A Rule Format for Associativity

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Outline

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SOS Rule Formats

SOS

$$(||_L)\frac{x_0 \xrightarrow{l} y_0}{x_0 \mid\mid x_1 \xrightarrow{l} y_0 \mid\mid x_1} \qquad (||_R)\frac{x_1 \xrightarrow{l} y_1}{x_0 \mid\mid x_1 \xrightarrow{l} x_0 \mid\mid y_1}$$

Basic Idea

Guarantee certain properties using syntactic conditions on the shape of SOS rules.

Instances

1. (Pre-)Congruence for various behavioral equivalences (pre-orders)
2. Generating complete axiomatization
3. Many others, see `http://www.win.tue.nl/~mousavi/sos_bibl.htm`.

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Definition

Binary operator f is associative w.r.t. equivalence $\sim$ when $f(p, f(q, r)) \sim f(f(p, q), r)$, for each p, q, r.

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Find a rule format for associativity w.r.t. all behavioral equivalences!

Means

By finding a rule format for associativity w.r.t. bisimilarity / graph isomorphism.

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Challenges

Associativity proofs are substantially more complex than commutativity or congruence proofs:

1. proof structures of depth 2 are to be investigated
2. the same proof structure may not lead to the alternative proof

Example: Congruence Proof

Assume that $p_i \underline{\leftrightarrow} q_i$; proof obligation: $p_0 \mid\mid p_1 \underline{\leftrightarrow} q_0 \mid\mid q_1$

$$\frac{p_0 \xrightarrow{l} p_0'}{p_0 \mid\mid p_1 \xrightarrow{l} p_0' \mid\mid p_1}$$

$p_0 \mid\mid p_1 \xrightarrow{l} p'$

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$$p_0 \mid\mid p_1 \xrightarrow{l} p' \qquad\qquad q_0 \mid\mid q_1 \xrightarrow{l} q'$$

Challenge (Cont'd)

Example: Associativity Proof

Proof obligation: $f(p_0, f(p_1, p_2)) \leftrightarrows f(f(p_0, p_1), p_2))$

$$(||_R)\frac{(||_L)\dfrac{x_1 \xrightarrow{l} y_1}{x_1 \,||\, x_2 \xrightarrow{l} y_1 \,||\, x_2}}{x_0 \,||\, (x_1 \,||\, x_2) \xrightarrow{l} x_0 \,||\, (y_1 \,||\, x_2)} \qquad (||_L)\frac{(||_R)\dfrac{x_1 \xrightarrow{l} y_1}{x_0 \,||\, x_1 \xrightarrow{l} x_0 \,||\, y_1}}{(x_0 \,||\, x_1) \,||\, x_2 \xrightarrow{l} (x_0 \,||\, y_1) \,||\, x_2}$$

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$$p_0 \,||\, (p_1 \,||\, p_2) \xrightarrow{l} p' \qquad (p_0 \,||\, p_1) \,||\, p_2 \xrightarrow{l} q'$$

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$$(||_R)\dfrac{(||_L)\dfrac{p_1 \xrightarrow{l} p_1'}{p_1 \;||\; p_2 \xrightarrow{l} p_1' \;||\; p_2}}{p_0 \;||\; (p_1 \;||\; p_2) \xrightarrow{l} p_0 \;||\; (p_1' \;||\; p_2)} \qquad (||_L)\dfrac{(||_R)\dfrac{p_1 \xrightarrow{l} p_1}{p_0 \;||\; p_1 \xrightarrow{l} p_0 \;||\; p_1'}}{(p_0 \;||\; p_1) \;||\; p_2 \xrightarrow{l} (p_0 \;||\; p_1') \;||\; p_2}$$

$$p_0 \;||\; (p_1 \;||\; p_2) \xrightarrow{l} p' \qquad (p_0 \;||\; p_1) \;||\; p_2 \xrightarrow{l} q'$$

Challenge (Cont'd)

Our Solutions

- Restricting syntax to reduce combinatorial complexity
- Finding patterns in associativity proofs

