Alternative discretization in the aperiodic Fourier modal method leading to reduction in computational costs

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ABSTRACT

The Fourier modal method (FMM), also referred to as Rigorous Coupled-Wave Analysis (RCWA), is based on Fourier-mode expansions and is inherently built for periodic structures such as diffraction gratings. When the infinite periodicity assumption is not realistic, the finiteness of the structure has to be incorporated into the model. In this paper we discuss the recent extensions of the FMM for finite structures. First, we explain how an efficient FMM-based method for finite structures is obtained by a reformulation of the governing equations and incorporation of perfectly matched layers (PMLs). Then we show that the computational cost of the method can be further reduced by employing an alternative discretization instead of the classical one. Numerical results demonstrate the characteristics of the discussed FMM-based methods and include a discussion of computational complexities.

Keywords: Fourier-modal method, FMM, rigorous coupled-wave analysis, RCWA, perfectly matched layers, PML, computational costs, aperiodic Fourier-modal method, AFMM-CFF, alternative discretization

1. INTRODUCTION

As the dimension of photonic structures decreases below the diffraction limit, inspection and design rely more intensely on rigorous modeling. In this paper we focus on efficient rigorous modeling of diffraction gratings while taking into account their finite extent. Maxwell’s equations with radiation boundary conditions lie at the heart of the considered mathematical model for which a cost-efficient numerical solution is required. During the last decades many numerical methods for solving Maxwell equations have been developed. Several of them are widely used: the finite-element method (FEM),\textsuperscript{1-3} the finite-difference time-domain method (FDTD)\textsuperscript{4-6} and the integral equation methods (IEM), which include the boundary element method (BEM)\textsuperscript{7, 8} and the volume integral method (VIM)\textsuperscript{9-13}.

The enumerated methods are derived from general-purpose numerical discretization schemes. In the field of diffractive optics one specialized method has gained a large popularity due to its simplicity and natural interpretation of the field expansions. It is known under several names and abbreviations among which are: rigorous coupled-wave analysis (RCWA), modal method with Fourier expansion (MMFE), and the Fourier-modal method (FMM). The latter name is used in this paper. The FMM\textsuperscript{14} was proposed in 1978 by Knop.\textsuperscript{15} In the past decades the method has matured due to fundamental studies and improvements to its stability\textsuperscript{16, 17} and convergence.\textsuperscript{18-20} Other important contributions to the evolution of the method are the techniques of adaptive spatial resolution\textsuperscript{21} and normal vector fields.\textsuperscript{22-24} Ref. 25 gives a mathematical perspective of the challenges that have been overcome in the FMM and of the open problems still to be addressed. For possible improvements of the method see Chapter 7 in Ref. 26.

In the process of modeling periodic structures with this method an important assumption is routinely made: it is considered that the periodic structure, which is finite in reality, is approximated reasonably well by an idealized infinite periodic structure. In this case the field is quasi-periodically repeating from one period to another and it suffices to compute the solution in a single period of the structure as shown in Figure 1 (a). Obviously this leads to very low computational costs. The disadvantage however lies in the “infinite periodicity”
Figure 1. Overview of FMM-based methods for modeling finite periodic structures. (a) Classical FMM employing the "infinite periodicity" assumption. Only the field in the highlighted cell needs to be computed. (b) Supercell FMM. (c) AFMM-CFF with classical discretization. (d) AFMM-CFF with alternative discretization.

We summarize the FMM-based approaches which incorporate finiteness in Figures 1 (b), (c), and (d). The most straightforward extension of the FMM to finite structures is the so-called supercell approach, depicted in Figure 1 (b). The computational domain is extended to include all periods of the structure. Moreover, extra space is left on both sides of the structure in order to decrease the coupling/reflections via the periodic boundary conditions which are built-in in the method due to the use of Fourier-mode expansions. The more empty space is left on the sides, the better the model represents finiteness. The computational cost significantly increases both as the number of periods in the structure becomes larger and as the empty spaces are chosen wider. In practice, this means that supercell FMM can only be used for small structures (with a size of several tens of wavelengths).

