Equational Theory of Timed CCS

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Motto

Negative results are the only possible self-contained theoretical results in Computer Science.

– – Christos Papadimitriou

Outline

1. CCS
   - Syntax and Semantics
   - Equational Theory
2. Timed CCS
   - Syntax and Semantics
   - Gap Theorem
   - Equational Theory of TCCS
3. Conclusions

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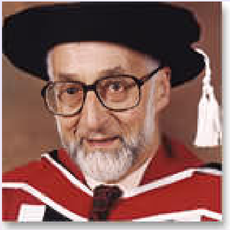
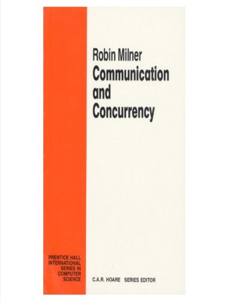
History

1. CCS (Calculus of Communicating Systems), proposed by Robin Milner in late 70's
2. Main concepts:
   1. Communication (message-passing, synchronization) and
   2. Concurrency (logical parallelism)
   3. prefixing (sequence) and non-determinism (choice, conditional).





CCS

Syntax

$$P \quad ::= \quad 0 \mid \alpha.P \mid P + P \mid P \,||\, P$$

$$\alpha \in \mathcal{A}$$
$$\mathcal{A} = \{\tau, a, \bar{a}b, \ldots\}$$

Semantics

$$\frac{}{\alpha.x \xrightarrow{\alpha} x} \qquad \frac{x_0 \xrightarrow{\alpha} y_0}{x_0 + x_1 \xrightarrow{\alpha} y_0} \qquad \frac{x_1 \xrightarrow{\alpha} y_1}{x_0 + x_1 \xrightarrow{\alpha} y_1}$$

$$\frac{x_0 \xrightarrow{\alpha} y_0}{x_0 \,||\, x_1 \xrightarrow{\alpha} y_0 \,||\, x_1} \qquad \frac{x_1 \xrightarrow{\alpha} y_1}{x_0 \,||\, x_1 \xrightarrow{\alpha} x_0 \,||\, y_1} \qquad \frac{x_0 \xrightarrow{a} y_0 \quad x_1 \xrightarrow{\bar{a}} y_1}{x_0 \,||\, x_1 \xrightarrow{\tau} y_0 \,||\, y_1}$$

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Syntax

$$P \quad ::= \quad 0 \mid \alpha.P \mid P + P \mid P \mid\mid P$$

Notational Conventions

1. Binding power: $\alpha.\_ > \_ + \_ > \_ \mid\mid \_$, i.e., $a.0 + b.0 \mid\mid c.0$ is $((a.0) + (b.0)) \mid\mid (c.0)$;
2. Omit the trailing 0: write $a.b + c$ for $(a.b.0) + (c.0)$
3. Assume $\mathcal{A} = \{a\}$! (Thus, no synchronization and no τ-prefixing.)

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Behavioral Equivalence

1. Distinguish less: e.g., $p + p = p$ and $p + q = q + p$;
2. Calculate with processes: e.g., rewriting parallel processes into a sequential process
3. Relating implementation to specification: e.g., p is an implementation of q when $p + q = q$

Bisimulation

Bisimulation relation R:

$$p \; R \; q \quad \Rightarrow \quad \forall_{\alpha,p'} p \xrightarrow{\alpha} p' \Rightarrow \exists_{q'} q \xrightarrow{\alpha} q' \wedge p' \; R \; q'$$

and vice versa...

- $p \; \underline{\leftrightarrow} \; q$ when there exists a bisim. rel. R s.t. $p \; R \; q$;
- $s \; \underline{\leftrightarrow} \; t$ when for all closing subst. σ, $\sigma(s) \; \underline{\leftrightarrow} \; \sigma(t)$.

CCS

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Equational Theory

The framework for calculation:

1. a set E of equations;
2. a simple logic, called equational logic for calculations.

