Congruence for Structural Congruences

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Outline

(Strong) Bisimilarity:	an important notion of behavioral equivalence
Congruence:	Replacing equals by equals does not matter, essential for algebraic treatment of bisimulation

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Congruence:	Replacing equals by equals does not matter, essential for algebraic treatment of bisimulation

1. Can we guarantee congruence for strong bisimilarity by looking at the operational semantic specification?
2. And what if the semantic specification is a mixture of operational and equational styles?
3. When does this mixture induce a unique semantics?

Outline

Strong Bisimilarity: an important notion of behavioral equivalence

Congruence: Replacing equals by equals does not matter, essential for algebraic treatment of bisimulation

1. Can we guarantee congruence for strong bisimilarity by looking at the operational semantic specification? Yes!

2. And what if the semantic specification is a mixture of operational and equational styles? Almost yes!

3. When does this mixture induce a unique semantics? Sometimes!

Overview

→ Bisimilarity and Congruence

1. Congruence
2. Well-Definedness
3. Conclusions

Nomenclature

- Signature Σ: a set of function symbols a, $f(\_)$, $g(\_)$, $\_ \,||\, \_$, etc. with fixed arities: $ar(f)$.
- Variables $\mathcal{V} = \{x, y, \ldots\}$.
- Terms: defined inductively based on Σ and $\mathcal{V}$, denoted by $\mathcal{T} = \{t, t', \ldots\}$.
- Closed terms: mention no variables, denoted by $\mathcal{C} = \{p, q, p_i, \ldots\}$.
- Labels: $\mathcal{L} = \{l, l_i, \ldots\}$.
- Transition relation: $\rightarrow \subseteq \mathcal{C} \times \mathcal{L} \times \mathcal{C}$
  write $p \xrightarrow{l} p'$ for $(p, l, p') \in \rightarrow$.

Bisimilarity

Relation R on closed terms is a bisimulation relation iff for all $(p, q) \in R$:

1. $\forall_{l,p'} p \xrightarrow{l} p' \Rightarrow \exists_{q'} q \xrightarrow{l} q' \ \wedge \ (p', q') \in R$;
2. $\forall_{l,q'} q \xrightarrow{l} q' \Rightarrow \exists_{p'} p \xrightarrow{l} p' \ \wedge \ (p', q') \in R$.

p and q are bisimilar, denoted by $p \leftrightarroweq q$, iff there exists a bisimulation relation, relating p and q.

Congruence

Relation R on terms is a congruence iff for all function symbols f:

1. $\forall_{0 \leq i < ar(f)} \forall_{p_i, q_i} (p_i, q_i) \in R \Rightarrow$
   $(f(p_0, \ldots, p_{ar(f)-1}), f(q_0, \ldots, q_{ar(f)-1})) \in R$.

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Congruence: Operational Paradigm

$$\frac{}{a \xrightarrow{l} 0} \qquad \frac{}{b \xrightarrow{l} 0}$$

Signature $\Sigma = \{0, a, b\}$

$$\{a \xrightarrow{l} 0, b \xrightarrow{l} 0\}$$

Congruence: Operational Paradigm

$$\frac{}{a \xrightarrow{l} 0} \qquad \frac{}{b \xrightarrow{l} 0}$$

$$\frac{x_0 \xrightarrow{l} y}{x_0 \mid\mid x_1 \xrightarrow{l} y \mid\mid x_1} \qquad \frac{x_1 \xrightarrow{l} y}{x_0 \mid\mid x_1 \xrightarrow{l} x_0 \mid\mid y}$$

Signature $\Sigma = \{0, a, b, \_ \mid\mid \_\}$

$$\{a \xrightarrow{l} 0, b \xrightarrow{l} 0, a \mid\mid b \xrightarrow{l} 0 \mid\mid b, \ldots\}$$

Congruence Format: Tyft

$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\}}{f(x_0, \ldots, x_{ar(f)-1}) \xrightarrow{l} t}$$

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$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\}}{f(x_0, \ldots, x_{ar(f)-1}) \xrightarrow{l} t}$$

Theorem [Groote&Vaandrager92]:
If a specification comprises deduction rules in tyft format, then bisimilarity for its induced transition relation is a congruence.

Motivating Equational Style

$$\frac{x_0 \xrightarrow{l} y}{x_0 \mid\mid x_1 \xrightarrow{l} y \mid\mid x_1} \qquad \frac{x_1 \xrightarrow{l} y}{x_0 \mid\mid x_1 \xrightarrow{l} x_0 \mid\mid y}$$

Motivating Equational Style

$$\frac{x_0 \xrightarrow{l} y}{x_0 \parallel x_1 \xrightarrow{l} y \parallel x_1} \qquad x_1 \parallel x_0 \equiv x_0 \parallel x_1$$

Connecting the Two Paradigms

$$\frac{x_0 \xrightarrow{l} y}{x_0 \,||\, x_1 \xrightarrow{l} y \,||\, x_1}$$

$$x_1 \,||\, x_0 \equiv x_0 \,||\, x_1$$

$$\frac{x \equiv x' \quad x' \xrightarrow{l} y' \quad y' \equiv y}{x \xrightarrow{l} y}$$

Congruence for the Mix

$$\frac{}{a \xrightarrow{l_0} a} \qquad \frac{}{b \xrightarrow{l_0} b}$$

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$$a \equiv f(b)$$

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$$\frac{}{a \xrightarrow{l_0} a} \qquad \frac{}{b \xrightarrow{l_0} b}$$

$$a \equiv f(b) \qquad \frac{x \equiv x' \quad x' \xrightarrow{l} y' \quad y' \equiv y}{x \xrightarrow{l} y}$$

$$f(b) \xrightarrow{l_0} a$$

$$f(a) \stackrel{l_0}{\not\rightarrow}$$

Congruence Format: CFSC

$f(\overrightarrow{x}) \equiv g(\overrightarrow{y})$

$f(\overrightarrow{x}) \equiv t$ where f is a fresh function symbol.

