

Software Specification

Supplementary Exercise Hennessy-Milner Logic

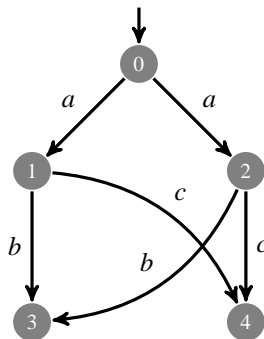
December 7, 2010

Note that it is assumed that the operators $[_]_-$ and $\langle _ \rangle_-$ bind stronger than the logical connectives \wedge and \vee .

1 Hennessy-Milner Logic

1.1 Check validity of HML formula

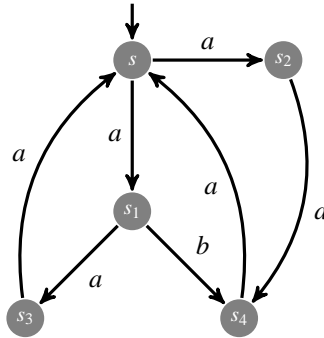
Exercise 1 Consider the following labeled transition system.



Verify whether or not the following HML formulas are satisfied by this labeled transition system:

1. $[a]([a]true \vee [a]false)$
2. $[a]([b]true \wedge [c]true)$
3. $[a]([a]false \wedge [c]true)$

Exercise 2 (Exercise 5.3.1 of Aceto) Consider the following labeled transition system.



Verify whether or not the following HML formulas are satisfied by this labeled transition system:

1. $\langle a \rangle true$
2. $\langle b \rangle true$
3. $[a] false$
4. $[b] false$
5. $[a] \langle b \rangle true$
6. $\langle a \rangle \langle b \rangle true$
7. $[a] \langle a \rangle [a] [b] false$
8. $\langle a \rangle (\langle a \rangle true \wedge \langle b \rangle true)$
9. $[a] (\langle a \rangle true \vee \langle b \rangle true)$
10. $\langle a \rangle ([b] [a] false \wedge \langle b \rangle true)$
11. $\langle a \rangle ([a] (\langle a \rangle true \wedge [b] false) \wedge \langle b \rangle false)$

Exercise 3 Consider the following labeled transition system.

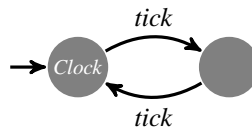


Verify whether or not the following HML formula are satisfied by this process:

1. $\langle tick \rangle true$
2. $[tick] false$
3. $[tick] (\langle tick \rangle true \wedge [tock] false)$
4. $[tick] (\langle tock \rangle true \vee [tick] false)$

(part of Exercise 5.4 of Aceto)

Consider additionally the following labeled transition system:



Is it possible to give an HML formula that is satisfied by one of these labeled transition systems and not by the other? If so, give such a formula; if not, prove this.

1.2 Give LTS for HML formula

Exercise 4 For each of the following HML formulas, if possible, give a labeled transition system that satisfies it:

1. $\langle a \rangle true \wedge [a] false$
2. $\langle a \rangle [b] (\langle a \rangle true \wedge [a] false)$
3. $\langle a \rangle [b] (\langle a \rangle true \wedge [a] false) \wedge [Act] \langle b \rangle true$
4. $\langle a \rangle [b] (\langle a \rangle true \wedge [a] false) \wedge [Act] \langle Act \rangle true$

Exercise 5 What is the interpretation of the HML formula

$$\langle Act \rangle true \wedge [Act] (\langle Act \rangle true \wedge [Act] (\langle Act \rangle true \wedge [Act \setminus \{a\}] false)) ?$$

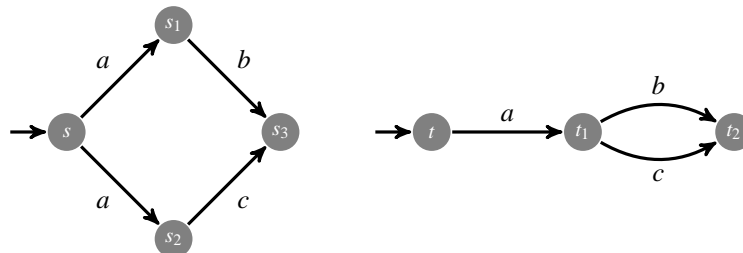
Give a labeled transition system that satisfies this formula. Also give a labeled transition system with a minimal number of states that satisfies this formula.

