Being Promoted Makes a Difference

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Once Upon a Time

De Simone

1. How to prove two programs (with holes in them) equal?
2. Is equality a congruence? I.e., Can we replace equals by equals?
3. Semantics is given in the SOS style.

Once Upon a Time

SOS Frameworks (Congruence Formats)

de Simone

$$\frac{\{x_i \xrightarrow{l_i} y_i | i \in I\}}{f(\overrightarrow{x}) \xrightarrow{l} t'}$$

Bisimilarity

Symmetric relation R on closed terms is a bisimulation relation iff for all $(p, q) \in R$:

- $\forall_{l,p'} p \xrightarrow{l} p' \Rightarrow \exists_{q'} q \xrightarrow{l} q' \ \wedge \ (p', q') \in R$;

p and q are bisimilar, denoted by $p \leftrightarrow q$, iffthere exists a bisimulation relation, relating p and q.

Congruence

Equivalence relation R on closed terms is a congruence when for all function symbols f:

$$\forall_{0 \leq i < ar(f)} \forall_{p_i, q_i} (p_i, q_i) \in R \Rightarrow (f(p_0, \ldots, p_{ar(f)-1}), f(q_0, \ldots, q_{ar(f)-1})) \in R.$$

SOS Frameworks (Congruence Formats)

GSOS

$$\frac{\{x_i \xrightarrow{l_i} y_i | i \in I\} \cup \{x_j \stackrel{l_j}{\nrightarrow} \ | j \in J\}}{f(\overrightarrow{x}) \xrightarrow{l} C[x_k, y_i]}$$

$\nwarrow$

de Simone

$$\{x_i \xrightarrow{l_i} y_i | i \in I\}$$

SOS Frameworks (Congruence Formats)

GSOS

$$\frac{\{x_i \xrightarrow{l_i} y_i | i \in I\} \cup \{x_j \stackrel{l_j}{\nrightarrow} \ | \ j \in J\}}{f(\overrightarrow{x}) \xrightarrow{l} C[x_k, y_i]} \qquad \frac{t \xrightarrow{l} y}{X \xrightarrow{l} y} \quad X \doteq t$$

↖

de Simone

$$\frac{\{x_i \xrightarrow{l_i} y_i | i \in I\}}{}$$

SOS Frameworks (Congruence Formats)

GSOS

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$$\frac{x \xrightarrow{\tau} y \quad y \xrightarrow{l} z}{x \xrightarrow{l} z}$$

↖

de Simone

$$\frac{\{x_i \xrightarrow{l_i} y_i | i \in I\}}{}$$

SOS Frameworks (Congruence Formats)

SOS: Structured Op. Sem

GSOS

$$\frac{\{x_i \xrightarrow{l_i} y_i | i \in I\} \cup \{x_j \stackrel{l_j}{\nrightarrow} \ | j \in J\}}{f(\overrightarrow{x}) \xrightarrow{l} C[x_k, y_i]}$$

↖

Tyft/Tyxt

$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\}}{f(\overrightarrow{x}) \ / \ x \xrightarrow{l} t}$$

↗

de Simone

$$\frac{\{x_i \xrightarrow{l_i} y_i | i \in I\}}{f(\overrightarrow{x}) \xrightarrow{l} t'}$$

Variable Dependency Graph

The variable dependency graph of a rule is a graph of which

- the nodes are variables and
- there is an edge from x to y when y appears in the target of a premise and x in its source.

Well-foundedness

1. well-founded rule: no infinite backward chain in the variable dependency graph;
2. well-founded TSS: consisting of well-founded rules

Congruence for Well-founded Tyft [Groote&Vaandrager'92] [Fokkink'94]

For a well-founded TSS in the tyft format, bisimilarity is a congruence.

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SOS Frameworks (Congruence Formats)

NTyft (PANTH, NTree)

$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\} \cup \{t_j \stackrel{l_j}{\nrightarrow} \ | j \in J\}}{f(\overrightarrow{x}) \xrightarrow{l} t}$$

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Tyft

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de Simone

$$\{x_i \xrightarrow{l_i} y_i | i \in I\}$$

Promoted Frameworks (Congruence Formats)

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$\nwarrow$

$$\frac{}{c!p.x \xrightarrow{p} x}$$

Tyft

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Promoted Frameworks (Congruence Formats)

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↖

Promoted Tyft

$$\frac{\{t_i \xrightarrow{g_i(\overrightarrow{t_i})} y_i | i \in I\}}{f(\overrightarrow{x}) \xrightarrow{g(\overrightarrow{z})} t}$$

↗

Tyft

$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\}}{f(\overrightarrow{x}) \xrightarrow{l} t}$$

Variable Dependency Graph

The variable dependency graph of a rule is a graph of which

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Well-foundedness

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2. well-founded TSS: consisting of well-founded rules

Congruence for Well-Founded Promoted Tyft [Bernstein'98]

For a well-founded TSS in the promoted tyft format, bisimilarity is a congruence.

