Causality in the Semantics of Esterel: Revisited

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Esterel

1. Developed in early 80's by Gérard Berry
2. Targeted at embedded reactive systems
3. Extensively used in industrial applications
4. Imperative language based on the synchronous assumption

A Cute Subset of Esterel

$$
\begin{array}{rcl}
p, q & ::= & 0 \mid \texttt{emit}\ s \mid \texttt{pres}\ s\ \texttt{?}\ p \diamond q\ \texttt{end} \mid \\
 & & p\ \texttt{;}\ q \mid p\ \texttt{||}\ q
\end{array}
$$

Intuitive Semantics of Esterel

$\mathsf{P0} \doteq \mathtt{emit}\ \mathrm{s}\ ;\ \mathtt{pres}\ \mathrm{s}\ ?\ \mathtt{emit}\ \mathrm{o} \diamond 0\ \mathtt{end}$

Semantics (global view): final state w.r.t. presence/absence of each signal (at each time step)

Intuitive Semantics of Esterel

$\mathsf{P0} \doteq \mathtt{emit}\ \mathrm{s}\ ;\ \mathtt{pres}\ \mathrm{s}\ ?\ \mathtt{emit}\ \mathrm{o} \diamond 0\ \mathtt{end}$

Semantics (global view): final state w.r.t. presence/absence of each signal (at each time step)

$P0 \uparrow^{\mathrm{s}}$

Intuitive Semantics of Esterel

$\mathsf{P0} \doteq \mathtt{emit}\ \mathrm{s}\ ;\ \mathtt{pres}\ \mathrm{s}\ ?\ \mathtt{emit}\ \mathrm{o} \diamond \mathtt{0}\ \mathtt{end}$

Semantics (global view): final state w.r.t. presence/absence of each signal (at each time step)

$P0 \uparrow^{\mathrm{s}}\ P0 \uparrow^{\mathrm{o}}$

Intuitive Semantics of Esterel

P0 $\doteq$ `emit` s ; `pres` s ? `emit` o $\diamond$ 0 `end`

Semantics (global view): final state w.r.t. presence/absence of each signal (at each time step)

$P0 \uparrow^{\mathrm{s}} \; P0 \uparrow^{\mathrm{o}}$

logical coherency: at least one `emit` for each present signal and no `emit` for absent signal

A Program Chasing Its Tale

$\mathtt{P1} \doteq \mathtt{pres\ s\ ?\ emit\ s} \diamond \mathtt{0\ end}$

A Program Chasing Its Tale

$\mathtt{P1} \doteq \mathtt{pres\ s\ ?\ emit\ s} \diamond \mathtt{0\ end}$

A Program Chasing Its Tale

P1 $\doteq$ pres s ? emit s $\diamond$ 0 end

C1 logical determinism: at most one logically coherent global status.

A Confused Program

$\mathsf{P2} \doteq \texttt{pres s ? 0} \diamond \texttt{emit s end}$

A Confused Program

P2 $\doteq$ `pres s ? 0` $\diamond$ `emit s end`

A Confused Program

P2 $\doteq$ `pres s ? 0` $\diamond$ `emit s end`

C2 logical reactivity: at least one logically coherent global status.

All Roads Lead to Rome

$\mathrm{P3} \doteq \texttt{pres s ? emit s} \diamond \texttt{emit s end}$

All Roads Lead to Rome

$\mathtt{P3} \doteq \mathtt{pres\ s\ ?\ emit\ s} \diamond \mathtt{emit\ s\ end}$

All Roads Lead to Rome

P3 $\doteq$ `pres s ? emit s` $\diamond$ `emit s end`

S Constructiveness: each signal either must be emitted or cannot be emitted

Pathological SOS Rules

$$\frac{\neg P \uparrow^{s}}{P \uparrow^{s}} \qquad\qquad \frac{P \uparrow^{s}}{P \uparrow^{s}}$$

Pathological SOS Rules

$\dfrac{\neg P \uparrow^{s}}{P \uparrow^{s}}$	$\dfrac{P \uparrow^{s}}{P \uparrow^{s}}$
pres s ? emit s $\diamond$ 0 end	pres s ? 0 $\diamond$ emit s end
	

