# Towards SOS for Higher Order Processes

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# Overview

0. Introducing Myself
1. SOS Frameworks for Processes: An Overview
2. Congruence for Strong Bisimilarity
3. Congruence for Higher Order Bisimilarity
4. Conclusions

# Wondering How to Pigeonhole Me?

| Degree | Subject | Date |
|---|---|---|
| Born | Mohammad Mousavi | Jul. 1978 |
| B.Sc. | | Sep. 1999 |
| M.Sc. | Nondeterminism in CZ | Sep. 2001 |
| Ph.D. | Structuring SOS | Sep. 26th 2005 |

# I Have Done Work in:

Formal Semantics of Different Languages:

0. Additive extension of GAMMA and its coordination language with timing [ACSD'03] and timing and distribution [HLDVT'02]
1. Incremental extension of an ACP-style real-time process algebra with scheduling constructs [FORMATS'03]
3. Operational Semantics of Reo [CSR-04-015] (Eindhoven Tech. Rep.)
4. Asynchronous Designs in Synchronous Languages [DATE'04]

But then I got into...

# Work in Structural Operational Semantics

0. Notions of Bisimulation and Congruence Formats for Systems with Data [LICS'04,I&C'05]
1. Commutativity Format [IPL'05]
2. Structural Congruences and SOS [FOSSACS'05]
3. Orthogonality of Language Extensions [ICALP'05] (to appear)
4. SOS for Higher Order Processes [?'05]
5. Nominal SOS (ongoing)
6. SOS for Language-Based Security (ongoing)

N.B. Most of the above are joint works with Michel Reniers and some with Jan Friso Groote and Jamie Gabbay.

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# SOS Frameworks

A (single- or multi-sorted) signature $\Sigma$

A fixed set of labels $\mathcal{L}$

A set of deduction rules $D$:

$$\frac{\{t_i \xrightarrow{l_i}_{r_i} t_i' | i \in I\}}{t \xrightarrow{l}_{r} t'}$$

# SOS Frameworks (Congruence Formats)

de Simone

$$\frac{\{x_i \xrightarrow{l_i} y_i | 0 \leq i < ar(f)\}}{f(\overrightarrow{x}) \xrightarrow{l} t'}$$

# SOS Frameworks (Congruence Formats)

$$\frac{x \xrightarrow{l} y \quad \forall_{l' > l}\, x \stackrel{l'}{\nrightarrow}}{\theta(x) \xrightarrow{l} \theta(y)}$$

de Simone

$$\frac{\{x_i \xrightarrow{l_i} y_i \mid 0 \leq i < ar(f)\}}{f(\overrightarrow{x}) \xrightarrow{l} t'}$$

# SOS Frameworks (Congruence Formats)

GSOS

$$\frac{\{x_i \xrightarrow{l_i} y_i | i \in I\} \cup \{x_j \overset{l_j}{\nrightarrow} \ | \ j \in J\}}{f(\overrightarrow{x}) \xrightarrow{l} C[x_k, y_i]}$$

de Simone

$$\frac{\{x_i \xrightarrow{l_i} y_i | 0 \leq i < ar(f)\}}{f(\overrightarrow{x}) \xrightarrow{l} t'}$$

# SOS Frameworks (Congruence Formats)

GSOS

$$\frac{\{x_i \xrightarrow{l_i} y_i | i \in I\} \cup \{x_j \overset{l_j}{\nrightarrow} \mid j \in J\}}{f(\overrightarrow{x}) \xrightarrow{l} C[x_k, y_i]}$$

$$\frac{t \xrightarrow{l} y}{X \xrightarrow{l} y} \quad X \doteq t$$

↖

de Simone

$$\frac{\{x_i \xrightarrow{l_i} y_i | 0 \leq i < ar(f)\}}{f(\overrightarrow{x}) \xrightarrow{l} t'}$$