Equational Logic

Assume a set E of equations of the form $t = t'$:

$$(\textbf{refl})\frac{}{E \vdash t = t} \quad (\textbf{symm})\frac{E \vdash t = t'}{E \vdash t' = t} \quad (\textbf{trans})\frac{E \vdash t_0 = t_1 \quad E \vdash t_1 = t_2}{E \vdash t_0 = t_2}$$

$$(\textbf{cong})\frac{E \vdash t_0 = t_0' \quad \ldots \quad E \vdash t_n = t_n'}{E \vdash f(t_0, \ldots, t_n) = f(t_0', \ldots, t_n')} \quad (\textbf{E})\frac{t = t' \in E}{E \vdash \sigma(t) = \sigma(t')}$$

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Equational Theory

Soundness and (ω-)Completeness

E is

1. sound when $E \vdash p = q \Rightarrow p \underline{\leftrightarrow} q$;
   (calculation is correct)
2. complete when $p \underline{\leftrightarrow} q \Rightarrow E \vdash p = q$;
   (calculation is sufficient; forget about models)
3. ω-complete when $s \underline{\leftrightarrow} t \Rightarrow E \vdash s = t$.
   (calculations is sufficient even for the most general results; the relevant notion in universal algebra)

CCS without Parallel

$$
\begin{array}{llll llll}
\text{A0} & x + y & = & y + x & \text{A1} & (x + y) + z & = & x + (y + z) \\
\text{A2} & x + x & = & x & \text{A3} & 0 + x & = & x
\end{array}
$$

Parallel?

$$a \mid\mid b \underline{\leftrightarrow} a.b + b.a$$

Sum Notation

Write $\sum_{i \in \{0,\ldots,n\}} P_i$ for $P_0 + \ldots + P_n$. ($\sum_{i \in \emptyset} = 0$)

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Expansion Theorem

$$\begin{array}{rcl} (\sum_{i\in I} a_i.x_i) \parallel (\sum_{j\in J} b_j.y_j) & = & \sum_{i\in I} a_i.(x_i \parallel (\sum_{j\in J} b_j.y_j)) + \\ & & \sum_{j\in J} b_j.((\sum_{i\in I} a_i.x_i) \parallel y_j) \end{array}$$

Infinitely many equations!

Source of the problem: $(x+y) \parallel z = (x \parallel z) + (y \parallel z)$ is not sound!

CCS

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Expansion Theorem

$$\begin{aligned} (\textstyle\sum_{i\in I} a_i.x_i) \,||\, (\sum_{j\in J} b_j.y_j) \;=\; & \textstyle\sum_{i\in I} a_i.(x_i \,||\, (\sum_{j\in J} b_j.y_j))+ \\ & \textstyle\sum_{j\in J} b_j.((\sum_{i\in I} a_i.x_i) \,||\, y_j) \end{aligned}$$

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Left Merge

$$\frac{x_0 \xrightarrow{a} y_0}{x_0 \,\lfloor\!\lfloor\, x_1 \xrightarrow{a} y_0 \,||\, x_1}$$

Equations for Parallel Composition

$$x \,||\, y = (x \,\lfloor\!\lfloor\, y) + (y \,\lfloor\!\lfloor\, x) \qquad \text{(M0)}$$

$$(x \,||\, y) \,||\, z = x \,||\, (y \,||\, z) \qquad \text{(M1)}$$

$$0 \,\lfloor\!\lfloor\, x = 0 \;\text{(LM0)} \quad x \,\lfloor\!\lfloor\, 0 = x \qquad \text{(LM9)}$$

$$a.x \,\lfloor\!\lfloor\, y = a.(x \,||\, y) \qquad \text{(LM1)}$$

$$(x + y) \,\lfloor\!\lfloor\, z = (x \,\lfloor\!\lfloor\, z) + (y \,\lfloor\!\lfloor\, z) \qquad \text{(LM2)}$$

Equations for Parallel Composition

$$x \,||\, y = (x \lfloor\lfloor y) + (y \lfloor\lfloor x) \text{ (M0)}$$
$$(x \,||\, y) \,||\, z = x \,||\, (y \,||\, z) \text{ (M1)}$$
$$0 \lfloor\lfloor x = 0 \text{ (LM0)} \quad x \lfloor\lfloor 0 = x \text{ (LM0)}$$
$$a.x \lfloor\lfloor y = a.(x \,||\, y) \text{(LM1)}$$
$$(x + y) \lfloor\lfloor z = (x \lfloor\lfloor z) + (y \lfloor\lfloor z) \text{ (LM2)}$$

Soundness and ω-Completeness[Milner'80]

The equational theory of CCS+$\lfloor\lfloor$ is sound and ω-complete.