Pathological SOS Rules (Cont'd)

$$\frac{\neg P \uparrow^s}{P \uparrow^s} \qquad \frac{P \uparrow^s}{P \uparrow^s}$$

Pathological SOS Rules (Cont'd)

$$\frac{\neg P \uparrow^{s}}{P \uparrow^{s}} \qquad \frac{P \uparrow^{s}}{P \uparrow^{s}}$$

`pres s ? emit s ⋄ emit s end`

Solution 1

Supported Model

⇒ if the positive premises of a rule are in the model and the negative ones are not contradicted by the model, then the conclusion is also in the model,

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- ⇐ each formula in the model is due to an instance of a rule, of which the positive premises are in the model and negative ones are not contradicted in the model.

Solution 1

Supported Model

⇒ if the positive premises of a rule are in the model and the negative ones are not contradicted by the model, then the conclusion is also in the model,

⇐ each formula in the model is due to an instance of a rule, of which the positive premises are in the model and negative ones are not contradicted in the model.

Logical Semantics

A program is meaningful if it has a unique supported model.

Pathological SOS Rules: Revisited

$$\frac{\neg P \uparrow^{s}}{P \uparrow^{s}} \qquad \frac{P \uparrow^{s}}{P \uparrow^{s}}$$

Pathological SOS Rules: Revisited

$\dfrac{\neg P \uparrow^{s}}{P \uparrow^{s}}$ ×	$\dfrac{P \uparrow^{s}}{P \uparrow^{s}}$
no supported model	

Pathological SOS Rules: Revisited

$\dfrac{\neg P \uparrow^s}{P \uparrow^s}$ ×	$\dfrac{P \uparrow^s}{P \uparrow^s}$ ×
no supported model	$\emptyset$ and $\{P \uparrow^s\}$

Pathological SOS Rules: Revisited

$$\frac{\neg P \uparrow^{s}}{P \uparrow^{s}} \qquad \frac{P \uparrow^{s}}{P \uparrow^{s}}$$

Pathological SOS Rules: Revisited

$$\frac{\neg P \uparrow^{s}}{P \uparrow^{s}} \qquad \frac{P \uparrow^{s}}{P \uparrow^{s}} \checkmark$$

Unique supported model: $\{P \uparrow^{s}\}$

Solution 2

Supported Proof (Refutation)

1. for a positive formula: proof tree using instances of rules
2. for negative formula: proof of a negated premise for each contradicting rule

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1. for a positive formula: proof tree using instances of rules
2. for negative formula: proof of a negated premise for each contradicting rule

Constructive Semantics

A program is meaningful if each formula concerning it has either a supported proof or a supported refutation.

Pathological SOS Rules: Re-Revisited

$$\frac{P \uparrow^{s}}{P \uparrow^{s}} \qquad \frac{\neg P \uparrow^{s}}{P \uparrow^{s}}$$

Pathological SOS Rules: Re-Revisited

$$\frac{P \uparrow^{s}}{P \uparrow^{s}} \; {\color{red}\times} \qquad\qquad \frac{\neg P \uparrow^{s}}{P \uparrow^{s}}$$

$$\frac{\dfrac{\vdots}{P \uparrow^{s}}}{P \uparrow^{s}} \quad \textit{and} \quad \frac{\dfrac{\vdots}{\neg P \uparrow^{s}}}{\neg P \uparrow^{s}}$$

Pathological SOS Rules: Re-Revisited

$$\frac{P \uparrow^{s}}{P \uparrow^{s}} \; \times \qquad\qquad \frac{\neg P \uparrow^{s}}{P \uparrow^{s}} \; \times$$

$$\frac{\dfrac{\vdots}{P \uparrow^{s}}}{P \uparrow^{s}} \; \textit{and} \; \frac{\dfrac{\vdots}{\neg P \uparrow^{s}}}{\neg P \uparrow^{s}} \qquad \frac{\dfrac{\dfrac{\vdots}{P \uparrow^{s}}}{\neg P \uparrow^{s}}}{P \uparrow^{s}} \; \text{and} \; \frac{\dfrac{\dfrac{\vdots}{\neg P \uparrow^{s}}}{P \uparrow^{s}}}{\neg P \uparrow^{s}}$$

Pathological SOS Rules: Revisited

$$\frac{\neg P \uparrow^{s}}{P \uparrow^{s}} \qquad \frac{P \uparrow^{s}}{P \uparrow^{s}}$$