Promoted Frameworks (Congruence Formats)

Promoted PANTH

$$\frac{\{t_i \xrightarrow{\overrightarrow{t_i''}} y_i | i \in I\} \cup \{t_j \stackrel{\overrightarrow{t_j''}}{\nrightarrow} \; | j \in J\}}{f(\overrightarrow{x}) \xrightarrow{\overrightarrow{t''}} t'}$$

↗ ↖

NTyft (PANTH, NTree)

$$\frac{\{t_i \xrightarrow{l_i} y_i | i \in I\} \cup \{t_j \stackrel{l_j}{\nrightarrow} \; | j \in J\}}{f(\overrightarrow{x}) \xrightarrow{l} t}$$

Promoted Tyft

$$\frac{\{t_i \xrightarrow{g_i(\overrightarrow{t_i''})} y_i | i \in I\}}{f(\overrightarrow{x}) \xrightarrow{g(\overrightarrow{z})} t}$$

↖ ↗

Tyft

$$\{t_i \xrightarrow{l_i} y_i | i \in I\}$$

Bernstein's Conclusions

In this paper, we have described a rule format that is a simple but expressive generalization of Groote and *Vaandrager's tyft/tyxt rule format. ...*
There are several open questions related to the work in this paper.

1. *It is not clear that the well-foundedness property is necessary for the congruence result.*
2. *We are not sure how the extensions to tyft/tyxt format that allow negative premises are compatible with our extensions.*
3. *It is not clear whether promoted tyft/tyxt format is strictly more expressive than tyft/tyxt format.*

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Variable Dependency Graph (recap)

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Well-foundedness (recap)

1. well-founded rule: no infinite backward chain in the variable dependency graph;
2. well-founded TSS: consisting of well-founded rules

Non-well-founded TSS 0

$$\frac{}{0 \xrightarrow{0} 1} \quad \frac{}{1 \xrightarrow{0} 0} \quad \frac{x \xrightarrow{0} y}{1 \xrightarrow{f(x)} x} \quad \frac{x \xrightarrow{0} y}{0 \xrightarrow{f(x)} y} \quad \frac{x \xrightarrow{f(y)} y}{f(x) \xrightarrow{1} y}$$

Provable Transitions

$$\{0 \xrightarrow{0} 1, \quad 1 \xrightarrow{0} 0,$$
$$1 \xrightarrow{f(0)} 0, \quad 1 \xrightarrow{f(1)} 1, \quad 0 \xrightarrow{f(0)} 1, \quad 0 \xrightarrow{f(1)} 0,$$
$$f(1) \xrightarrow{1} 0, \quad f(1) \xrightarrow{1} 1\}$$

Congruence is ruined

$0 \underline{\leftrightarrow} 1$ but
$f(0) \stackrel{1}{\not\rightarrow}$ while $f(1) \xrightarrow{1} 0; 1$.

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$$\frac{}{0 \xrightarrow{0} 1} \quad \frac{}{1 \xrightarrow{0} 0} \quad \frac{x \xrightarrow{0} y}{1 \xrightarrow{f(x)} x} \quad \frac{x \xrightarrow{0} y}{0 \xrightarrow{f(x)} y} \quad \frac{x \xrightarrow{f(y)} y}{f(x) \xrightarrow{1} y}$$

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Non-well-founded TSS 2

$$\frac{}{0 \xrightarrow{0} 1} \quad \frac{}{1 \xrightarrow{0} 0} \quad \frac{}{0 \xrightarrow{1} 0} \quad \frac{}{1 \xrightarrow{1} 1}$$

$$\frac{x \xrightarrow{0} y}{1 \xrightarrow{f(x)} x} \quad \frac{x \xrightarrow{0} y}{0 \xrightarrow{f(x)} y} \quad \frac{x \xrightarrow{f(y)} y' \quad y' \xrightarrow{1} y}{f(x) \xrightarrow{1} y}$$

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Tyft

$$\frac{\{t_i \xrightarrow{l_i} y_i \mid i \in I\}}{f(\overrightarrow{x}) \xrightarrow{l} t}$$

Promoted Tyft

$$\frac{\{t_i \xrightarrow{g_i(\overrightarrow{t_i})} y_i \mid i \in I\}}{f(\overrightarrow{x}) \xrightarrow{g(\overrightarrow{z})} t}$$

Tyft Labels: Constants or Closed Terms

1. l is a constant: Tyft syntactically fits in the Promoted Tyft;
2. l is a closed term: ?