CCS

Faron Moller [ICALP'90] (Put Informally)

Assume you could do without B&K, i.e., there is a sound and complete equational theory E for CCS, then

- there is a maximum size for equations in E, say n
- for $m > n$, one cannot prove

$$E \vdash \sum_{1 \le i \le m} a.a^{\le i} \parallel a \;=\; a.(\sum_{1 \le i \le m} a.a^{\le i}) + \sum_{1 \le i \le m} a.(a^{\le i} \parallel a)$$

where $a^{\le i} = a + \ldots + a^i$, $a^0 = 0$ and and $a^i = a.a^{i-1}$

Faron's Argument (Put VERY Informally)

- for $m > n$, one cannot prove

$$E \vdash \sum_{1\leq i\leq m} a.a^{\leq i} \mid\mid a \quad = \quad a.(\sum_{1\leq i\leq m} a.a^{\leq i}) + \sum_{1\leq i\leq m} a.(a^{\leq i} \mid\mid a)$$

- because
  1. all $a.a^{\leq i}$'s are different (they cannot be fitted to the size of equation), and
  2. $+$ does not distribute over $\mid\mid$ (cannot push $\mid\mid$ inside "structurally"), and
  3. the equations are small (they cannot push $\mid\mid$ inside "manually").

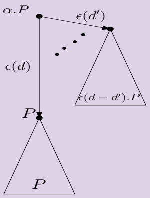
Outline

TCCS

Syntax

$$P \quad ::= \quad 0 \mid \epsilon(d).P \mid \alpha.P \mid P + P \mid P \parallel P$$

Semantics (Informal)



Semantics (SOS)

$$(\textbf{td0})\ \frac{}{\epsilon(d).x \stackrel{\epsilon(d)}{\longrightarrow} x} \qquad (\textbf{td1})\ \frac{}{\epsilon(d+e).x \stackrel{\epsilon(d)}{\longrightarrow} \epsilon(e).x}$$

$$(\textbf{td2})\ \frac{x \stackrel{\epsilon(e)}{\longrightarrow} y}{\epsilon(d).x \stackrel{\epsilon(d+e)}{\longrightarrow} y}$$

TCCS

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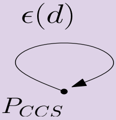
Informal



Timed Semantics(SOS)

$$(\textbf{tn})\frac{}{0 \stackrel{\epsilon(d)}{\longrightarrow} 0} \qquad (\textbf{ta})\frac{}{\alpha.x \stackrel{\epsilon(d)}{\longrightarrow} \alpha.x}$$

$$(\textbf{tc})\frac{x_0 \stackrel{\epsilon(d)}{\longrightarrow} y_0 \quad x_1 \stackrel{\epsilon(d)}{\longrightarrow} y_1}{x_0 + x_1 \stackrel{\epsilon(d)}{\longrightarrow} y_0 + y_1} \qquad (\textbf{tp})\frac{x_0 \stackrel{\epsilon(d)}{\longrightarrow} y_0 \quad x_1 \stackrel{\epsilon(d)}{\longrightarrow} y_1}{x_0 \parallel x_1 \stackrel{\epsilon(d)}{\longrightarrow} y_0 \parallel y_1}$$

Time Insensitivity

p is initially time insensitive:

- if $p \stackrel{\epsilon(d)}{\longrightarrow} p_d$, then $p_d \underline{\leftrightarrow} p$.

p is time insensitive processes:

1. if $p \stackrel{\epsilon(d)}{\longrightarrow} p_d$, then $p_d \underline{\leftrightarrow} p$ and p_d is time insensitive and
2. if $p \stackrel{a}{\longrightarrow} p_a$ then p_a is time insensitive.

Small Theorem [Aceto et.al.'07]

p is time insensitive when there is a CCS process q, such that $p \underline{\leftrightarrow} q$.

Outline

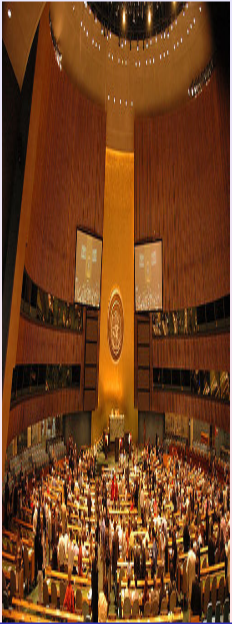
TCCS: Equational Theory

A Ph.D. Student's Nightmare!