Pathological SOS Rules: Revisited

$$\frac{\neg P \uparrow^{s}}{P \uparrow^{s}} \qquad \frac{P \uparrow^{s}}{P \uparrow^{s}} \; \times$$

$$\frac{\dfrac{\vdots}{P \uparrow^{s}} \;\text{ or }\; \dfrac{\dfrac{\vdots}{P \uparrow^{s}} \quad \dfrac{\vdots}{\neg P \uparrow^{s}}}{\neg P \uparrow^{s}}}{P \uparrow^{s}} \qquad \frac{\dfrac{\vdots}{\neg P \uparrow^{s}} \quad \dfrac{\dfrac{\vdots}{\neg P \uparrow^{s}} \;\text{ or }\; \dfrac{\vdots}{P \uparrow^{s}}}{P \uparrow^{s}}}{\neg P \uparrow^{s}}$$

Unified Semantics of Esterel

$$(\mathrm{e0})\,\frac{}{\texttt{emit}\ x \uparrow^{c,x}}$$

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$$(\mathrm{e0})\frac{}{\texttt{emit}\ x \uparrow^{c,x}}$$

$$(\mathrm{f0})\frac{c \uparrow^{c,s} \quad p \uparrow^{c,x}}{\texttt{pres}\ s\ ?\ p \diamond q\ \texttt{end} \uparrow^{c,x}} \qquad (\mathrm{f1})\frac{\neg c \uparrow^{c,s} \quad q \uparrow^{c,x}}{\texttt{pres}\ s\ ?\ p \diamond q\ \texttt{end} \uparrow^{c,x}}$$

Unified Semantics of Esterel

$$(\mathrm{e0})\ \frac{}{\texttt{emit}\ x \uparrow^{c,x}}$$

$$(\mathrm{f0})\ \frac{c \uparrow^{c,s} \quad p \uparrow^{c,x}}{\texttt{pres}\ s\ \texttt{?}\ p \diamond q\ \texttt{end} \uparrow^{c,x}} \qquad (\mathrm{f1})\ \frac{\neg c \uparrow^{c,s} \quad q \uparrow^{c,x}}{\texttt{pres}\ s\ \texttt{?}\ p \diamond q\ \texttt{end} \uparrow^{c,x}}$$

$$(\mathrm{s0})\ \frac{p \uparrow^{c,x}}{p\ \texttt{;}\ q \uparrow^{c,x}} \qquad (\mathrm{s1})\ \frac{p\checkmark_c \quad q \uparrow^{c,x}}{p\ \texttt{;}\ q \uparrow^{c,x}} \qquad (\mathrm{s2})\ \frac{p \stackrel{c,x'}{\rightarrow} p' \quad p'\checkmark_c \quad q \uparrow^{c,x}}{p\ \texttt{;}\ q \uparrow^{c,x}}$$

Unified Semantics of Esterel

$$(\text{e0})\,\frac{}{\texttt{emit}\ x \uparrow^{c,x}}$$

$$(\text{f0})\,\frac{c \uparrow^{c,s} \quad p \uparrow^{c,x}}{\texttt{pres}\ s\ ?\ p \diamond q\ \texttt{end} \uparrow^{c,x}} \qquad (\text{f1})\,\frac{\neg c \uparrow^{c,s} \quad q \uparrow^{c,x}}{\texttt{pres}\ s\ ?\ p \diamond q\ \texttt{end} \uparrow^{c,x}}$$

$$(\text{s0})\,\frac{p \uparrow^{c,x}}{p\ ;\ q \uparrow^{c,x}} \qquad (\text{s1})\,\frac{p\checkmark_c \quad q \uparrow^{c,x}}{p\ ;\ q \uparrow^{c,x}} \qquad (\text{s2})\,\frac{p \xrightarrow{c,x'} p' \quad p'\checkmark_c \quad q \uparrow^{c,x}}{p\ ;\ q \uparrow^{c,x}}$$

$$(\text{p0})\,\frac{p \uparrow^{c,x}}{p\ ||\ q \uparrow^{c,x}} \qquad (\text{p1})\,\frac{q \uparrow^{c,x}}{p\ ||\ q \uparrow^{c,x}}$$

Solution 1: Revisited

Logical Semantics of Esterel

A Esterel program p is meaningful if it has a unique supported model for all its subterms and for each label $\uparrow^{p,x}$.