A more efficient approach is to replace the wide (and expensive) empty spaces by narrow artificial absorbers as done in the aperiodic Fourier modal method in contrast-field formulation (AFMM-CFF) and depicted in Figure 1 (c). The AFMM-CFF\cite{27,28,29} is a recent extension of the FMM to aperiodic structures. It builds upon the aperiodic Fourier modal method developed and refined by Lalanne and co-workers\cite{30,31,32,33,34} with the key difference that the AFMM-CFF solves Maxwell’s equations formulated in terms of a contrast (scattered) field instead of a total field. This reformulation allows prescription of arbitrary incoming fields onto the structure of interest.\cite{27} Aperiodicity is achieved by using perfectly matched layers (PMLs)\cite{35} on the vertical boundaries in order to annihilate the periodic boundary condition.

Finally, it turns out that the computational cost for AFMM-CFF can be further reduced by applying an alternative discretization,\cite{36} which can also be viewed as a classical FMM discretization applied to a rotated geometry as shown in Figure 1 (d). This alternative discretization is chosen based on complexity arguments and will be explained in Section 5.
We describe the four approaches in Sections 2 - 5. Numerical results and conclusions are presented in Section 6.

2. CLASSICAL FMM

We are concerned with solving the time-harmonic Maxwell equations for non-magnetic materials on the domain $(x, y, z) \in [0, \Lambda] \times \mathbb{R} \times \mathbb{R}$.

\[
\nabla \times \mathbf{e}(\mathbf{x}) = -k_0 \mathbf{h}(\mathbf{x}), \\
\nabla \times \mathbf{h}(\mathbf{x}) = -k_0 \epsilon(x, z) \mathbf{e}(\mathbf{x}),
\]

where $\mathbf{x} = (x, y, z)$ is the position vector, $\mathbf{e} = (e_x, e_y, e_z)$ is the electric field and $\mathbf{h} = (h_x, h_y, h_z)$ is the magnetic field scaled by $-i\sqrt{\epsilon \mu}/\mu_0$. The temporal frequency $\omega$ is incorporated into the constant $k_0 = \omega\sqrt{\epsilon \mu_0}$. The electric permittivity $\epsilon$ is assumed $y$-invariant. See Figure 2 for a sample geometry. The incident field is given by

\[
e^{inc}(\mathbf{x}) = a e^{-ik^{inc} \cdot \mathbf{x}},
\]

where $k^{inc} = (k^{inc}_x, k^{inc}_y, k^{inc}_z)$ is the wavevector and $a = (a_x, a_y, a_z)$ is the amplitude vector. Note that the Maxwell equations require that $k^{inc} \cdot a = 0$.

For simplicity, we consider the special case of a planar TE incident wave

\[
e^{inc}_y(x, z) = a_y e^{-i(k^{inc}_x x + k^{inc}_z z)},
\]

such that Maxwell’s equations are reduced to a single scalar equation for the $y$ component of the electric field,

\[
\frac{\partial^2}{\partial z^2} e_y + k_0^2 \epsilon(x, z) e_y = 0.
\]

We now describe the classical FMM for this equation. The first step is to divide the computational domain into layers such that the permittivity may be considered $z$-independent in each single layer. Thus, the profile of the scatterer is approximated by a staircase as in Figure 2. Now in each layer $j = 1, ..., M$ the permittivity $\epsilon_j(x)$ is independent of $z$.

\[
\frac{\partial^2}{\partial z^2} e_{y,j}(x, z) = -\Sigma_j e_{y,j}(x, z), \quad \text{with} \quad \Sigma_j = \frac{\partial^2}{\partial x^2} + k_0^2 \epsilon_j(x).
\]
with matching conditions
\[ B_j e_{y,j}(x, h_j) = B_{j+1} e_{y,j+1}(x, h_j), \quad \text{with } B_j = \begin{pmatrix} 1 \\ k-1 \\ 0 \\ \partial/\partial z \end{pmatrix}. \] (5)

where \( h_j \) is the z-coordinate of the top interface of layer \( j \). The next step is to discretize the equations in the \( x \)-direction by using a Galerkin approach with "shifted" Fourier harmonics as basis functions and test functions,
\[ \phi_n(x) = e^{-ik_{xn}x}, \quad \text{where } k_{xn} = k_{x_{inc}} + n\frac{2\pi}{\Lambda}, \quad \text{for } n = -N, ..., +N. \] (6)

In each layer the electric fields are expanded as
\[ e_{y,j}(x, z) = \sum_{n=-N}^{N} u_{j,n}(z)\phi_n(x). \] (7)