1. Wang Yi presented TCCS and the following expansion law in CONCUR'90:

$$\sum_{i\in I}\epsilon(d_i).\mu_i.p_i \parallel \sum_{j\in J}\epsilon(d'_j).\mu'_j.p'_j = \\ \sum_{i\in I}\epsilon(d_i).\mu_i.(p_i \parallel \sum_{j\in J}\epsilon(d'_j - d_i).\mu'_j.p'_j) + \\ \sum_{j\in J}\epsilon(d'_j).\mu'_j.(p'_j \parallel \sum_{i\in I}\epsilon(d_i - d'_j).\mu_i.p_i)$$

2. two other Ph.D. students (Faron Moller and Chris Tofts) shot a hole in the law, on the spot!



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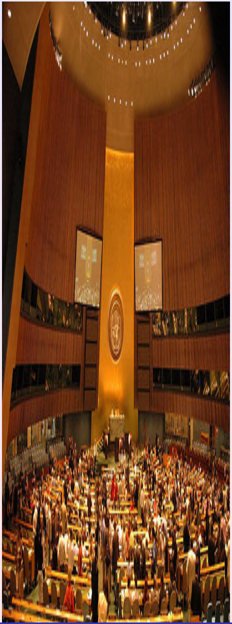


TCCS: Equational Theory

Hole

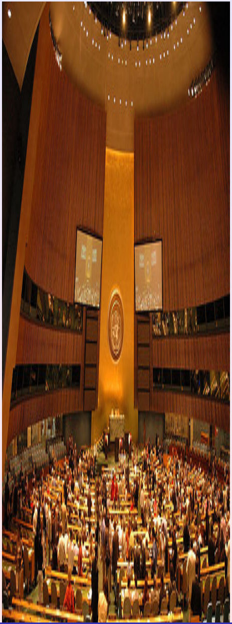
$$P_0 \equiv \epsilon(1).a.0$$

$$Q_0 \equiv b.0$$

$$P_0 \mid\mid Q_0 =^? (\epsilon(1).a.b.0) + b.\epsilon(1).a.0$$

Research Questions

1. Is there an expansion law for TCCS?
   No! Gap Theorem [Godskesen&Larsen'92]
2. Is there a finite axiomatization for TCCS?
   No! Our Theorem! [Aceto et. al.'07]



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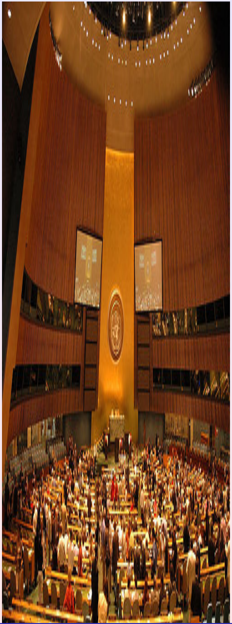

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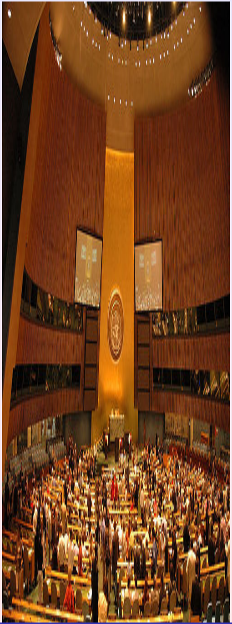

Gap Theorem

Idea

In TCCS, parallel composition cannot be resolved.

Simple Presentation

There is no sequential p such that

$p \underline{\leftrightarrow} \epsilon(d).a \parallel a.\epsilon(d).a.$

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In TCCS, parallel composition cannot be resolved.

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There is no sequential p such that
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Gap Theorem

Simple Presentation

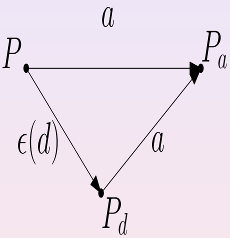
There is no sequential p without parallel composition such that $p \leftrightarrow \epsilon(d).a \mid\mid a.\epsilon(d).a$.