Exercise 6 (Exercise 5.7 of Aceto) Find one labeled transition system that satisfies all the following properties:

- $\langle a \rangle (\langle b \rangle \langle c \rangle true \wedge \langle c \rangle true)$
- $\langle a \rangle \langle b \rangle ([a] false \wedge [b] false \wedge [c] false)$
- $[a] \langle b \rangle ([c] false \wedge \langle a \rangle true)$

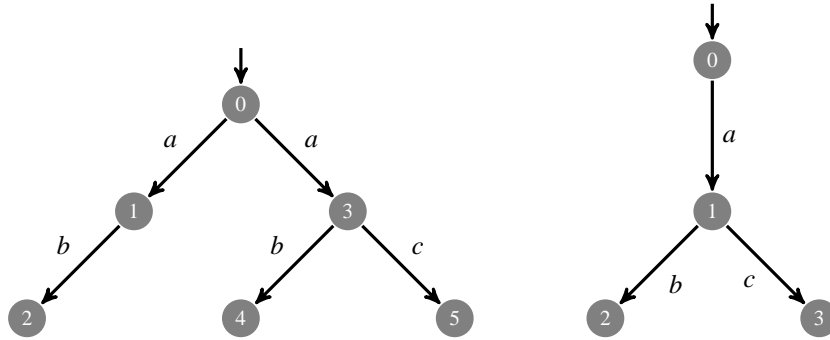
1.3 Discriminate LTSs

Exercise 7 (Exercise 5.5 of Aceto) Consider the following transition systems:

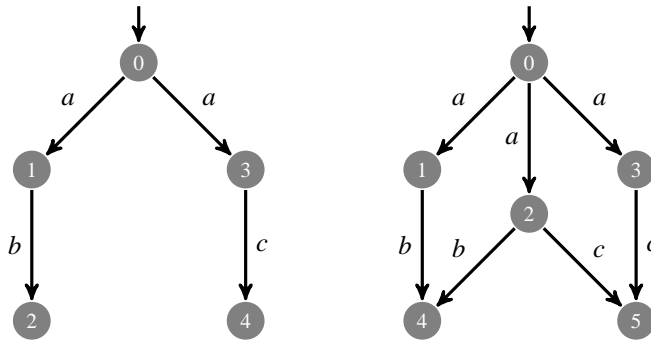


Give an HML formula that is satisfied by the labeled transition system on the left, but not by the one on the right.

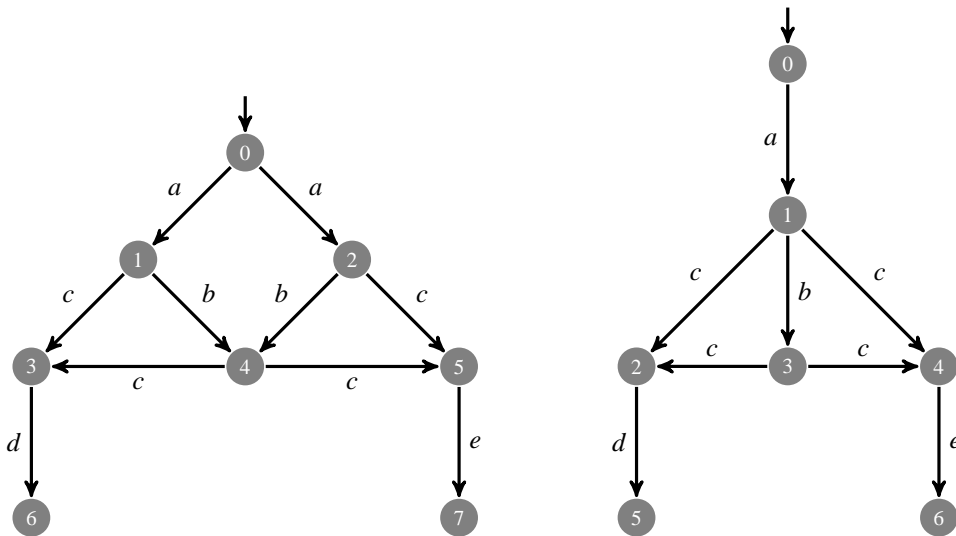
Exercise 8 Give a formula that discriminates the labeled transition systems given below.