Solution 1: Revisited

$$(\mathrm{e0})\frac{}{\texttt{emit}\ x \uparrow^{c,x}} \qquad (\mathrm{s0})\frac{p \uparrow^{c,x}}{p\ \texttt{;}\ q \uparrow^{c,x}}$$

$$(\mathrm{f0})\frac{c \uparrow^{c,s} \quad p \uparrow^{c,x}}{\texttt{pres}\ s\ \texttt{?}\ p \diamond q\ \texttt{end} \uparrow^{c,x}} \qquad (\mathrm{f1})\frac{\neg c \uparrow^{c,s} \quad q \uparrow^{c,x}}{\texttt{pres}\ s\ \texttt{?}\ p \diamond q\ \texttt{end} \uparrow^{c,x}}$$

Chasing...

$\mathrm{P1} \doteq \texttt{pres s ? emit s} \diamond \texttt{0 end}$

Solution 1: Revisited

$$(\mathrm{e0})\ \frac{}{\texttt{emit}\ x \uparrow^{c,x}} \qquad (\mathrm{s0})\ \frac{p \uparrow^{c,x}}{p\ ;\ q \uparrow^{c,x}}$$

$$(\mathrm{f0})\ \frac{c \uparrow^{c,s} \quad p \uparrow^{c,x}}{\texttt{pres}\ s\ ?\ p \diamond q\ \texttt{end} \uparrow^{c,x}} \qquad (\mathrm{f1})\ \frac{\neg c \uparrow^{c,s} \quad q \uparrow^{c,x}}{\texttt{pres}\ s\ ?\ p \diamond q\ \texttt{end} \uparrow^{c,x}}$$

Chasing...

P1 $\doteq$ pres s ? emit s ◇ 0 end ×

$\{P1 \uparrow^{P1,s}, \texttt{emit}\ s \uparrow^{P1,s}\}$

and

$\{\texttt{emit}\ s \uparrow^{P1,s}\}$

Solution 1: Revisited

$$(\text{e0})\ \frac{}{\texttt{emit}\ x \uparrow^{c,x}} \qquad (\text{s0})\ \frac{p \uparrow^{c,x}}{p\ ;\ q \uparrow^{c,x}}$$

$$(\text{f0})\ \frac{c \uparrow^{c,s} \quad p \uparrow^{c,x}}{\texttt{pres}\ s\ ?\ p \diamond q\ \texttt{end} \uparrow^{c,x}} \qquad (\text{f1})\ \frac{\neg c \uparrow^{c,s} \quad q \uparrow^{c,x}}{\texttt{pres}\ s\ ?\ p \diamond q\ \texttt{end} \uparrow^{c,x}}$$

Confused...

P2 $\doteq$ `pres s ? 0` $\diamond$ `emit s end` ×

$P2 \uparrow^{P2,s}$?

Solution 1: Revisited

$$(\text{e0})\frac{}{\texttt{emit}\ x \uparrow^{c,x}} \qquad (\text{s0})\frac{p \uparrow^{c,x}}{p\ \texttt{;}\ q \uparrow^{c,x}}$$

$$(\text{f0})\frac{c \uparrow^{c,s} \qquad p \uparrow^{c,x}}{\texttt{pres}\ s\ \texttt{?}\ p \diamond q\ \texttt{end} \uparrow^{c,x}} \qquad (\text{f1})\frac{\neg c \uparrow^{c,s} \qquad q \uparrow^{c,x}}{\texttt{pres}\ s\ \texttt{?}\ p \diamond q\ \texttt{end} \uparrow^{c,x}}$$

Rome...

P3 $\doteq$ `pres s ? emit s` $\diamond$ `emit s end` ✓

$\{P3 \uparrow^{P3,s}, \texttt{emit}\ s \uparrow^{P3,s}\}$.

Solution 2

Constructive Semantics

An Esterel program p is meaningful if each formula of the form $p \uparrow^{p,x}$ has either a supported proof or a supported refutation.