Informal Argument

Think of

- $\epsilon(d).a$ as a clock;
- think of α-transitions as resets.

All clocks of a sequential process have one reset. (After each action, all clocks are reset.)



Gap Theorem

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There is no sequential p without parallel composition such that $p \underline{\leftrightarrow} \epsilon(d).a \parallel a.\epsilon(d).a$.

Formal Argument

- Assume $p \underline{\leftrightarrow} q \equiv \sum_{i \in I} a.q_i + \sum_{j \in J} \epsilon(d_j).q_j$;
- $p \xrightarrow{a} a(d) \parallel a(d)$, hence, $q_i \xrightarrow{a} q_i' \underline{\leftrightarrow} a(d) \parallel a(d)$;
- $q \overset{\epsilon(d')}{\longrightarrow} . \xrightarrow{a} q_i'$
- but $p \overset{\epsilon(d')}{\longrightarrow} a(d'-d) \parallel a.a(d)$ $\xrightarrow{a} a(d-d') \parallel a(d)$.

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- $q \xrightarrow{\epsilon(d')} . \xrightarrow{a} q_i'$
- but $p \xrightarrow{\epsilon(d')} a(d' - d) \parallel a.a(d) \xrightarrow{a} a(d - d') \parallel a(d)$.

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Two Interpretations of Time-Delay

- Single-sorted: $\epsilon(d).\_$ is a unary operators for each $d \in D$.
- Two-sorted: $\epsilon(\_).\_$ is a binary operator.

Single-sorted TCCS has no finite basis

Lemma

If $E \vdash s = t$, then $vars(s) = vars(t)$.

Corollary: If d is greater than all delays in E, then $\epsilon(d).\epsilon(d).a = \epsilon(d + d).a$ is not provable from E.

TCCS

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Lemma

If $E \vdash s = t$, then $vars(s) = vars(t)$.

Corollary: If d is greater than all delays in E, then $\epsilon(d).\epsilon(d).a = \epsilon(d + d).a$ is not provable from E.

Two-Sorted TCCS: Faron Moller Can Be On Time

- The basic case holds essentially by the same argument;
- B&K cannot help: no term containing $\Bot$ (as a summand) can be bisimilar to a TCCS process.

Giving $\lfloor\!\lfloor$ a Sense of Time

$$\frac{x_0 \stackrel{\epsilon(d)}{\longrightarrow} y_0 \quad x_1 \stackrel{\epsilon(d)}{\longrightarrow} y_1}{x_0 \lfloor\!\lfloor x_1 \stackrel{\epsilon(d)}{\longrightarrow} y_0 \lfloor\!\lfloor y_1}$$

B&K Could Not Help In Time

- $x \mid\mid q = x \lfloor\!\lfloor y + y \lfloor\!\lfloor x$ holds, but
- $a.x \lfloor\!\lfloor y = a.(x \mid\mid y)$ does not hold:
  $a \lfloor\!\lfloor \epsilon(d).a =^{?} a.(\epsilon(d).a)$
- hence, for a finite E, one cannot prove that

$$E \vdash a \lfloor\!\lfloor \sum_{1 \le i \le n} a.a^{\le i} =$$

$$a.(\sum_{1 \le i \le n} a.a^{\le i})$$

Crux Lemma

Assume that $E \vdash p = q$,

- $p \underline{\leftrightarrow} a.\Phi_n$, for $n > 2n_0 + 1$,
- p contains a summand of the form $p_i \lfloor\lfloor p'_i$ where $p \underline{\leftrightarrow} a$ and $p'_i \underline{\leftrightarrow} \Phi_n$,

then q contains a summand of the form $q_j \lfloor\lfloor q'_j$ where $q_j \underline{\leftrightarrow} a$ and $q'_j \underline{\leftrightarrow} \Phi_n$.

Conclusions

World Before Our Paper

1. CCS affords no finite axiomatization.
2. There is a finite axiomatization for CCS+$\lfloor\!\lfloor$.
3. In TCCS with inifnitely many actions, parallelism cannot be resolved.

World After Our Paper

1. TCCS has no finite axiomatization.
2. TCCS+$\lfloor\!\lfloor$ has no finite axiomatization.
3. TCCS+$\lfloor\!\lfloor_t$ has no finite axiomatization.
4. In TCCS, parallelism cannot be resolved.