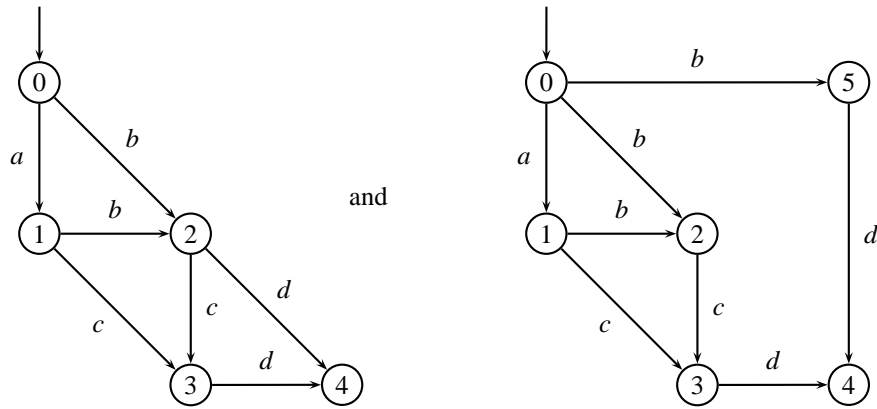
Exercise 9 Give a formula that discriminates the labeled transition systems given below.



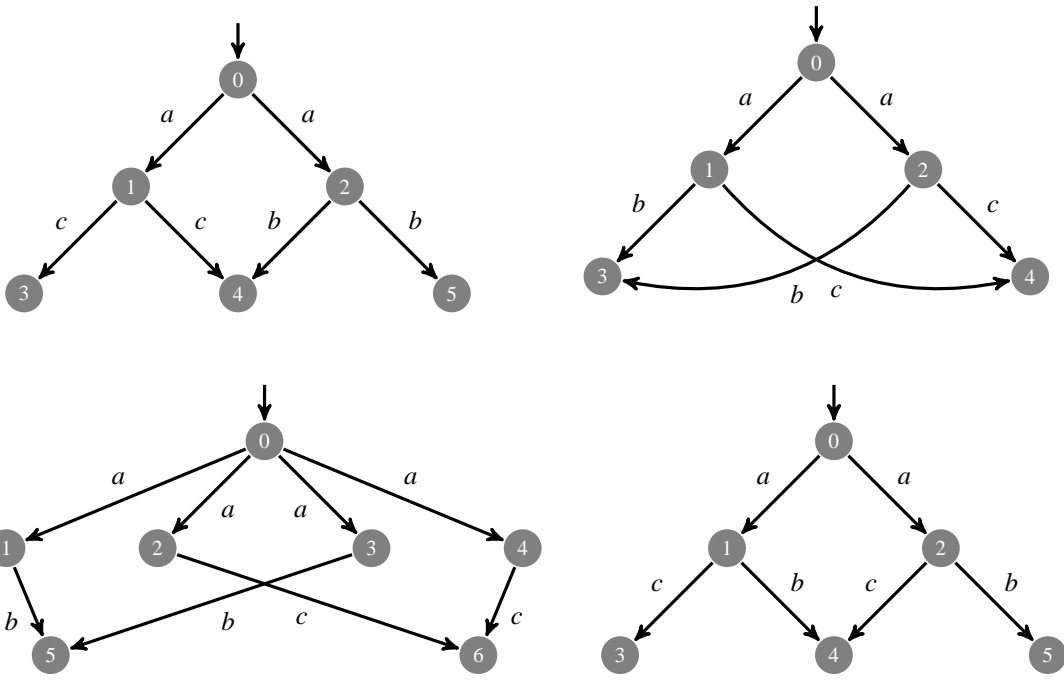
Exercise 10 Give a formula that discriminates the labeled transition systems given below.