# SOS Frameworks (Congruence Formats)

GSOS

$$\frac{\{x_i \xrightarrow{l_i} y_i | i \in I\} \cup \{x_j \stackrel{l_j}{\not\rightarrow} \mid j \in J\}}{f(\overrightarrow{x}) \xrightarrow{l} C[x_k, y_i]}$$

$$\frac{x \xrightarrow{\tau} y \quad y \xrightarrow{l} z}{x \xrightarrow{l} z}$$

↖

de Simone

$$\frac{\{x_i \xrightarrow{l_i} y_i | 0 \leq i < ar(f)\}}{f(\overrightarrow{x}) \xrightarrow{l} t'}$$

# SOS Frameworks (Congruence Formats)

GSOS

$$\frac{\{x_i \xrightarrow{l_i} y_i | i \in I\} \cup \{x_j \stackrel{l_j}{\not\rightarrow} \ | \ j \in J\}}{f(\overrightarrow{x}) \xrightarrow{l} C[x_k, y_i]}$$

Tyft/Tyxt

$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\}}{f(\overrightarrow{x}) \ / \ x \xrightarrow{l} t}$$

de Simone

$$\frac{\{x_i \xrightarrow{l_i} y_i | 0 \leq i < ar(f)\}}{f(\overrightarrow{x}) \xrightarrow{l} t'}$$

# SOS Frameworks (Congruence Formats)

NTyft (PANTH, NTree)

$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\} \cup \{t_j \stackrel{l_j}{\nrightarrow} \ | \ j \in J\}}{f(\overrightarrow{x}) \xrightarrow{l} t}$$

GSOS

$$\frac{\{x_i \xrightarrow{l_i} y_i | i \in I\} \cup \{x_j \stackrel{l_j}{\nrightarrow} \ | \ j \in J\}}{f(\overrightarrow{x}) \xrightarrow{l} C[x_k, y_i]}$$

Tyft

$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\}}{f(\overrightarrow{x}) \xrightarrow{l} t}$$

de Simone

$$\frac{\{x_i \xrightarrow{l_i} y_i | 0 \leq i < ar(f)\}}{f(\overrightarrow{x}) \xrightarrow{l} t'}$$

# SOS Frameworks (Congruence Formats)

NTyft (PANTH, NTree)

$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\} \cup \{t_j \overset{l_j}{\not\rightarrow} \mid j \in J\}}{f(\overrightarrow{x}) \xrightarrow{l} t}$$

$$\frac{}{c!p.x \xrightarrow{p} x}$$

Tyft

$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\}}{f(\overrightarrow{x}) \xrightarrow{l} t}$$

# SOS Frameworks (Congruence Formats)

NTyft (PANTH, NTree)

$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\} \cup \{t_j \stackrel{l_j}{\nrightarrow} \mid j \in J\}}{f(\overrightarrow{x}) \xrightarrow{l} t}$$

Promoted Tyft

$$\frac{\{t_i \xrightarrow{g_i(\overrightarrow{t_i})} y_i | i \in I\}}{f(\overrightarrow{x}) \xrightarrow{g(\overrightarrow{z})} t}$$

Tyft

$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\}}{f(\overrightarrow{x}) \xrightarrow{l} t}$$

# SOS Frameworks (Congruence Formats)

Promoted PANTH

$$\frac{\{t_i \xrightarrow{\overrightarrow{t_i''}} y_i | i \in I\} \cup \{t_j \stackrel{\overrightarrow{t_j''}}{\nrightarrow} \mid j \in J\}}{f(\overrightarrow{x}) \xrightarrow{\overrightarrow{t''}} t'}$$

↗ ↖

NTyft (PANTH, NTree)

$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\} \cup \{t_j \stackrel{l_j}{\nrightarrow} \mid j \in J\}}{f(\overrightarrow{x}) \xrightarrow{l} t}$$

Promoted Tyft

$$\frac{\{t_i \xrightarrow{g_i(\overrightarrow{t_i''})} y_i | i \in I\}}{f(\overrightarrow{x}) \xrightarrow{g(\overrightarrow{z})} t}$$

↖ ↗

Tyft

$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\}}{f(\overrightarrow{x}) \xrightarrow{l} t}$$

# SOS Frameworks (Congruence Formats)

Henceforth, I will *not* talk about:

1. Predicates (syntactic sugar, no technicality);
2. Negative premises (complicating factor but can be dealt with);
3. A variable as the source of conclusion (Tyxt, syntactic sugar);
4. Lists of terms as labels (may be helpful, no technicality).