The standard inner product on the interval \([0, \Lambda]\) is used. After the application of the Galerkin method, Equations (4) become
\[ u_j''(z) = -L_j u_j(z). \] (8)

Here, matrix \( L_j \in \mathbb{R}^{(2N+1)\times(2N+1)} \) and vector \( u_j \in \mathbb{R}^{(2N+1)} \). This is a homogeneous second-order ordinary differential equations whose general solution is given by
\[ u_j(z) = W_j(e^{-Q_j(z-h_j)}c_j^+ + e^{Q_j(z-h_{j+1})}c_j^-), \] (9a)

where \( W_j \) is the matrix of eigenvectors of \( L_j \) and \( Q_j \) is a diagonal matrix with square roots of the corresponding eigenvalues on its diagonal. An eigenvalue decomposition costs \( O(N^3) \) operations. For all layers, the cost of computing analytical solutions is \( O(MN^3) \).

As final step, the matching conditions (5) are applied to the analytical solution (9). Care has to be taken such that the application of the matching condition does not introduce instabilities in the algorithm.\(^{17} \)

After some algebraic manipulations we arrive at expressions of this form
\[ \begin{pmatrix} c_{j+1}^+ \\ c_j^- \end{pmatrix} = S_{j,j+1} \begin{pmatrix} c_j^+ \\ c_{j+1}^- \end{pmatrix}, \] (10)

where the matrix \( S_{j,j+1} \) depends on the layer matrices \( W_j, W_{j+1}, Q_j, Q_{j+1} \). This is a system of recursive linear equations. It can be solved by eliminating the unknown coefficients for the intermediary layers or equivalently by computing the global S-matrix for all layers
\[ S_{1,M} = S_{1,2} * S_{2,3} * \ldots * S_{M-1,M}. \] (11)

Here the * symbol denotes the (homogeneous) Redheffer star product.\(^{37} \) Since a Redheffer star product is computed in \( O(N^3) \) operations, computing the global S-matrix will cost \( O(MN^3) \). Given the global S-matrix we directly evaluate the expression
\[ \begin{pmatrix} c_M^- \\ c_1^+ \end{pmatrix} = S_{1,M} \begin{pmatrix} c_M^+ \\ c_M^- \end{pmatrix}. \] (12)

where the vectors \( c_1^+ \) and \( c_M^- \) on the right-hand side represented the incident field above and below the stack respectively.

Finally, if the near field is required, the intermediary coefficients can be computed recursively starting from the first or last layer. If the computed inverses computed for the Redheffer star product are properly reused, then only matrix-vector products are required in this step. Therefore, the computational cost scales with \( O(MN^2) \).

The main steps of the method are summarized below.
1. Slice the computational domain into layers, so that $\epsilon$ does not depend on $z$ in each layer (negligible cost).
2. Apply Galerkin approach for discretisation in the $x$-direction (negligible cost).
3. Derive/compute the general solution in each layer, $O(MN^3)$.
4. Match the solutions at layer interfaces, $O(MN^2)$.
5. Compute intermediary coefficients, $O(MN^2)$.

Thus the estimated total cost of the method is given by

$$
T = (c^T_{\text{diag}} + c^T_S)M'N'^3 + c^TƦ M'N'^2,
$$
(13a)

$$
M = (c^M_{\text{diag}} + c^M_S)M'N'^2 + c^MƦ M'N'.
$$
(13b)

We have used the constants $N'$ and $M'$ which give respectively the number of harmonics and number of slices per period. This is done in order to facilitate the comparison of complexities of different methods. Since the classical FMM models a single period of the structure we have $M = M'$ and $N = N'$. For the other methods this will not be the case. For a reasonably large $N'$, the second term in the above expressions (being one order lower) can be neglected. We will also assume (supported by evidence from practical experiments) that $c^T_{\text{diag}} = c^S_{\text{diag}} = \frac{1}{2}c^T_{\text{diag}}$. Then estimate (13) reduces to

$$
T \approx c^T M'N'^3,
$$
(14a)

$$
M \approx c^M M'N'^2.
$$
(14b)

### 3. SUPERCELL FMM

The supercell version of the FMM requires the user only to define a different geometry without having to modify the method itself. The new geometry will include the entire grating as well as empty space on the sides (see Figure 1 (b)) where the field can decay such that the effect of the periodic boundary condition is minimized.