De Simone, Simplified

1_l. Left-conforming rules

$$\frac{x \xrightarrow{l} x'}{f(x,y) \xrightarrow{l} f(x',y)}$$

2_l. Right-conforming rules

$$\frac{y \xrightarrow{l} y'}{f(x,y) \xrightarrow{l} f(x,y')}$$

3_l. Left-choice rules

$$\frac{x \xrightarrow{l} x'}{f(x,y) \xrightarrow{l} x'}$$

4_l. Right-choice rules

$$\frac{y \xrightarrow{l} y'}{f(x,y) \xrightarrow{l} y'}$$

5_l. Left-choice axioms

$$\frac{}{f(x,y) \xrightarrow{l} x}$$

6_l. Right-choice axioms

$$\frac{}{f(x,y) \xrightarrow{l} y}$$

$7_{(l_0,l_1)}$. Communicating rules

$$\frac{x \xrightarrow{l_0} x' \quad y \xrightarrow{l_1} y'}{f(x,y) \xrightarrow{\gamma(l_0,l_1)} f(x',y')}$$

Towards a Format

Finding Patterns

T_r	T_l	c_r	c_l	further req.
1_l	$1_l \cdot 1_l$	$f(x', f(y, z))$	$f(f(x', y), z)$	
$2_l \cdot 1_l$	$1_l \cdot 2_l$	$f(x, f(y', z))$	$f(f(x, y'), z)$	
$2_l \cdot 2_l$	2_l	$f(x, f(y, z'))$	$f(f(x, y), z')$	
$2_l \cdot 3_l$	$3_l \cdot 2_l$	$f(x, y')$	$f(x, y')$	
$2_l \cdot 4_l$	$7_{(l',l)} \cdot 5_{l'}$	$f(x, z')$	$f(x, z')$	$\gamma(l', l) = l$
$2_l \cdot 5_l$	5_l	$f(x, y)$	$f(x, y)$	
$2_l \cdot 6_l$	$1_l \cdot 5_l$	$f(x, z)$	$f(x, z)$	
$2_{\gamma(l,l')} \cdot 7_{(l,l')}$	$7_{(l,l')} \cdot 2_l$	$f(x, f(y', z'))$	$f(f(x, y'), z')$	
$\vdots$				

Towards a Format

Constraints

1. $5_l \Rightarrow 2_l \wedge 3_l$,
2. $6_l \Rightarrow 1_l \wedge 4_l$,
3. $7_{(l,l')} \Rightarrow$
   $(1_l \Leftrightarrow 2_{l'}) \wedge (3_l \Leftrightarrow 4_{l'})$
   $\wedge (2_l \Leftrightarrow 2_{\gamma(l,l')}) \wedge (4_l \Leftrightarrow 4_{\gamma(l,l')}) \wedge (1_{l'} \Leftrightarrow 1_{\gamma(l,l')}) \wedge (3_{l'} \Leftrightarrow 3_{\gamma(l,l')})$,
4. $1_l \wedge 3_l \Leftrightarrow \exists_{l'}\ \gamma(l,l') = l \wedge 7_{(l,l')} \wedge 5_{l'} \wedge 6_{l'}$,
5. $2_l \wedge 4_l \Leftrightarrow \exists_{l'}\ \gamma(l',l) = l \wedge 7_{(l,l')} \wedge 5_{l'} \wedge 6_{l'}$,
6. $(1_l \vee 4_l) \wedge (2_l \vee 3_l) \Rightarrow (5_l \Leftrightarrow 6_l)$.

Towards a Format

Example: CSP's External Choice

$$1_\tau.\ \frac{x \xrightarrow{\tau} x'}{x \Box y \xrightarrow{\tau} x' \Box y} \qquad 2_\tau.\ \frac{y \xrightarrow{\tau} y'}{x \Box y \xrightarrow{\tau} x \Box y'}$$

$$3_l.\ \frac{x \xrightarrow{l} x'}{x \Box y \xrightarrow{l} x'} l \neq \tau \qquad 4_l.\ \frac{y \xrightarrow{l} y'}{x \Box y \xrightarrow{l} y'} l \neq \tau$$

Towards a Format

Example: CSP's External Choice

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Example: CSP's External Choice

1_τ. 2_τ. 3_l. 4_l.