Solution 1: Revisited

$$(\mathrm{e0})\frac{}{\texttt{emit}\ x \uparrow^{c,x}} \qquad (\mathrm{s0})\frac{p \uparrow^{c,x}}{p\ ;\ q \uparrow^{c,x}}$$

$$(\mathrm{f0})\frac{c \uparrow^{c,s} \quad p \uparrow^{c,x}}{\texttt{pres}\ s\ ?\ p \diamond q\ \texttt{end} \uparrow^{c,x}} \qquad (\mathrm{f1})\frac{\neg c \uparrow^{c,s} \quad q \uparrow^{c,x}}{\texttt{pres}\ s\ ?\ p \diamond q\ \texttt{end} \uparrow^{c,x}}$$

Rome...

$\mathrm{P3} \doteq \texttt{pres s ? emit s} \diamond \texttt{emit s end}$?

$\{P3 \uparrow^{P3,s}, \texttt{emit}\ s \uparrow^{P3,s}\}$.

Solution 1: Revisited

Rome...

$P3 \doteq$ `pres s ? emit s` $\diamond$ `emit s end` ?

$\{P3 \uparrow^{P3,s}$, `emit` $s \uparrow^{P3,s}\}$.

Rome...: Revisited

$$\mathtt{P3} \doteq \mathtt{pres\ s\ ?\ emit\ s} \diamond \mathtt{emit\ s\ end}$$

$$\frac{P3 \uparrow^{P3,s} \quad \frac{}{\mathtt{emit}\ s \uparrow^{P3,s}} \quad \text{or} \quad P3 \uparrow^{P3,s} \quad \frac{}{\mathtt{emit}\ s \uparrow^{P3,s}}}{P3 \uparrow^{P3,s}}$$

$$\frac{\neg P3 \uparrow^{P3,s} \quad \text{or} \quad P3 \uparrow^{P3,s} \quad \text{or} \quad \neg\mathtt{emit}\ s \uparrow^{P3,s}}{\neg P3 \uparrow^{P3,s}}$$

Rome...: Revisited

$$\mathtt{P3} \doteq \mathtt{pres\ s\ ?\ emit\ s} \diamond \mathtt{emit\ s\ end} \ \times$$

$$\frac{P3 \uparrow^{P3,s} \quad \frac{}{\mathtt{emit}\ s \uparrow^{P3,s}} \quad \text{or} \quad P3 \uparrow^{P3,s} \quad \frac{}{\mathtt{emit}\ s \uparrow^{P3,s}}}{P3 \uparrow^{P3,s}}$$

$$\frac{\neg P3 \uparrow^{P3,s} \quad \text{or} \quad P3 \uparrow^{P3,s} \quad \text{or} \quad \neg\mathtt{emit}\ s \uparrow^{P3,s}}{\neg P3 \uparrow^{P3,s}}$$

Conclusions

Done:

1. An analogy between causality in Esterel and cyclic rules and negative premises in SOS,
2. A unified framework for both causal and constructive semantics Esterel

To be done:

1. Extend the framework to the full language
2. Formal comparison among different semantics of Esterel
3. Extending the framework to other synchronous languages: Signal, Lustre, StateFlow

Grazie

Ci sono domande?

Unified Semantics of Esterel (Part II - Transition)

$$(\text{nil})\frac{}{\mathtt{0}\checkmark_c} \qquad (\text{em})\frac{}{\mathtt{emit}\ x \xrightarrow{c,x} \mathtt{0}}$$

Unified Semantics of Esterel (Part II - Transition)

$$(\text{nil})\,\frac{}{0\checkmark_c} \qquad (\text{em})\,\frac{}{\texttt{emit}\ x \xrightarrow{c,x} 0}$$

$$(\text{seq0})\,\frac{p \xrightarrow{c,x} p' \quad p'\checkmark_c \quad q \xrightarrow{c,x'} q'}{p\ ;\ q \xrightarrow{c,x} q'} \qquad (\text{seq1})\,\frac{p \xrightarrow{c,x} p' \quad p'\checkmark_c \quad q \xrightarrow{c,x'} q'}{p\ ;\ q \xrightarrow{c,x'} q'}$$

$$(\text{seq2})\,\frac{p\checkmark_c \quad q \xrightarrow{c,x} q'}{p\ ;\ q \xrightarrow{c,x} q'} \qquad (\text{seq3})\,\frac{p \xrightarrow{c,x} p' \quad p'\checkmark_c \quad q\checkmark_c}{p\ ;\ q \xrightarrow{c,x} p'} \qquad (\text{seq4})\,\frac{p\checkmark_c \quad q\checkmark_c}{p\ ;\ q\checkmark_c}$$