We introduce the domain expansion factor $S$ as the ratio of the widths of the computational domain and of the grating. For an accurate representation of finiteness this factor should be large, $S \gg 1$. Also let $R$ be the number of periods of the grating. In order to achieve the same accuracy per period as in the standard FMM, we choose $N = SRN'$ and the computational costs are given by

$$
T \approx c^T S^3R^3M'N'^3,
$$
(15a)

$$
M \approx c^M S^2R^2M'N'^2.
$$
(15b)

### 4. AFMM-CFF WITH CLASSICAL DISCRETIZATION

The periodic basis functions (6) force the solution to be periodic. For an aperiodic scatterer we need to implement the radiation BC at the lateral boundaries. One way of achieving this without changing the basis is to place perfectly matched layers (PMLs)\cite{35} of a certain thickness just before the boundary. This approach has been previously used to apply the FMM to waveguide problems.\cite{30,31,33} The PML changes the $x$-derivative in the differential equations (4) as follows

$$
\frac{\partial}{\partial x} \rightarrow \frac{1}{f'(x)} \frac{\partial}{\partial x}, \text{ with } f(x) = x + i\beta(x).
$$
(16)

The function $\beta(x)$ is continuous and non-zero only in the PMLs which are placed in the intervals $[0, x_{i}]$ and $[x_{r}, \Lambda]$. An example of such a function is shown in Figure 3. The modified version of the $\Sigma_{j}$ operator where the $x$-derivative has been changed according to (16) will be denoted by $\hat{\Sigma}_{j}$. In order to avoid modifications of the incident field by the PML, the computed solution should consist of only outgoing waves.\cite{27} Therefore the
equations are reformulated such that the incident field, i.e. the known part of the solution, is moved into the
source. In addition to the total field problem
\[ \frac{\partial^2}{\partial x^2} e_{y,j}(x,z) = -\tilde{\Sigma}_j e_{y,j}(x,z), \quad e_{y}^{\text{inc}}(x,z) = a_y e^{-i(k_x^{\text{inc}} x + k_z^{\text{inc}} z)}, \text{ where } \tilde{\Sigma}_j = \frac{1}{f'(x)} \frac{\partial}{\partial x} \frac{1}{f'(x)} \frac{\partial}{\partial x} + \epsilon_j(x), \]  \tag{17} \]
we define a background problem with PMLs
\[ \frac{\partial^2}{\partial x^2} e_{y,j}(x,z) = -\tilde{\Sigma}_j e_{y,j}(x,z), \quad e_{y}^{\text{inc}}(x,z) = a_y e^{-i(k_x^{\text{inc}} x + k_z^{\text{inc}} z)}, \text{ where } \tilde{\Sigma}_j = \frac{1}{f'(x)} \frac{\partial}{\partial x} \frac{1}{f'(x)} \frac{\partial}{\partial x} + \epsilon_j(x), \]  \tag{18} \]
and a background problem without PMLs
\[ \frac{\partial^2}{\partial x^2} e_{y,j}(x,z) = -\tilde{\Sigma}_j e_{y,j}(x,z), \quad e_{y}^{\text{inc}}(x,z) = a_y e^{-i(k_x^{\text{inc}} x + k_z^{\text{inc}} z)}, \text{ where } \tilde{\Sigma}_j = \frac{1}{f'(x)} \frac{\partial}{\partial x} \frac{1}{f'(x)} \frac{\partial}{\partial x} + \epsilon_j(x). \]  \tag{19} \]
Note that \( e_j^{\text{b}} \) is chosen to be \( x \)-independent, i.e. it represents a multilayer stack. This implies that the background problem without PMLs has an analytical solution. Since the PML effectively implements the radiation conditions, for an ideal PML the two background problems have equal solutions in the physical domain (the region between the PMLs).
\[ \tilde{e}_j^{\text{b}}(x,z) = e_j^{\text{b}}(x,z), \text{ for } x \in [x_l, x_r] \times \mathbb{R}. \]  \tag{20} \]
Subtracting (18) from (17) and defining the contrast field \( \tilde{e}^c = \tilde{e} - e_j^{\text{b}} \), we get
\[ \frac{\partial^2}{\partial x^2} e_{y,j}^c(x,z) = -\tilde{\Sigma}_j e_{y,j}^c(x,z) - (\tilde{\Sigma}_j - \tilde{\Sigma}_j^{\text{b}}) e_{y,j}^{\text{b}}(x,z), \quad e_{y}^{\text{inc}}(x,z) = 0. \]  \tag{21} \]
The incident field has been removed, and a non-homogeneous term appears on the right-hand side. The background permittivity is chosen such that \( \epsilon(x,z) - \epsilon_j^{\text{b}}(z) \) vanishes in the PML and has compact support. As a consequence, \( \tilde{\Sigma}_j - \tilde{\Sigma}_j^{\text{b}} \neq 0 \) only for \( x \in [x_l, x_r] \), which according to (20) entitles the substitution of \( e_{y,j}^{\text{b}}(x) \) by \( e_{y,j}^c(x) \) in (21).