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Example: CSP's External Choice

$1_\tau.$ $2_\tau.$ $3_l.$ $4_l.$

Predicates

Example: Sequential Composition

$$\frac{x \xrightarrow{l} x'}{x \cdot y \xrightarrow{l} x' \cdot y} \qquad \frac{x \downarrow \quad y \xrightarrow{l} y'}{x \cdot y \xrightarrow{l} y'} \qquad \frac{x \downarrow \quad y \downarrow}{x \cdot y \downarrow}$$

Predicates

Coding

$\dfrac{\mathcal{P}_x}{\mathcal{P}_{f(x,y)}}$	$\dfrac{\mathcal{P}_y}{\mathcal{P}_{f(x,y)}}$	$\dfrac{\mathcal{P}_x \quad \mathcal{P}_y}{\mathcal{P}_{f(x,y)}}$
$1_{\mathcal{P}}.\dfrac{x \xrightarrow{\mathcal{P}} x'}{f(x,y) \xrightarrow{\mathcal{P}} f(x',y)}$	$2_{\mathcal{P}}.\dfrac{y \xrightarrow{\mathcal{P}} y'}{f(x,y) \xrightarrow{\mathcal{P}} f(x,y')}$	$7_{(\mathcal{P},\mathcal{P})}.\dfrac{x \xrightarrow{\mathcal{P}} x' \quad y \xrightarrow{\mathcal{P}} y'}{f(x,y) \xrightarrow{\mathcal{P}} f(x',y')}$

Testing

Example: Coding of Sequential Composition

$1_l.\ \dfrac{x \xrightarrow{l} x'}{x \cdot y \xrightarrow{l} x' \cdot y}$ $\quad 9_{(\downarrow,l)}.\ \dfrac{x \xrightarrow{\downarrow} x' \quad y \xrightarrow{l} y'}{x \cdot y \xrightarrow{l} y'}$ $\quad 7_{(\downarrow,\downarrow)}.\ \dfrac{x \xrightarrow{\downarrow} x' \quad y \xrightarrow{\downarrow} y'}{x \cdot y \xrightarrow{\gamma(\downarrow,\downarrow)} x' \cdot y'}$

Testing Rules

$8_{(l,l')}$. Left-choice + test

$$\dfrac{x \xrightarrow{l} x' \quad y \xrightarrow{l'} y'}{f(x,y) \xrightarrow{l} x'}$$

$9_{(l',l)}$. Right-choice + test

$$\dfrac{x \xrightarrow{l'} x' \quad y \xrightarrow{l} y'}{f(x,y) \xrightarrow{l} y'}$$

Testing

Example: Coding of Sequential Composition

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Extra Constraints

1. $8_{(l,l')} \wedge X_p \Rightarrow 3_l$ and $9_{(l',l)} \wedge X_p \Rightarrow 4_l$, for each $X_p \in \{1_{l'}, \ldots, 6_{l'}, 7_{(l_0,l_1)} \mid \gamma(l_0, l_1) = l' \wedge l_0 \neq l' \vee l_1 \neq l'\}$,
2. $(8_{(l,l')} \wedge 1_l) \Rightarrow \exists_{l''} \gamma(l'', l) = l \wedge 7_{(l'',l)} \wedge 5_{l''}$ and $(9_{(l',l)} \wedge 2_l) \Rightarrow \exists_{l''} \gamma(l'', l) = l \wedge 7_{(l'',l)} \wedge 5_{l''}$,
3. $8_{(l,l')} \wedge 6_l \Rightarrow 5_l$ and $9_{(l',l)} \wedge 5_l \Rightarrow 6_l$,
4. $7_{(l_0,l_1)} \Rightarrow (8_{(l_1,l')} \Leftrightarrow 8_{(\gamma(l_0,l_1),l')}) \wedge (8_{(l_0,l')} \Leftrightarrow 9_{(l',l_1)}) \wedge (9_{(l',\gamma(l_0,l_1))} \Leftrightarrow 9_{(l',l_0)})$,
5. $8_{(l,l')} \wedge 8_{(l,l'')} \Rightarrow 8_{(l',l'')} \vee (7_{(l',l'')} \wedge 8_{(l,\gamma(l',l''))})$ and $9_{(l',l)} \wedge 9_{(l'',l)} \Rightarrow 9_{(l',l'')} \vee (7_{(l',l'')} \wedge 9_{(\gamma(l',l''),l)})$.