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# Nomenclature

- Signature $\Sigma$: a set of function symbols $a$, $f(\_)$, $g(\_)$, $\_ \,||\, \_$, etc. with fixed arities: $ar(f)$.
- Variables $\mathcal{V} = \{x, y, \ldots\}$.
- Terms: defined inductively based on $\Sigma$ and $\mathcal{V}$, denoted by $\mathcal{T} = \{t, t', \ldots\}$.
- Closed terms: mention no variables, denoted by $\mathcal{C} = \{p, q, p_i, \ldots\}$.
- Transition relation: $\rightarrow \; \subseteq \mathcal{C} \times \mathcal{C} \times \mathcal{C}$
  write $p \xrightarrow{p''} p'$ for $(p, p'', p') \in \; \rightarrow$.

# Strong Bisimilarity

Relation $R$ on closed terms is a strong bisimulation relation iff for all $(p, q) \in R$:

1. $\forall_{p',p''} p \xrightarrow{p''} p' \Rightarrow \exists_{q'} q \xrightarrow{p''} q' \ \wedge \ (p', q') \in R$;
2. $\forall_{q',q''} q \xrightarrow{q''} q' \Rightarrow \exists_{p'} p \xrightarrow{q''} p' \ \wedge \ (p', q') \in R$.

$p$ and $q$ are strongly bisimilar, denoted by $p \underline{\leftrightarrow} q$, iff there exists a strong bisimulation relation, relating $p$ and $q$.

# Congruence

Relation $R$ on terms is a congruence iff for all function symbols $f$:

1. $\forall_{0 \leq i < ar(f)} \forall_{p_i, q_i} (p_i, q_i) \in R \Rightarrow$
   $(f(p_0, \ldots, p_{ar(f)-1}), f(q_0, \ldots, q_{ar(f)-1})) \in R$.

# Promoted PANTH: Starting Point

$$\frac{\{t_i \xrightarrow{t_i''}_{r_i} y_i \mid i \in I\}}{f(\overrightarrow{x}) \xrightarrow{t''}_{r} t'}$$

# First Motivating Examples

$$\frac{}{a \xrightarrow{a} a} \qquad \frac{}{b \xrightarrow{a} b}$$

$$\frac{}{f(x) \xrightarrow{x} y}$$

# First Motivating Examples

$$\frac{}{a \xrightarrow{a} a} \qquad \frac{}{b \xrightarrow{a} b}$$

$$\frac{}{f(x) \xrightarrow{x} y}$$

$a \,\underline{\leftrightarrow}\, b$ but $f(a) \,\underline{\not\leftrightarrow}\, f(b)$:

$f(a) \xrightarrow{a} a$ but $f(b) \stackrel{a}{\not\rightarrow}$

# First Step Towards Promoted PANTH

$$\frac{}{a \xrightarrow{l} \epsilon} \qquad \frac{}{b \xrightarrow{l} \epsilon}$$

$$\frac{}{f(x) \xrightarrow{x} y}$$

$$\frac{\{t_i \xrightarrow{t''_i}_{r_i} y_i | i \in I\}}{f(\overrightarrow{x}) \xrightarrow{t''}_{r} t'}$$

$$vars(f(\overrightarrow{x})) \cap vars(t'') = \emptyset$$

# Second Motivating Example

$$\frac{}{a \xrightarrow{g(a)} \epsilon} \qquad \frac{}{b \xrightarrow{g(a)} \epsilon}$$

$$\frac{x \xrightarrow{g(x)} y}{f(x) \xrightarrow{g(a)} y}$$

# Second Motivating Example

$$\frac{}{a \xrightarrow{g(a)} \epsilon} \qquad \frac{}{b \xrightarrow{g(a)} \epsilon}$$

$$\frac{x \xrightarrow{g(x)} y}{f(x) \xrightarrow{g(a)} y}$$

$a \underline{\leftrightarrow} b$ but $f(a) \not\underline{\leftrightarrow} f(b)$
since $f(a) \xrightarrow{g(a)} a$ but $f(b) \stackrel{g(a)}{\not\rightarrow}$.

