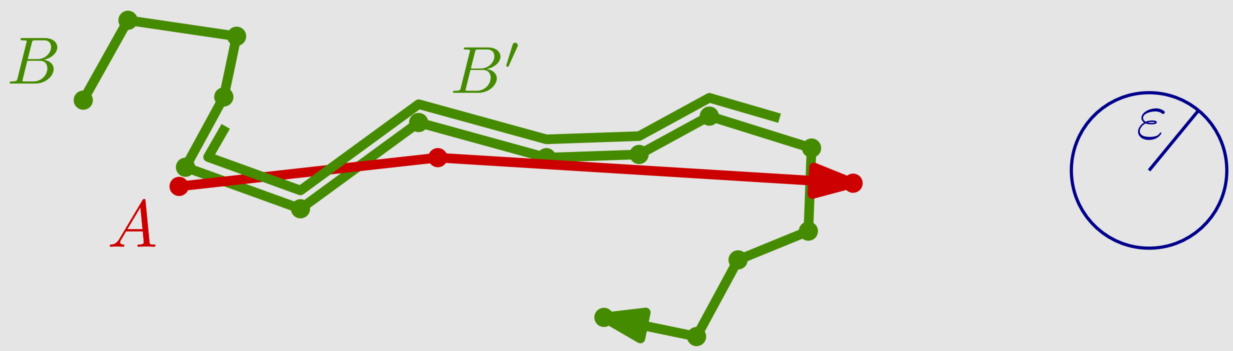


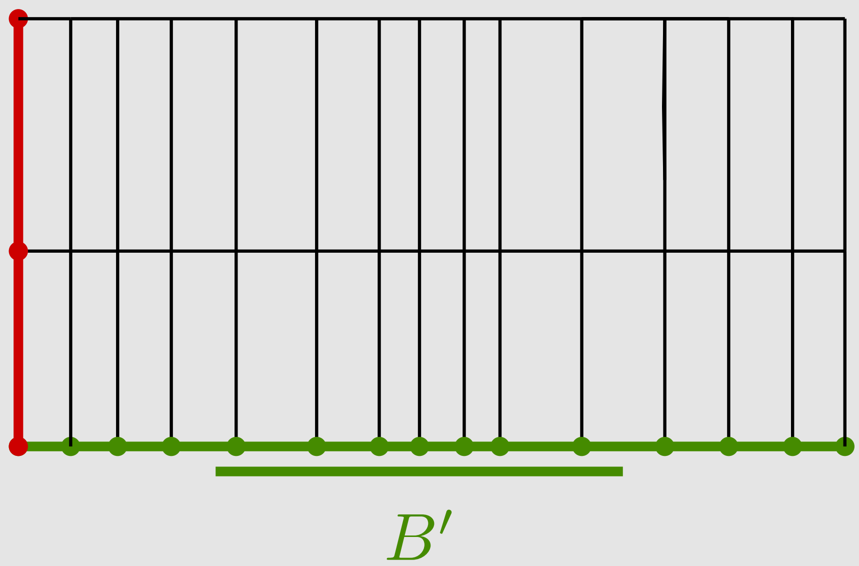
Partial matching: what if we want to decide if there is a connected **subcurve** of B similar to A?





Partial matching: what if we want to decide if there is a connected subcurve of B similar to A?

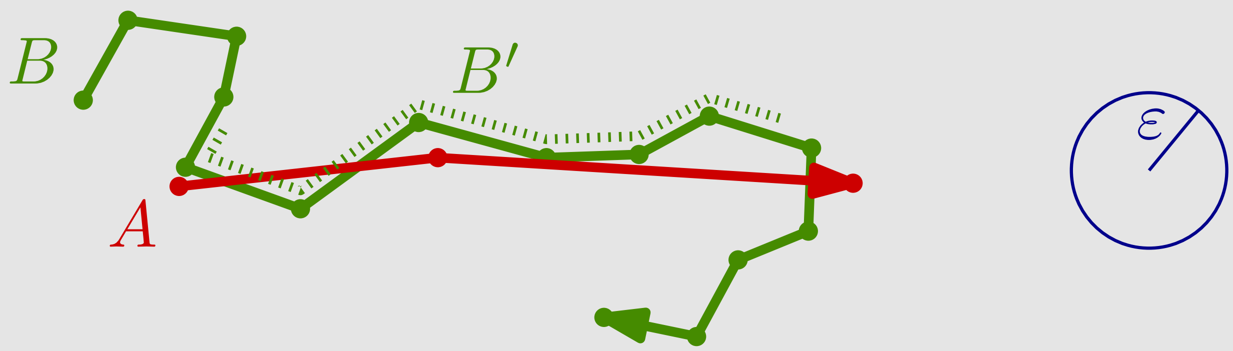


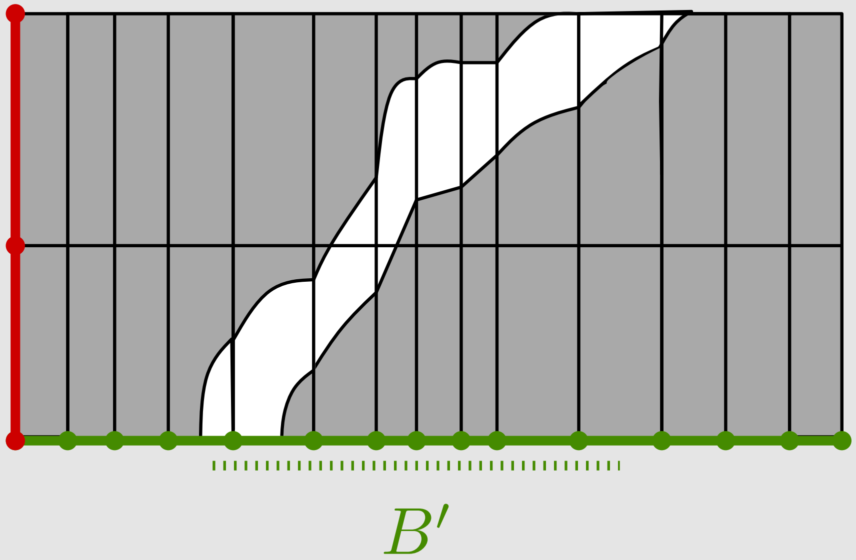


mapping from $B' \subseteq B$ to A

path from bottom to top in diagram

Partial matching: what if we want to decide if there is a connected subcurve of B that is similar to A?