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Promoted Tyft Does *NOT* Promote Tyft!

Tyft (with Closed Terms as Labels)

$$\frac{\{t_i \xrightarrow{p_i} y_i \mid i \in I\}}{f(\overrightarrow{x}) \xrightarrow{p} t}$$

Promoted Tyft

$$\frac{\{t_i \xrightarrow{g_i(\overrightarrow{t_i})} y_i \mid i \in I\}}{f(\overrightarrow{x}) \xrightarrow{g(\overrightarrow{z})} t}$$

Lemma [Bernstein'98]

For a TSS in Promoted Tyft,

if $p \xrightarrow{f(\overrightarrow{p'})} p'' \wedge p \,\underline{\leftrightarrow}\, q \wedge \overrightarrow{p'} \,\underline{\leftrightarrow}\, \overrightarrow{q'}$ then $\exists_{q''} q \xrightarrow{f(\overrightarrow{q'})} q'' \wedge p'' \,\underline{\leftrightarrow}\, q''$.

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Tyft (with Closed Terms as Labels)

$$\frac{\{t_i \xrightarrow{p_i} y_i | i \in I\}}{f(\overrightarrow{x}) \xrightarrow{p} t}$$

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For a TSS in Promoted Tyft,

if $p \xrightarrow{f(\overrightarrow{p'})} p'' \wedge p \underline{\leftrightarrow} q \wedge \overrightarrow{p'} \underline{\leftrightarrow} \overrightarrow{q'}$ then

$\exists_{q''} q \xrightarrow{f(\overrightarrow{q'})} q'' \wedge p'' \underline{\leftrightarrow} q''$.

$$\frac{}{2 \xrightarrow{f(0)} 2}$$

$0 \underline{\leftrightarrow} 1$ and $2 \xrightarrow{f(0)} 2$ but $2 \overset{f(1)}{\not\rightarrow}$.

Tyft is *NOT* Promoted

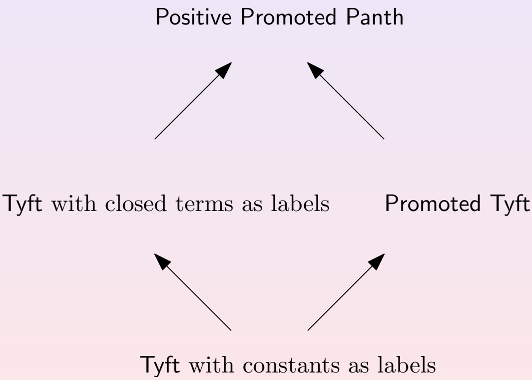
$$\frac{}{0 \xrightarrow{0} 0} \quad \frac{}{1 \xrightarrow{0} 0} \quad \frac{}{0 \xrightarrow{0} 1} \quad \frac{}{1 \xrightarrow{0} 1}$$

$$\frac{}{0 \xrightarrow{1} 0} \quad \frac{}{1 \xrightarrow{1} 0} \quad \frac{}{0 \xrightarrow{1} 1} \quad \frac{}{1 \xrightarrow{1} 1}$$

$$\frac{x \xrightarrow{0} y}{0 \xrightarrow{f(x)} 1} \quad \frac{x \xrightarrow{0} y}{1 \xrightarrow{f(x)} 0} \quad \frac{x \xrightarrow{f(x)} y}{0 \xrightarrow{f(x)} y} \quad \frac{x \xrightarrow{f(x)} y}{1 \xrightarrow{f(x)} y} \quad \frac{x \xrightarrow{f(x)} y}{f(x) \xrightarrow{1} y}$$

$$\{0 \xrightarrow{0} 0; 1\ ,\ 1 \xrightarrow{0} 0; 1\ ,\ 0 \xrightarrow{1} 0; 1\ ,\ 1 \xrightarrow{1} 0; 1\ ,$$
$$0 \xrightarrow{f(0)} 1\ ,\ 0 \xrightarrow{f(1)} 0; 1\ ,\ 1 \xrightarrow{f(0)} 0; 1\ ,\ 1 \xrightarrow{f(1)} 0\ ,$$
$$f(0) \xrightarrow{1} 1\ ,\ f(1) \xrightarrow{1} 0\}$$

Expressiveness Lattice



"Nitpicking Corner Cases"?

Well-foundedness

- Goal: To find "minimal" constraints on the congruence format for promoted SOS;
- Conjecture: To disallow only "unsafe" cycles in the variable dependency graph.

Expressiveness

- Goal: Show that there is a fundamental gap between promoted and traditional SOS;
- Conjecture: The gap shown by the last counter-example cannot be bridged by adding an arbitrary number of auxiliary operators.

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