# Second (Last) Restriction

$$\frac{}{a \xrightarrow{g(a)} \epsilon} \qquad \frac{}{b \xrightarrow{g(a)} \epsilon}$$

$$\frac{x \xrightarrow{g(x)} y}{f(x) \xrightarrow{g(a)} y}$$

"Such $g$ operators",
if are in the label of conclusion of any rule,
appear in the form $g(\overrightarrow{z})$ ...

# Example: Modified CHOCS!

$$\Sigma = \{0,\ a, \tau.\_, c!p.\_, c?a.\_, \_ + \_, \ldots\}$$

# Example: Modified CHOCS!

$$\frac{}{a \xrightarrow{z}_{/a} z} \qquad \frac{}{b \xrightarrow{z}_{/a} b} a \neq b \qquad \frac{x_0 \xrightarrow{z}_{/a} y_0 \quad x_1 \xrightarrow{z}_{/a} y_1}{c!x_0.x_1 \xrightarrow{z}_{/a} c!y_0.y_1} \qquad \frac{x \xrightarrow{z}_{/b} y}{c?a.x \xrightarrow{z}_{/b} c?a.y} \, a \neq b$$

$$\frac{x_0 \xrightarrow{z}_{/a} y_0 \quad x_1 \xrightarrow{z}_{/a} y_1}{x_0 + x_1 \xrightarrow{z}_{/a} y_0 + y_1}$$

# Example: Modified CHOCS!

$$\frac{}{a \xrightarrow{z}_{/a} z} \qquad \frac{}{b \xrightarrow{z}_{/a} b} a \neq b \qquad \frac{x_0 \xrightarrow{z}_{/a} y_0 \quad x_1 \xrightarrow{z}_{/a} y_1}{c!x_0.x_1 \xrightarrow{z}_{/a} c!y_0.y_1} \qquad \frac{x \xrightarrow{z}_{/b} y}{c?a.x \xrightarrow{z}_{/b} c?a.y} a \neq b$$

$$\frac{x_0 \xrightarrow{z}_{/a} y_0 \quad x_1 \xrightarrow{z}_{/a} y_1}{x_0 + x_1 \xrightarrow{z}_{/a} y_0 + y_1}$$

$$\frac{x_1 \xrightarrow{z}_{/a} y_1}{c?a.x_1 \xrightarrow{z}_{c?} y_1} \qquad \frac{}{c!p.x_1 \xrightarrow{p}_{c!} x_1} \qquad \frac{}{\tau.x \longrightarrow_{\tau} x}$$

$$\frac{x_0 \xrightarrow{z}_{c!} y_0}{x_0 + x_1 \xrightarrow{z}_{c!} y_0} \qquad \frac{x_0 \xrightarrow{z}_{c?} y_0}{x_0 + x_1 \xrightarrow{z}_{c?} y_0} \qquad \frac{x_0 \longrightarrow_{\tau} y_0}{x_0 + x_1 \longrightarrow_{\tau} y_0}$$

Theorem:
If an SOS specification is in promoted PANTH then
for its induced transition relation strong bisimilarity is a congruence.

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# Motivating Higher Order Bisimilarity: CHOCS

$$\frac{}{c!x.y \xrightarrow{x}_{c!} y}$$

# Motivating Higher Order Bisimilarity: CHOCS

$$\frac{}{c!x.y \xrightarrow{x}_{c!} y}$$

$$c!(a+b).0 \not\leftrightarrow c!(b+a).0$$

since $c!(a+b).0 \xrightarrow{a+b}_{c!} 0$

but $c!(b+a).0 \not\xrightarrow{a+b}_{c!}$ .

# Higher Order Bisimilarity

Relation $R$ on closed terms is a ho-bisimulation relation iff for all $(p, q) \in R$:

1. $\forall_{p',p''} p \xrightarrow{p''} p' \Rightarrow \exists_{q',q''} q \xrightarrow{q''} q' \ \wedge \ (p', q'), (p'', q'') \in R$;
2. $\forall_{q',q''} q \xrightarrow{q''} q' \Rightarrow \exists_{p',p''} p \xrightarrow{p''} p' \ \wedge \ (p', q'), (p'', q'') \in R$.