Exercise 11 Give a formula that discriminates the labeled transition systems given below.

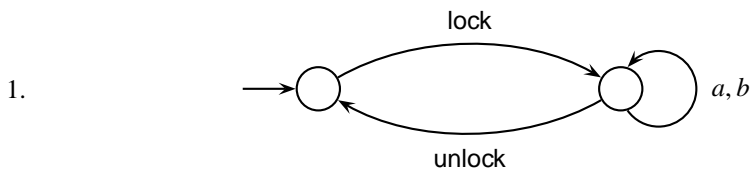


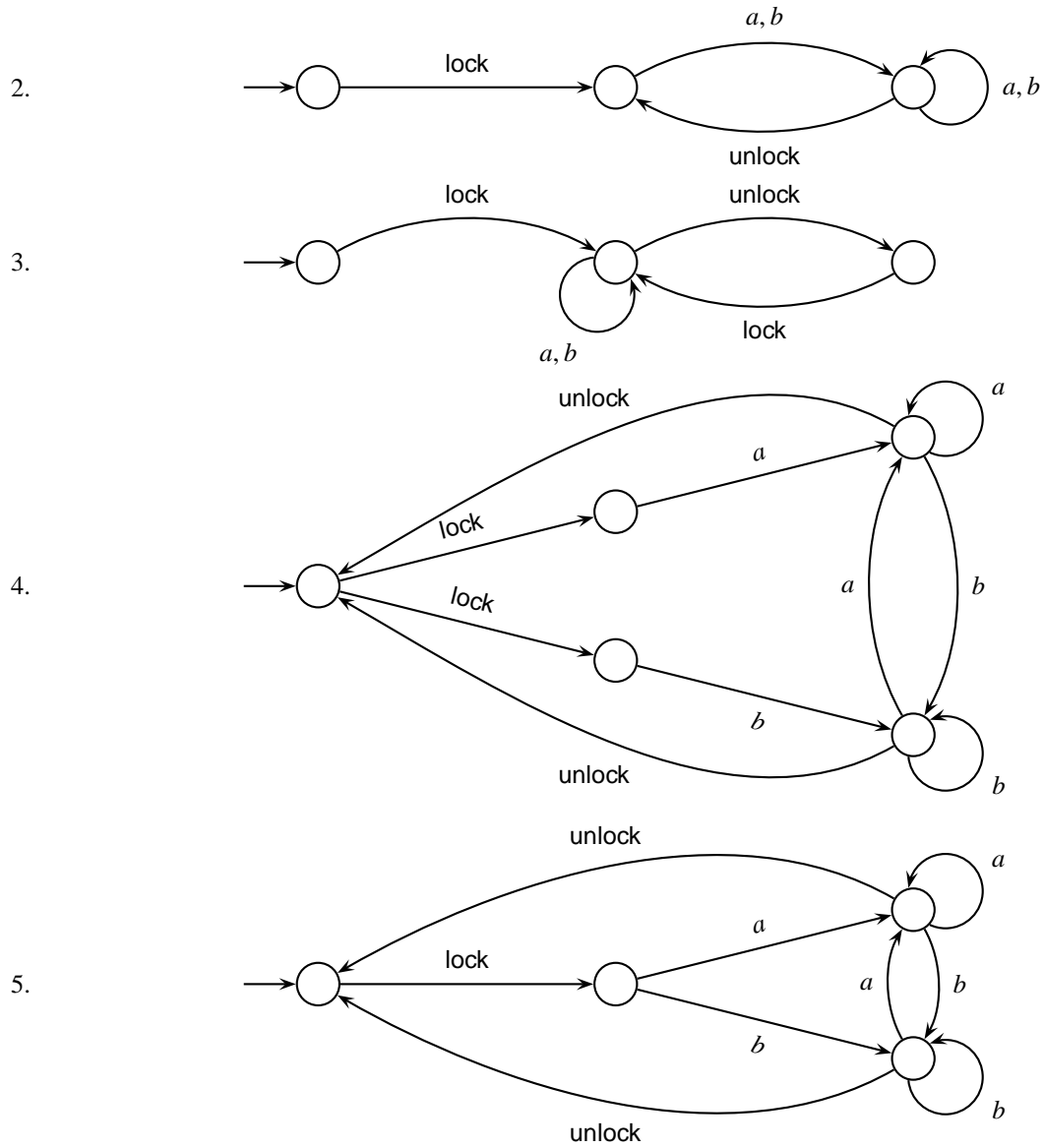
Exercise 12 Consider the labeled transition systems given below. For each pair of labeled transition systems, give an HML formula that discriminates between them, if possible.



