ABSTRACTS GAMM PDE MEETING

EINDHOVEN UNIVERSITY OF TECHNOLOGY, 27–29 SEPTEMBER 2017

Fabian Christowiak – University of Regensburg.

Homogenization of layered materials with diverging elastic constants

We consider variational models for bilayered elastoplastic materials featuring one component with a large elastic constant. Our goal is to determine the effective material response as the layer thickness tends to zero in terms of Γ-convergence. Since the individual thin layers are easy to deform elastically, it is key to understand the antithetical relation between layer thickness and stiffness.

In this work we determine the optimal scaling relation, identifying two regimes. For slow divergence of the elastic constant we discuss explicit examples illustrating the effects of bending and wrinkling of layers on the macroscopic material behavior. In contrast, if the component becomes stiff sufficiently fast, sequences of uniformly bounded elastic energy lead to a rather resticted class of limit deformations. In the presence of a local volume preservation condition, they can be characterized as globally rotated shear deformations in layer direction. To quantify the stiffness of the individual layers we exploit the geometric rigidity estimate by Friesecke, James and Müller in 2002.

For a variety of models with a specific elastic or plastic softer component this characterization of limit deformation forms the basis to determine their Γ-limit. While knowing the particular limit structure is crucial for lower bound estimates, the restricted class of macroscopic defomations requires tailor-made recovery sequences for which we give explicit constructions.

This project is joint work with Carolin Kreisbeck, Universiteit Utrecht.

Patrick Dondl – University of Freiburg.

Gradient flow in a Phase Field Model for Dislocations with Forest Hardening

We consider a phase field model for dislocations introduced by Koslowski, Cuitino, and Ortiz in 2002. The model describes a single slip plane and consists of a Peierls potential penalizing non-integer slip and a long range interaction modeling elasticity. Forest dislocations are introduced as a restriction to the allowable phase field functions: they have to vanish at the union of a number of small disks in the plane. Garroni and Müller proved large scale limits of these models in terms of Gamma-convergence, obtaining a line-tension energy for the dislocations and a bulk term penalizing slip. This bulk term is a capacity stemming from the forest dislocations.

In the present work, we show that the contribution of the forest dislocations to the the viscous gradient flow evolution is small. In particular it is much slower than the timescale for other effects like elastic attraction/repulsion of dislocations, which, by a recent result due to del Mar Gonzales and Monneau is already slower than the time scale from line tension energy. Overall, this leads to an effective behavior like a gradient flow in a wiggly potential. On the other hand, of course, when adding a driving force in the direction of increasing slip, one needs to spend the energy to overcome the obstacles. The forest dislocations thus act like a dissipation for increasing slip, but their effect on the propagation is absent for decreasing slip.

Clemens Förster – University of Leipzig.

Piecewise constant subsolutions for the incompressible Porous Media Equation

A recent result by Castro, Córdoba and Faraco in [1] showed the existence of infinitely many weak solutions of the incompressible porous media equations for all Muskat type initial data with H^5-regularity of the interface and in the unstable regime. The main ingredient for the proof was the construction of a suitable subsolution, which was done analogously to the case of the flat interface by Székelyhidi in [2] and leads to a nonlinear evolution equation for the sheet. In this talk I present an alternative proof of this result. The density of the subsolution will be defined to be piecewise constant. In particular, we can choose an arbitrary fine approximation of the continuous density in [1]. The second main step is the observation that we can avoid solving the full nonlinear equation, since a power series ansatz up to order two is sufficient for the construction. Moreover, this method leads to a necessary regularity for the initial interface in the class $W^{4,1}(\mathbb{R}) \cap C^{4,\alpha}(\mathbb{R})$. As already mentioned in [1], we obtain a similar result for Muskat type initial data in the stable regime except of the at case.

[1] Castro. A., Cordoba. D., Faraco. D. "Mixing solutions for the Muskat problem," arXiv:1605.04822[math.AP]., (2016).

[2] Székelyhidi Jr.. L., "Relaxation of the incompressible Porous Media Equation," Ann. Sci. Éc. Norm. Supér., 45, No. 3, 491–509 (2012).

Gero Friesecke – TU Munich.