Once the source-term is determined, Equation (21) may be solved. For this purpose, the source term must also be expanded into Fourier modes. After truncation a non-homogeneous system of ordinary differential equations is obtained for each layer. The field is found by matching the general solutions at the layer interfaces. This leads to non-homogeneous equations of the form
\[ \begin{pmatrix} c^{j+1}_j \\ c^{j+1}_j \end{pmatrix} = S_{j,j+1} \begin{pmatrix} c^{j}_j \\ c^{j}_j \end{pmatrix} + \begin{pmatrix} f^{j+1}_{j,j+1} \\ f^{j+1}_{j,j+1} \end{pmatrix}. \]  \tag{22} \]
The system of recursive linear equations is solved by computing the so-called global matrix-vector pair
\[ (S_{1,M}, f_{1,M}) = (S_{1,2}, f_{1,2}) \ast (S_{2,3}, f_{2,3}) \ast \ldots \ast (S_{M-1,M}, f_{M-1,M}). \]  
(23)

Here the \( \ast \) symbol denotes the non-homogeneous Redheffer star product.\(^{28,36}\) The computation of the matrix-vector pair is dominated by the matrix computation. For all layers, the cost again amounts to \( O(MN^3) \). Note that the number of harmonics \( N \) required in the discretization will depend on the number of periods \( R \), i.e. \( N = RN' \). The time and memory complexity of the method is given by
\[ T \approx c^T R^3 M' N'^3, \]  
(24a)
\[ M \approx c^M R^2 M' N'^2. \]  
(24b)

5. AFMM WITH ALTERNATIVE DISCRETIZATION

It appears from (24) that the \( x \)-direction (for which harmonics are used) is "more expensive" both in terms of time and memory than the \( z \)-direction (for which slices are used). For rectangular scatterers/domains that are much longer in the \( x \)-direction it is reasonable to choose an alternative discretization: make the longer direction "cheaper" by using spatial discretization into layers and apply spectral discretization in the shorter direction.

An ideal PML yields an effective radiation boundary condition. Thus, unlike in the (periodic) FMM, effectively the same boundary condition is imposed on all boundaries. This fact facilitates the exchange of discretization directions. If the problem is first rotated from the \((x,y,z)\) coordinates to the \((z,-y,x)\) coordinates, the same algorithm as described in the previous section can be used. For a discussion on the projection of the analytically computed background field onto the new basis see Ref. 36.

Due to the periodicity in the vertical direction, the S-matrices are also periodically repeated. As before, the non-homogeneous S-matrix algorithm\(^ {28,36} \) uses the iteration
\[ (S_{1,j+1}, f_{1,j+1}) = (S_{1,j}, f_{1,j}) \ast (S_{j,j+1}, f_{j,j+1}). \]  
(25)
The global matrix-vector pair for \( M - 1 \) interfaces (\( M \) layers) is given by
\[ (S_{1,M}, f_{1,M}) = \ldots \ast ([S_{1,2}, f_{1,2}] \ast [S_{2,3}, f_{2,3}] \ast [S_{3,4}, f_{3,4}] \ast \ldots \ast (S_{M-1,M}, f_{M-1,M})]. \]  
(26)
The left side of Figure 4 gives a visual representation of the merging process. The cost of computing (26) scales linearly with the number of periods,
\[ T_S = O(RM' N'^3), \]  
(27a)
\[ M_S = O(RM' N'^2). \]  
(27b)