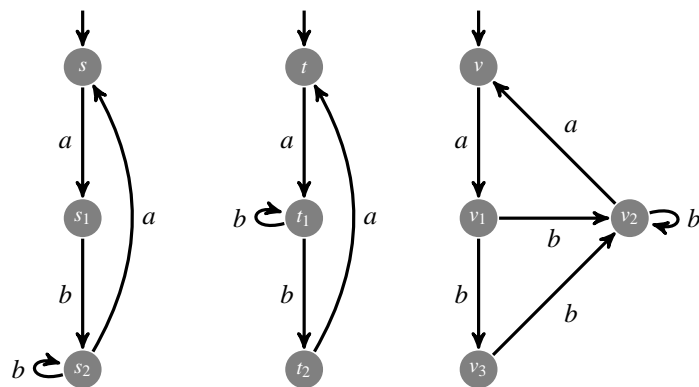
Exercise 13

Determine for any two of the following labeled transition systems whether it is possible to give a discriminating HML formula. If so, give the formula.





Exercise 14 (Exercise 5.10 of Aceto) Consider the following three labeled transition systems. Give a distinguishing HML formula for each pair of labeled transition systems.



Exercise 15 Give two labeled transition systems that cannot be discriminated with an HML formula of modal depth at most three, but for which it is possible to give an HML formula of bigger depth that discriminates them.

1.4 Formalize requirements

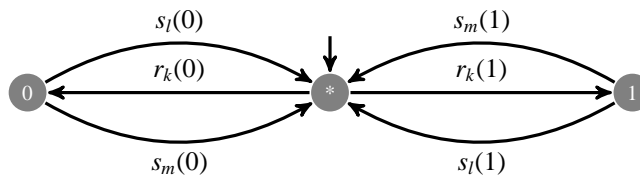
Exercise 16

1. Give an HML formula that says that an a can be done, followed by both a b and a c .
2. Give an HML formula that expresses that if an a action is possible, it cannot be followed by a b or by a c .
3. Give an HML formula that states that if an a action can be done and a b is also possible, the a cannot be followed by a b .
4. Give an HML formula that states that both actions a and b are enabled, but after any action a has been done, action b is no longer enabled.