Curse of dimension and nonlinearity in partial differential equations

The currently fashionable topic of 'big data' is concerned with guessing, modelling, or 'learning' the often low-dimensional but highly nonlinear structure of high-dimensional data sets a priori located in a big linear space. This type of strategy existed long before the contemporary hype, and has been followed for almost a century in my own field of interest, PDE models of electronic structure. Only three years after the many-electron Schroedinger equation (a linear PDE in many variables) had been introduced, physicists were already approximating it by the Thomas-Fermi equation (a nonlinear PDE in a single variable) or by the Hartree-Fock equations (a system of nonlinear PDEs in a single variable). The latter models remain, to this day, important templates for refined state-of-the-art low-dimensional models like Kohn-Sham density functional theory (still a PDE model) or the density matrix renormalization group method (almost a PDE model).

I will begin by describing some of the above models without assuming any background in many-body quantum theory and electronic structure. I will then focus on dimension reduction results in scaling limits. In particular, I will explain how in a high-density limit the many-electron Schroedinger equation tends to Hartree-Fock theory (folklore, but not widely known in mathematics) and in a low-density limit one rigorously obtains a Kantorovich optimal transport problem (recent work by Cotar, Friesecke and Klueppelberg, Comm. Pure Appl. Math. 66 548-599 (2013) and arXiv:1706.05676 (2017)) which, at least in special cases, is known to be equivalent to a (low-dimensional) Monge optimal transport problem.

Martin Heida – WIAS Berlin.

On G-convergence and stochastic two-scale convergences of the squareroot approximation scheme to the Fokker-Planck operator

We study the qualitative convergence properties of a finite volume scheme that recently was proposed by Lie, Fackeldey and Weber (2013) in the context of conformation dynamics. The scheme was derived from physical principles and is called the squareroot approximation (SQRA) scheme:

$$\partial_t u_i^\varepsilon = \frac{1}{\varepsilon^2} \sum_{(i,j)\in E^\varepsilon} \left(u_j^\varepsilon \frac{v_i^\varepsilon}{v_j^\varepsilon} - u_i^\varepsilon \frac{v_j^\varepsilon}{v_i^\varepsilon} \right) , \quad v_i^\varepsilon = \exp\left(-\frac{\beta}{2} V_i^\varepsilon \right) .$$

We show that solutions to the SQRA equation converge to solutions of the Fokker-Planck equation as $\varepsilon \to 0$ using a discrete notion of G-convergence. As an example, in the special case of stationary Voronoi tessellations we use stochastic two-scale convergence to prove that this setting satisfies the G-convergence property. In particular, the class of tessellations for which the G-convergence result holds is not empty.

Hermen Jan Hupkes – Leiden University.
Discretization Schemes vs Travelling Waves for Reaction-Diffusion Systems

We study various temporal and spatial discretization methods for the Nagumo and the FitzHugh-Nagumo equations. In particular, we consider infinite-range spatial discretizations of the Laplacian, adaptive grid methods and full spatial-temporal discretizations using BDF schemes.

Our main goal is to understand in what sense the well-known existence, uniqueness and stability results for travelling fronts and pulses transfer to these discretized settings.

The main focus is on the functional differential operators that arise after linearizing around travelling waves in various well-understood limits. In particular, we discuss how the discretization schemes affect the spectral properties of these operators.

These schemes give rise to highly singular perturbations that we attempt to understand via weak-limit methods based on the pioneering work of Bates, Chen and Chmaj (2003).
Joint work with: E. van Vleck (U. Kansas) and W. Schouten (U. Leiden)

Johannes Kampmann – University of Regensburg.
Analysis of a coupled bulk-surface model for lipid raft formation in cell membranes and its connection to the Ohta-Kawasaki model

We investigate a model for lipid raft formation and dynamics in biological membranes which was proposed in [1]. The model describes the lipid composition of the membrane and an interaction with cholesterol. To account for cholesterol exchange between cytosol and cell membrane we couple a bulk-diffusion to a surface PDE on the membrane. The latter describes a relaxation dynamics for an energy taking lipid-phase separation and lipid-cholesterol interaction energy into account. By allowing different constitutive laws for the exchange between cell membrane and cytosol, the resulting model can include nonequilibrium processes. The resulting equation on the surface takes the form of an extended Cahn-Hilliard equation. We discuss mathematical properties of the model, in particular the existence of stationary solutions. In the nonequilibrium case, we highlight a connection to the Ohta-Kawasaki equation by showing that weak solutions to this model converge to the solutions to the Ohta-Kawasaki equation if certain parameters in the

model tend to zero.
Joint work with: H. Abels, H. Garcke, A. Rätz and M. Röger.

