MRI course
Variational and Topological Methods for PDEs

Exercises 1

1. Let $\Omega \subset \mathbb{R}^n$ be open, and $f \in C(\mathbb{R})$. Consider

$$\begin{aligned} -\Delta u &= f(u) && \Omega \\ u &= 0 && \partial\Omega. \end{aligned}$$

a) Assume that f is non-increasing. Use Schauder's fixed point theorem to prove existence.

b) Does uniqueness hold?

c) Can you allow for increasing f? How far can you generalize?

d) Can you also use the contraction mapping theorem?

2. Show that Schauder's fixed point theorem is invalid as soon as either the convexity or the compactness condition is discarded.