1. $8_{(l,l')} \wedge X_p \Rightarrow 3_l$ and $9_{(l',l)} \wedge X_p \Rightarrow 4_l$, for each $X_p \in \{1_{l'}, \ldots, 6_{l'}, 7_{(l_0,l_1)} \mid \gamma(l_0,l_1) = l' \wedge l_0 \neq l' \vee l_1 \neq l'\}$,
2. $(8_{(l,l')} \wedge 1_l) \Rightarrow \exists_{l''} \gamma(l'',l) = l \wedge 7_{(l'',l)} \wedge 5_{l''}$ and $(9_{(l',l)} \wedge 2_l) \Rightarrow \exists_{l''} \gamma(l'',l) = l \wedge 7_{(l'',l)} \wedge 5_{l''}$,
3. $8_{(l,l')} \wedge 6_l \Rightarrow 5_l$ and $9_{(l',l)} \wedge 5_l \Rightarrow 6_l$,
4. $7_{(l_0,l_1)} \Rightarrow (8_{(l_1,l')} \Leftrightarrow 8_{(\gamma(l_0,l_1),l')}) \wedge (8_{(l_0,l')} \Leftrightarrow 9_{(l',l_1)}) \wedge (9_{(l',\gamma(l_0,l_1))} \Leftrightarrow 9_{(l',l_0)})$,
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$$1_l.\ \frac{x \xrightarrow{l} x'}{x \cdot y \xrightarrow{l} x' \cdot y} \qquad 9_{(\downarrow,l)}.\ \frac{x \xrightarrow{\downarrow} x' \quad y \xrightarrow{l} y'}{x \cdot y \xrightarrow{l} y'} \qquad 7_{(\downarrow,\downarrow)}.\ \frac{x \xrightarrow{\downarrow} x' \quad y \xrightarrow{\downarrow} y'}{x \cdot y \xrightarrow{\gamma(\downarrow,\downarrow)} x' \cdot y'}$$

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Changing Operators

Example: Communication Merge

$$\frac{x \xrightarrow{l} x' \quad y \xrightarrow{l'} y'}{x \mid y \xrightarrow{\gamma(l,l')} x' \parallel y'}$$

Extra Conditions

- all (f, l)-defining rules of type 7, have the same target operator $g \in \Sigma'$,
- no (f, l)-defining rule of type 1 or 2 has different source and target operators

De Simone, Simplified (with changing)

1_l. Left-conforming rules

$$\frac{x \xrightarrow{l} x'}{f(x,y) \xrightarrow{l} f(x',y)}$$

2_l. Right-conforming rules

$$\frac{y \xrightarrow{l} y'}{f(x,y) \xrightarrow{l} f(x,y')}$$

3_l. Left-choice rules

$$\frac{x \xrightarrow{l} x'}{f(x,y) \xrightarrow{l} x'}$$

4_l. Right-choice rules

$$\frac{y \xrightarrow{l} y'}{f(x,y) \xrightarrow{l} y'}$$

5_l. Left-choice axioms

$$\frac{}{f(x,y) \xrightarrow{l} x}$$

6_l. Right-choice axioms

$$\frac{}{f(x,y) \xrightarrow{l} y}$$

$7_{(l_0,l_1)}$. Communicating rules

$$\frac{x \xrightarrow{l_0} x' \quad y \xrightarrow{l_1} y'}{f(x,y) \xrightarrow{\gamma(l_0,l_1)} g(x',y')}$$

Graph Isomorphism

Extra Condition

All f-defining rules are either of types $\{1, 2, 7\}$ or $\{3, 4, 5, 6, 8, 9\}$.

Eliminating and preserving rules cannot be mixed.

Conclusions

Done

- A basic rule format guaranteeing associativity w.r.t. all equivalences including strong bisimilarity
- Extended with testing, predicates and changing operators
- Extra constraints to obtain associativity w.r.t. isomorphism (and all weaker equivalences)

To be done

- Extension to arbitrary testing and changing labels (full De Simone)
- Negative premises
- Bisimulation with data

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