[1] H. Garcke, J. Kampmann, A. Rätz, M. Röger, A coupled surface-Cahn-Hilliard bulk-diffusion system modeling lipid raft formation in cell membranes, Math. Models Methods Appl. Sci. 26 (2016), no. 6, 1149–1189. DOI: 10.1142/S0218202516500275

Herbert Koch – University of Bonn.
The analysis of nonlinear dispersive waves
Nonlinear dispersive waves arise as asymptotic equations in many physical contexts including nonlinear optics and water waves. They describe interacting waves at in a narrow frequency and velocity range. I plan to explain recent results and open questions.

Matthias Köhne – University of Düsseldorf.
On Global Strong Solutions for a Class of Heterogeneous Catalysis Models
We consider a model for heterogeneous catalysis in a finite three-dimensional cylinder-shaped pore, with the lateral walls acting as a catalytic surface. The system under consideration consists of an advection-diffusion system within the bulk phase and a reaction–diffusion–adsorption system modeling the processes on the catalytic wall and the exchange between bulk and surface. We assume Fickian diffusion with constant coefficients, sorption kinetics with linear growth bounds and a network of chemical reactions which possesses a certain triangular structure. Our main result gives sufficient conditions for the existence of a unique global strong solution to this model, thereby extending by now classical results on reactiondiffusion systems to the more complicated case of heterogeneous catalysis.
Joint work with: Dieter Bothe, Siegfried Maier and Jürgen Saal.

Christian Kuehn – TU Munich.
Dynamics of Nonlocal PDEs
Recently, nonlocality has emerged as a highly active theme in differential equations. In my talk, I shall give several examples of PDEs, where nonlocal effects play a, sometimes quite surprising, role. We start with dynamics of the FKPP equation with nonlocal convolution terms, then proceed to waves in the fractional-diffusion Nagumo equation, and then consider nonlocal modulation equations with convolution terms. Having illustrated the deterministic PDE case, we also provide two examples for SPDE nonlocality and briefly discuss stochastic neural fields

as well as fractional Allen-Cahn SPDEs. The individual results provide a broad illustration of many current research directions in nonlocal PDE theory.

Stephan Luckhaus – University of Leipzig.
Geometric rigidity and dislocations
Geometric rigidity refers to estimates of the variance of elastic strain in terms of either (linear theory) the deformation or (nonlinear theory) the distance of the strain to SO(n) - and possibly its curl. Here we are interested in the situation where the curl is concentrated on dislocations. The aim is to show that for small energy the dislocations combine in small angle grain boundaries leading to the formation of a subgrain structure during the process of annealing, which is part of the industrial hardening procedure. So far in a joint work with G. Lauteri we were able to prove this by a rigorous lower energy estimate for the case of a two dimensional configuration. The same estimate is conjectured to hold in the general 3-d situation too, this is ongoing work with Lauteri.

Alexander Mielke – WIAS Berlin.
Perspectives for gradient flows
We will discuss general question concerning the existence and construction/emergence of gradient structures for a particular classes of dissipative equation like reaction-diffusion systems of the quantum Markov equations, also called Lindblad equation.

In particular, we will introduce the notion of EDP convergence of gradient systems, which is based on the Energy-Dissipation Principle, which was introduced by De Giorgi for general metric gradient flows.

Throughout we will complement recent result with open questions.

Julia Orlik – Fraunhofer ITWM, Kaiserslautern.
Stability conditions and energy estimates for periodic frame structures made of thin beams
We consider a periodic frame structure consisting of thin beams (of radius r) and study the energy estimates for this problem as the period ε, and the radius r of the beams tend to zero. The decomposition of the displacement field into the extensional and bending components is used, to obtain a priori estimates for beams between the nodes and then the complete microscopic displacement field is extended by the multilinear interpolation. Two types of unfolding operators are introduced to deal with different parts of the decomposition. Different estimates w.r.t. the relation between the period of the frame and the beam's thickness, as well as w.r.t. design of the frame's graph and stability conditions

are discussed.
Joint work with: G. Griso and O. Sivak.

