Performance Analysis of χ models via Markov Chain Generation

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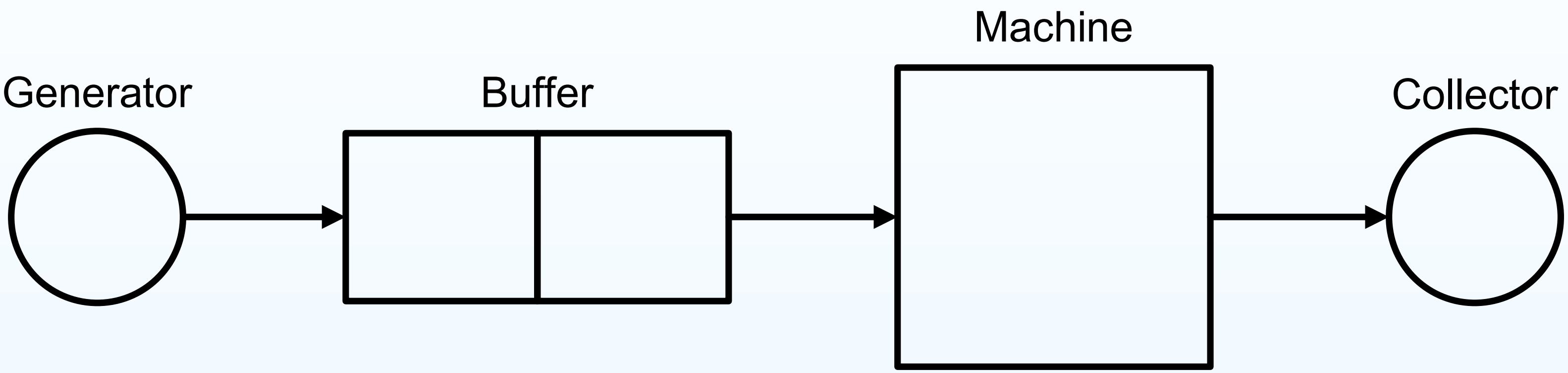
Overview

- Motivation
- Continuous Time Markov chains
- From χ to Markov chains
- Tool demonstration

Motivation - Manufacturing Line Example



Question: What is the utilization of the machine?

Motivation - Obtaining Performance by Simulation

How?

1. model the example in χ
2. add some bookkeeping information to the code
3. run a simulator

Drawbacks:

- χ code changed
- time consuming
- non-determinism problem

Motivation - Obtaining Performance Analytically

How?

1. model the example as a Markov Chain
2. use results from queuing theory or solve the MC directly

Drawbacks:

- non-compositional
- have to be an expert
- two different models (χ and MC)

