MRI course
Variational and Topological Methods for PDEs

Exercises 6

1. Show by example that without convexity in p of $L(p, z, x)$ the weak lower semicontinuity does not necessarily hold. (Can you even make a stronger statement: without convexity the weak lower semicontinuity *can not* hold?)

2. In the course (and in Evans, section 8.2) we used a Dirichlet boundary condition to deduce boundedness in $W^{1,q}$ of a minimizing sequence. Formulate a condition on L that allows to deduce the same boundedness property without using the boundary condition.

3. Let $F : \mathbb{R} \to \mathbb{R}$ be convex. Verify that the functional

$$I(u) := \int |\nabla u|^q + \int F(u)$$

   is strictly convex on $W_0^{1,q}(\Omega)$ if either $q > 1$ or F is strictly convex (or both).

4. Do exercise 8.6.8 of Evans

5. Do exercise 8.6.3 of Evans