$p$ and $q$ are ho-bisimilar, denoted by $p \underline{\leftrightarrow}_h q$, iff there exists a higher order bisimulation relation, relating $p$ and $q$.

# Higher Order Bisimilarity

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2. $\forall_{q',q''} q \xrightarrow{q''} q' \Rightarrow \exists_{p',p''} p \xrightarrow{p''} p' \ \wedge \ (p', q'), (p'', q'') \in R$.

$p$ and $q$ are ho-bisimilar, denoted by $p \underline{\leftrightarrow}_h q$, iff there exists a higher order bisimulation relation, relating $p$ and $q$.

$c!(a+b).0 \not\underline{\leftrightarrow} \ c!(b+a).0$

but $c!(a+b).0 \underline{\leftrightarrow}_h c!(b+a).0$.

# HO-PANTH: Starting Point

$$\frac{\{t_i \xrightarrow{t_i''}_{r_i} y_i \mid i \in I\}}{f(\overrightarrow{x}) \xrightarrow{t''}_{r} t'}$$

# First Motivating Examples

$$\frac{}{a \xrightarrow{a} a} \qquad \frac{}{b \xrightarrow{a} b}$$

$$\frac{x \xrightarrow{x} y}{f(x) \xrightarrow{x} y}$$

# First Motivating Examples

$$\frac{}{a \xrightarrow{a} a} \quad \frac{}{b \xrightarrow{a} b}$$

$$\frac{x \xrightarrow{x} y}{f(x) \xrightarrow{x} y}$$

$a \leftrightarrow_h b$ but $f(a) \not\leftrightarrow_h f(b)$
since $f(a) \xrightarrow{a} a$ while $f(b) \not\rightarrow$ .

# HO-PANTH (Already!?)

$$\dfrac{\dfrac{}{a \xrightarrow{a} a} \quad \dfrac{}{b \xrightarrow{a} b} \quad x \xrightarrow{x} y}{f(x) \xrightarrow{x} y}$$

$$\frac{\{t_i \xrightarrow{z_i}_{r_i} y_i \mid i \in I\}}{f(\overrightarrow{x}) \xrightarrow{t''}_{r} t'}$$

# Example: CHOCS

$$\frac{}{a \xrightarrow{z}_{/a} z} \qquad \frac{}{b \xrightarrow{z}_{/a} b} a \neq b \qquad \frac{x_0 \xrightarrow{z}_{/a} y_0 \quad x_1 \xrightarrow{z}_{/a} y_1}{c!x_0.x_1 \xrightarrow{z}_{/a} c!y_0.y_1} \qquad \frac{x \xrightarrow{z}_{/b} y}{c?a.x \xrightarrow{z}_{/b} c?a.y} a \neq b$$

$$\frac{x_0 \xrightarrow{z}_{/a} y_0 \quad x_1 \xrightarrow{z}_{/a} y_1}{x_0 + x_1 \xrightarrow{z}_{/a} y_0 + y_1}$$

$$\frac{x_1 \xrightarrow{z}_{/a} y_1}{c?a.x_1 \xrightarrow{z}_{c?} y_1} \qquad \frac{}{c!x_0.x_1 \xrightarrow{x_0}_{c!} x_1} \qquad \frac{}{\tau.x \longrightarrow_{\tau} x}$$

$$\frac{x_0 \xrightarrow{z}_{c!} y_0}{x_0 + x_1 \xrightarrow{z}_{c!} y_0} \qquad \frac{x_0 \xrightarrow{z}_{c?} y_0}{x_0 + x_1 \xrightarrow{z}_{c?} y_0} \qquad \frac{x_0 \longrightarrow_{\tau} y_0}{x_0 + x_1 \longrightarrow_{\tau} y_0}$$

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# Conclusions

Done:

- Defined a SOS framework for higher order processes;
- Proposed sufficient criteria for strong and higher-order bisimilarity to be a congruence.

To be done:

- To address “nominal concepts” (names, freshness, capture avoiding subst.).

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**Thank You!**