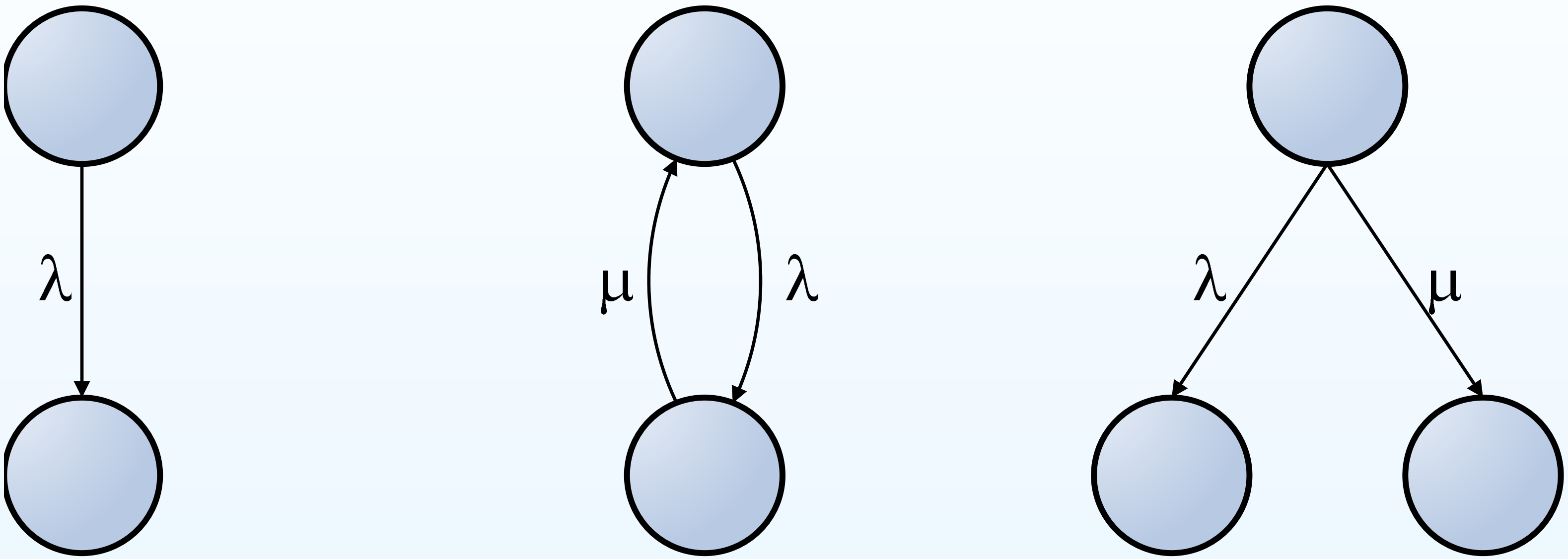
Motivation - Conclusion

It would be nice to obtain Markov Chains automatically from χ specifications.

Markov Chains - Exponential Distribution

- continuous distribution
- approximates the best if only expected value is known
- $P(X \leq t) = \begin{cases} 1 - e^{-\lambda t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$
- $P(X \leq t + \Delta t \mid X > t) = P(X \leq \Delta t)$ - memoryless property

Markov Chains - Examples

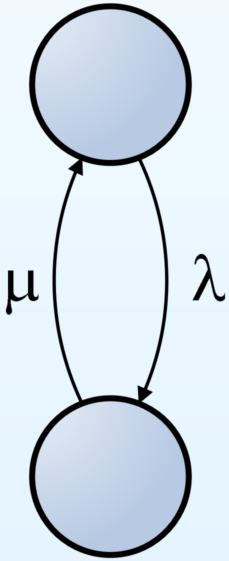


Markov Chains - Performance Analysis

Steady State Vector:

- probabilities of being in a state in a long run
- fast numerical methods

Example:

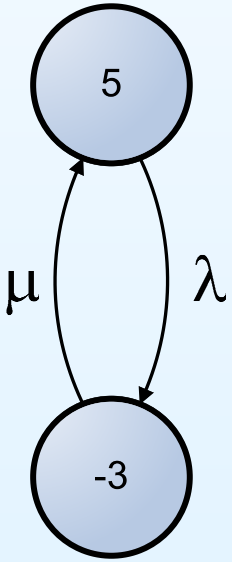


$$\pi = \left(\frac{\mu}{\lambda+\mu}, \frac{\lambda}{\lambda+\mu}\right).$$

Markov Chains - Rewards

- mechanism to obtain different measures from the steady state vector
- every state (i) is given a reward $r_i \geq 0$
- $R = \Sigma_{i=1}^{n} \pi_i \cdot r_i$ - total reward

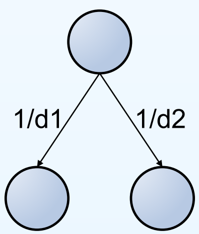
Example:



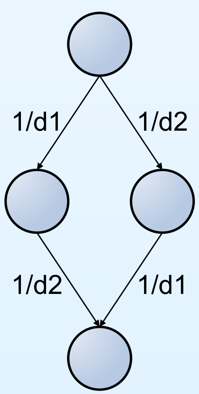
$R = \frac{5\mu - 3\lambda}{\lambda + \mu}$.

From χ to Markov Chains - Change in the Semantics

1. new meaning to delays
   - $\Delta 0$ is equivalent to ε
   - Δd is an exponential delay with rate $1/d$
2. change in SOS rules
   - $\Delta d_1 + \Delta d_2$ is represented with both transitions



   - $\Delta d_1 || \Delta d_2$ is with full interleaving



From χ to Markov Chains - Steps

1. generate a state space (IMC) using new SOS rules
2. hide all actions, leave only rates
3. apply the IMC minimizer of H. Hermanns

Tool Demonstration - χ Model of the Line

```
G(out:chan) = (out!0; delay 2.0)*; deadlock

B(in,out:chan) = |[xs:list[bool]=mtlist |
                      (    len(xs) < 2 :-> in?x; xs:=xs ++ (x:mtlist)
                        | len(xs) > 0 :-> out!hd(xs); xs:=tl(xs)
                      )*; deadlock ]|

M(in,out:chan)  = (in?x; delay 3.0; out!0)*;deadlock

E(in:chan) = in?x*; deadlock

sys()=  mp ( enc sa(*,*):ra(*,*):mtset (
            |[ ~gb,~bm, ~me |
                       G(~gb)
                       || B(~gb,~bm)
                       || M(~bm,~me)
                       || E(~me) ]| ))
```