Unified Semantics of Esterel (Part II - Transition)

$$(\text{nil})\frac{}{0\surd_c} \qquad (\text{em})\frac{}{\texttt{emit } x \stackrel{c,x}{\rightarrow} 0}$$

$$(\text{seq0})\frac{p \stackrel{c,x}{\rightarrow} p' \quad p'\surd_c \quad q \stackrel{c,x'}{\rightarrow} q'}{p \ ; \ q \stackrel{c,x}{\rightarrow} q'} \qquad (\text{seq1})\frac{p \stackrel{c,x}{\rightarrow} p' \quad p'\surd_c \quad q \stackrel{c,x'}{\rightarrow} q'}{p \ ; \ q \stackrel{c,x'}{\rightarrow} q'}$$

$$(\text{seq2})\frac{p\surd_c \quad q \stackrel{c,x}{\rightarrow} q'}{p \ ; \ q \stackrel{c,x}{\rightarrow} q'} \qquad (\text{seq3})\frac{p \stackrel{c,x}{\rightarrow} p' \quad p'\surd_c \quad q\surd_c}{p \ ; \ q \stackrel{c,x}{\rightarrow} p'} \qquad (\text{seq4})\frac{p\surd_c \quad q\surd_c}{p \ ; \ q\surd_c}$$

$$(\text{par0})\frac{p \stackrel{c,x}{\rightarrow} p' \quad q \stackrel{c,x'}{\rightarrow} q'}{p \ || \ q \stackrel{c,x}{\rightarrow} p' \ || \ q'} \qquad (\text{par2})\frac{p\surd_c \quad q \stackrel{c,x}{\rightarrow} q'}{p \ || \ q \stackrel{c,x}{\rightarrow} q'} \qquad (\text{par4})\frac{p\surd_c \quad q\surd_c}{p \ || \ q\surd_c}$$

Unified Semantics of Esterel (Part II - Transition)

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$$(\text{seq0})\frac{p \overset{c,x}{\rightarrow} p' \quad p'\checkmark_c \quad q \overset{c,x'}{\rightarrow} q'}{p\ ;\ q \overset{c,x}{\rightarrow} q'} \qquad (\text{seq1})\frac{p \overset{c,x}{\rightarrow} p' \quad p'\checkmark_c \quad q \overset{c,x'}{\rightarrow} q'}{p\ ;\ q \overset{c,x'}{\rightarrow} q'}$$

$$(\text{seq2})\frac{p\checkmark_c \quad q \overset{c,x}{\rightarrow} q'}{p\ ;\ q \overset{c,x}{\rightarrow} q'} \qquad (\text{seq3})\frac{p \overset{c,x}{\rightarrow} p' \quad p'\checkmark_c \quad q\checkmark_c}{p\ ;\ q \overset{c,x}{\rightarrow} p'} \qquad (\text{seq4})\frac{p\checkmark_c \quad q\checkmark_c}{p\ ;\ q\checkmark_c}$$

$$(\text{par0})\frac{p \overset{c,x}{\rightarrow} p' \quad q \overset{c,x'}{\rightarrow} q'}{p \mid\mid q \overset{c,x}{\rightarrow} p' \mid\mid q'} \qquad (\text{par2})\frac{p\checkmark_c \quad q \overset{c,x}{\rightarrow} q'}{p \mid\mid q \overset{c,x}{\rightarrow} q'} \qquad (\text{par4})\frac{p\checkmark_c \quad q\checkmark_c}{p \mid\mid q\checkmark_c}$$

$$(\text{if0})\frac{c \uparrow^{c,s} \quad p \overset{c,x}{\rightarrow} p'}{\texttt{pres}\ s\ ?\ p \diamond q\ \texttt{end} \overset{c,x}{\rightarrow} p'} \qquad (\text{if4})\frac{c \uparrow^{c,s} \quad p\checkmark_c}{\texttt{pres}\ s\ ?\ p \diamond q\ \texttt{end}\checkmark_c}$$