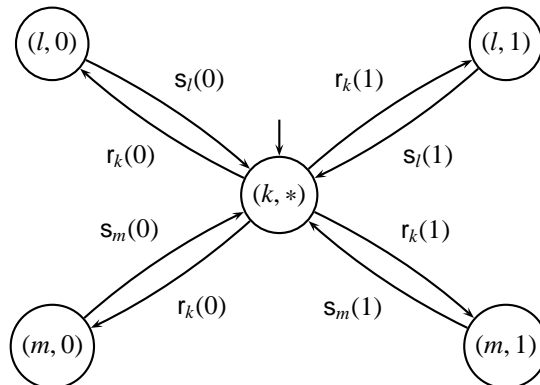
1.5 Cases

Exercise 17 We consider a split connection between nodes in a network. A split connection has one input port and two output ports. A datum that has been consumed at the input port can be delivered at either one of the output ports. That is, the choice of the output ports is resolved after the datum has been consumed.

The behaviour of a split connection with input port k and output ports l and m can be described as follows. We assume a set of data $D = \{0, 1\}$. As states of the split connection, we have \perp and the data $d \in D$, with \perp as initial state. As actions, we have $s_i(d)$ (send d at port i) and $r_i(d)$ (receive d at port i) for $i = k, l, m$ and $d \in D$. The labeled transition system for the split connection is as follows.



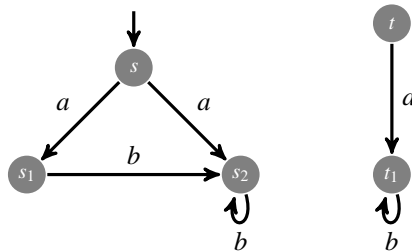
1. Give an HML formula that expresses that any datum put into the split connection via its input port, will be transferred to one of the output ports next.
2. Give an HML formula that expresses that it is impossible to have two consecutive input actions.
3. Similarly as the previous question, only this case for two consecutive output actions.
4. Give an HML formula that expresses that a datum that has been consumed can still be delivered at either of the output ports.
5. Check whether the labeled transition system satisfies the previous HML formulas.
6. Next we consider the following labeled transition system:



Establish whether the formulas in the previous items are satisfied by this labeled transition systems.

1.6 Denotational Semantics for HML

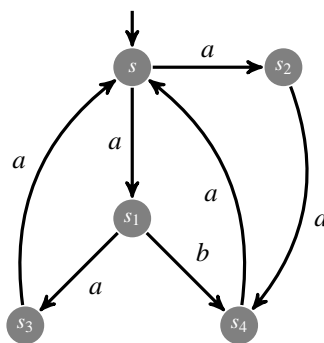
Exercise 18 (based on Exercise 5.1 of Aceto) Consider the following labeled transition system.



Compute $\langle \cdot a \cdot \rangle \{s_1, t_1\}$ and $[\cdot a \cdot] \{s_1, t_1\}$. Also compute $[\cdot b \cdot] \emptyset$ and $\langle \cdot a \cdot \rangle [\cdot b \cdot] \emptyset$. Explain the latter two in words.

Exercise 19 Show that $[\cdot a \cdot] Proc = Proc$ for any $a \in Act$.

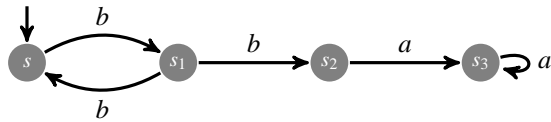
Exercise 20 (Exercise 5.3.1 of Aceto) Consider the following labeled transition system.



Compute the following sets using the alternative semantics for HML:

1. $\llbracket [a][b]false \rrbracket$
2. $\llbracket \langle a \rangle (\langle a \rangle true \wedge \langle b \rangle true) \rrbracket$
3. $\llbracket [a][a][b]false \rrbracket$
4. $\llbracket [a] (\langle a \rangle true \vee \langle b \rangle true) \rrbracket$

Exercise 21 Consider the following labeled transition system.



Determine for which states the formula $[b](\langle b \rangle [b]false \wedge \langle b \rangle [a]false)$ holds.

Exercise 22 Show that

$$T, s \models F \quad \text{iff} \quad s \in \llbracket F \rrbracket$$

for any labeled transition system T , state s (from T) and HML formula F . Hint: Use induction on the structure of formulae.