Due to the associativity property of the non-homogeneous Redheffer product, we can regroup the multiplication operations. We observe that once we have computed the matrix-vector corresponding to the stack \( \{1, \ldots, M' + 1\} \), the matrix-vector pair corresponding to the stack \( \{M' + 1, \ldots, 2M' + 1\} \) can be computed with a minimal effort
\[ (S_{M'+1,2M'+1}, f_{M'+1,2M'+1}) = (S_{1,M'+1,1}, \nu f_{1,M'+1}), \]  
(28)
where \( \nu = \exp(-k_0 q(h_{M'+1} - h_1)) \) is a phase factor which appears due to the scalar \( z \)-dependence of \( f \). Thus, we can directly compute the matrix-vector pair corresponding to the stack \( \{1, \ldots, 2M' + 1\} \)
\[ (S_{1,2M'+1}, f_{1,2M'+1}) = (S_{1,M'+1,1}, f_{1,M'+1}) \ast (S_{1,M'+1,1}, \nu f_{1,M'+1}). \]  
(29)
The iteration which computes the matrix-vector pair for an exponentially increasing number of layers is depicted on the right side of Figure 4 and defined by
\[ (S_{1,2^{k+1}M'+1}, f_{1,2^{k+1}M'+1}) = (S_{1,2^kM'+1}, f_{1,2^kM'+1}) \ast (S_{1,2^kM'+1}, \nu^{2^k} f_{1,2^kM'+1}), \]  
(30)
for \( k = 0, \ldots, R - 1 \). By exploiting the local periodicity the computational costs have been reduced down to
\[ T_S = O((M' + \log_2(R)) N'^3), \]  
(31a)
\[ M_S = O((M' + \log_2(R)) N'^2). \]  
(31b)
6. RESULTS AND CONCLUSIONS

In the previous sections we have briefly described four FMM-based approaches for modeling scattering from finite periodic structures. In this section their characteristics are summarized and some simulation results are discussed. Table 1 lists the time and memory complexity of the methods (based on (14), (15), (24), (31)), as well as their ability to model the effects of finiteness. The classical FMM appears to be the most efficient method in terms of complexity. It is however unable to incorporate finiteness in the model. The supercell FMM is the most simple approach among the ones able to model finiteness. The price for the simplicity is paid in terms of computational complexity. For instance, simulating a grating with \( R = 10 \) lines and a moderate domain expansion factor \( S = 5 \) would take \( S^3R^3 = 125000 \) more time than approximating the finite grating by an infinite one and using the FMM. The AFMM-CFF with classical and alternative discretization drastically reduce these costs. The AFMM-CFF with classical discretization removes the need for domain expansion by employing PMLs, such that for our example it will be 125 times faster than supercell FMM but still 1000 times slower than the FMM. Finally, the AFMM-CFF with alternative discretization also removes the cubic scaling with the number of periods by exploiting the repeating patterns in the geometry. In this way the complexity of the algorithm is brought very close to the complexity of the FMM while still being able to represent finiteness.

Figure 5 shows solutions computed with the four methods for a 32 line grating (resist on glass). We observe that the fields in the center are similar for all methods. This is explained by the fact that the effects of finiteness are most prominent on the edges as can be seen in Figure 5 (b), (c), (d). The classical FMM computes the solution on a single period. For comparison with the other methods it is repeated periodically over the extent of the finite grating. For the supercell FMM a larger domain \( (S = 5) \) is used and only the part of the
domain containing the grating is shown. The computational domains used in the AFMM-CFF with classical and alternative discretization coincides with the size of the grating and is shown in Figure 5 (c), (d) without modifications.

Figure 5. An example of solutions obtained with the FMM-based methods described in this paper for a 32 line grating (resist on glass). (a) FMM. (b) Supercell FMM. (c) AFMM-CFF. (d) AFMM-CFF with alternative discretization.

We conclude that FMM-based methods offer a range of approaches for simulating scattering from finite periodic structures. When the grating is large and the edge effects due to finiteness are not of interest the standard FMM should be applied for a single period of the grating. If the edge effects are important, then one of the three other methods should be used. The choice depends on the desired computational cost, on one hand, and the effort on the implementation, on the other hand. The supercell FMM is trivially obtained from the FMM by redefining the geometry. The AFMM-CFF requires a certain implementation effort, with the reward that computational costs are significantly reduced. If the alternative discretization is used these costs approach the ones of the standard FMM.

REFERENCES