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$f(\overrightarrow{x}) \equiv g(\overrightarrow{y})$
$f(\overrightarrow{x}) \equiv t$ if f is a fresh function symbol.

Theorem [Mousavi&Reniers04]:
If a semantic specification consists of
deduction rules in tyft format,
and structural congruences in CFSC
then bisimilarity with respect to its induced transition relation
is a congruence.

Motivating Equational Style (revisited)

$$\frac{x_0 \xrightarrow{l} y}{x_0 \,||\, x_1 \xrightarrow{l} y \,||\, x_1}$$

$$x_1 \,||\, x_0 \equiv x_0 \,||\, x_1$$

$$\frac{x \equiv x' \quad x' \xrightarrow{l} y' \quad y' \equiv y}{x \xrightarrow{l} y}$$

Motivating Equational Style (revisited)

$$\frac{x_0 \xrightarrow{l} y}{x_0 \,||\, x_1 \xrightarrow{l} y \,||\, x_1}$$

$$x_1 \,||\, x_0 \equiv x_0 \,||\, x_1 \qquad !x \equiv x \,||\, !x$$

$$\frac{x \equiv x' \quad x' \xrightarrow{l} y' \quad y' \equiv y}{x \xrightarrow{l} y}$$

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Congruence Formats: NTyft

$$\frac{\{t_i \xrightarrow{l_i} y_i \mid i \in I\} \quad \{t_j \stackrel{l_j}{\not\rightarrow} \mid j \in J\}}{f(x_0, \ldots, x_{ar(f)-1}) \xrightarrow{l} t}$$

Induced Transition Relation?

$$\frac{b \stackrel{l_0}{\not\rightarrow}}{a \xrightarrow{l_0} a} \qquad \frac{a \stackrel{l_0}{\not\rightarrow}}{b \xrightarrow{l_0} b}$$

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$$\frac{b \stackrel{l_0}{\not\rightarrow}}{a \xrightarrow{l_0} a} \qquad \frac{a \stackrel{l_0}{\not\rightarrow}}{b \xrightarrow{l_0} b}$$

$$\{a \xrightarrow{l_0} a\}? \quad or \quad \{b \xrightarrow{l_0} b\}?$$

Stratification [Groote93] (simplified)

A function $\mathcal{S}$ is a stratification for a set of rules in ntyft format if for all rules:

$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\} \quad \{t_j \xrightarrow{l_j}\!\!\!\!\!/ \; | j \in J\}}{f(\overrightarrow{x}) \xrightarrow{l} t}$$

and for all closed substitutions σ:

1. $\forall_{i \in I} \mathcal{S}(\sigma(f(\overrightarrow{x}))) \leq \mathcal{S}(\sigma(t_i))$
2. $\forall_{j \in I} \mathcal{S}(\sigma(f(\overrightarrow{x}))) < \mathcal{S}(\sigma(t_j))$

Stratification

Theorem [Bol&Groote96]:

If a set of deduction rules is in ntyft format

and it is stratified

then it has a unique stable model

and for this model, bisimilarity is a congruence.

Well-definedness for the Mix

$$\frac{b \stackrel{l_0}{\not\longrightarrow}}{a \stackrel{l_0}{\longrightarrow} a}$$

Well-definedness for the Mix

$$\frac{b \stackrel{l_0}{\not\to}}{a \stackrel{l_0}{\longrightarrow} a}$$

$$a \equiv b \qquad \frac{x \equiv x' \quad x' \stackrel{l}{\longrightarrow} y' \quad y' \equiv y}{x \stackrel{l}{\longrightarrow} y}$$

Stratification (extended and simplified)

A combination of rules in ntyft format and SCs in CFSC format is stratified if rules are stratified by function $\mathcal{S}$ and

1. for all equations of the form $f(\overrightarrow{x}) \equiv g(\overrightarrow{y})$
   $\mathcal{S}(\sigma(f(\overrightarrow{x}))) = \mathcal{S}(\sigma(g(\overrightarrow{y})))$
2. for all equations of the form $f(\overrightarrow{x}) \equiv t$ then
   $\mathcal{S}(\sigma(t)) \leq \mathcal{S}(\sigma(f(\overrightarrow{x})))$

Stratification (extended and simplified)

Theorem [Mousavi&Reniers04]:

If a combination of a rules and SCs is stratified then

it has a unique stable model

and bisimilarity with respect to this model is a congruence.

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Conclusion

Done:

- Structural congruences may harm well-definedness of the semantics and congruence of strong bisimilarity, if not used with care;
- Syntactic criteria for making them safe were proposed;
- Conservative (operational and equational) extensions of semantics with structural congruences were studied (not presented).

To be done:

- Studying congruence for other notions of bisimilarity;
- Investigating structural congruences in the categorical model of [Turi&Plotkin97].

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Thank You!