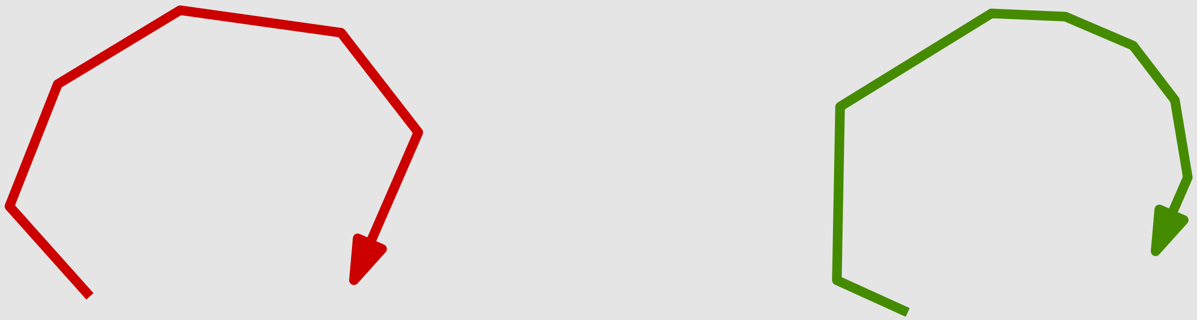


mapping from $B' \subseteq B$ to A

$\Updownarrow$

path from bottom to top in diagram

what if exact locations are not important?

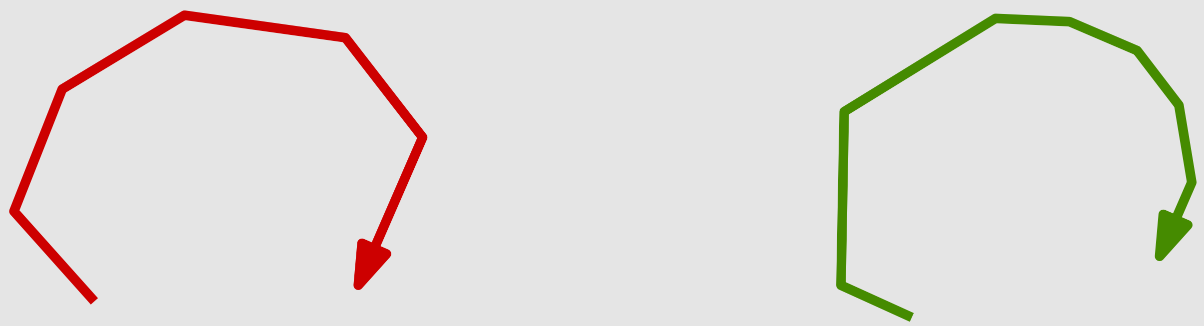


Recall definition of Fréchet distance:

$$\text{Fréchet-dist}(A, B) = \min_{\mu} \max_{a \in A} d(a, \mu(a)),$$

where min is taken over all continuous 1-to-1 mappings

what if exact locations are not important?



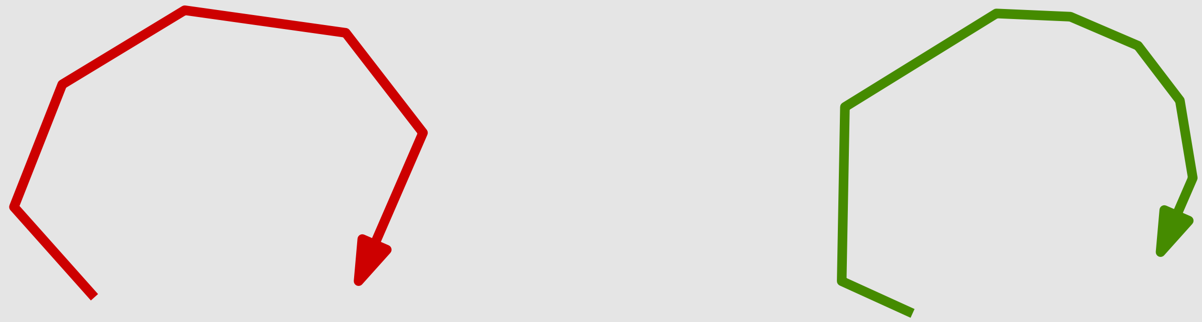
Recall definition of Fréchet distance:

$$\text{Fréchet-dist}(A, B) = \min_{\mu} \max_{a \in A} \boxed{d(a, \mu(a))},$$

where min is taken over all continuous 1-to-1 mappings

Euclidean distance

what if exact locations are not important?



Recall definition of Fréchet distance:

$$\text{Fréchet-dist}(A, B) = \min_{\mu} \max_{a \in A} d(a, \mu(a)),$$

where min is taken over all continuous 1-to-1 mappings

difference in direction of movement ~~Euclidean distance~~

Fr'echet distance: summary

- good measuring similarity of curves (e.g. trajectories)
- can be computed fairly efficiently for polygonal curves
- many variations (partial matching, directional Fréchet distance, . . .)