Felix Otto – MPI Leipzig.
Parallelisms between stochastic PDE and stochastic homogenization
Pushing the limits of (elliptic or parabolic) regularity theory is both an important ingredient in the recent advances in (quantitative) stochastic homogenization and – much more prominently – in stochastic PDE. In the second case, one fights against the roughness on small scales, while in the first case, one seeks to take advantage of cancellations on large scales. In both cases, the approach is characterized by a clear separation between the stochastic control of certain model problems and an augmented but completely deterministic regularity theory. These developments offer new perspectives for PDE analysis.

Georg Prokert – TU Eindhoven.
Well-posedness for a moving boundary model of an evaporation front in a porous medium
We consider a two-phase elliptic-parabolic moving boundary problem modeling an evaporation front in a porous medium. Our main result is a proof of short-time existence and uniqueness of strong solutions to the corresponding nonlinear evolution problem in an L^p-setting. It relies critically on nonstandard optimal regularity results for a linear elliptic-parabolic system with dynamic boundary condition.
Joint work with: F. Lippoth.

Matthias Röger – TU Dortmund.
Charged elastic drops
We consider a functional on sets in the plane or in three dimensional space, given by contributions from a surface area energy, a bending energy, and a Riesz self-interaction energy. We present results for corresponding minimization problems in classes of fixed total mass and in parts under constraints on the topology.
This is joint work with Michael Goldman and Matteo Novaga.

Athanasios Stylianou – University of Kassel.
Modelling pattern formation on the surface of a ferrofluid
The talk deals with patterns appearing on the free surface of a ferromagnetic fluid placed in a vertical magnetic field, undergoing a so-called Rosensweig instability. We present some old results concerning existence and stability of periodic structures as well as a new existence theory for static solitons and for the associated free boundary problem.

Details

Consider an infinite slab separated into two simply connected subdomains: Ω_F is occupied by a ferrofluid and Ω_A is occupied by air, both subjected to an upwards pointing vertical magnetic field of strength $H > 0$. Maxwell's equations of magnetostatics: div $\mathbf{B} = 0$ and curl $\mathbf{H} = 0$, are satisfied, where $\mathbf{B} : \Omega \to \mathbb{R}^3$ is the magnetic induction and $\mathbf{H} : \Omega \to \mathbb{R}^3$ is the (irrotational) magnetic field strength satisfying $\mathbf{B} = \mu_0 \mathbf{H}$ in Ω_A, and $\mathbf{B} = \mu_0 \big(\mathbf{H} + \mathbf{M}(\mathbf{H})\big)$ in Ω_F. Moreover, μ_0 is the permeability of vacuum, the relative permeability of the air is normalized to be equal to 1 (i.e., there is no magnetization) and the function $\mathbf{M} : \mathbb{R}^3 \to \mathbb{R}^3$ describes the corresponding magnetization law for the ferrofluid. Since the field is irrotational there exist two magnetic potentials ϕ, χ such that $\mathbf{H} = -\nabla\phi$ in Ω_F and $\mathbf{H} = -\nabla\chi$ in Ω_A. Maxwell's equations imply that χ is harmonic in Ω_A and ϕ is "almost" harmonic in Ω_F.

Assuming an arbitrary nonlinear magnetisation law, we extract a free boundary problem involving the magnetic potentials in the ferrofluid and the air together with compatibility and dynamic conditions on the free interface. First, we stay in the planar case and assume that the surface of the ferrofluid can be described by the graph of a function. Using spatial dynamic techniques and an appropriate center manifold reduction we show existence of static solitons on the surface. Next, going back to 3d and dropping the "graph" assumption, we solve the free boundary problem by applying techniques developed for the study of convex-concave functionals.

Joint work with: M. Groves, D. Lloyd and E. Parini

[1] M. D. Groves, D. J. B. Lloyd, and A. Stylianou. Pattern formation on the free surface of a ferrofluid: Spatial dynamics and homoclinic bifurcation. *Physica D: Nonlinear Phenomena*, 350:1–12, 2017.

[2] E. Parini and A. Stylianou. A free boundary approach to the Rosensweig instability of ferrofluids. 2017. arXiv:1704.05722 [math.AP].

[3] E. E. Twombly and J. W. Thomas. Bifurcating instability of the free surface of a ferrofluid. *SIAM J. Math. Anal.*, 14(4):736–766, 1983.

Mark Veraar – TU Delft.

Polynomial stability of semigroups and classical solutions to PDEs

Since the groundbreaking work of Lebeau, it is well-known that there is a connection between the stability of classical solutions to damped wave equations and the growth rate of the resolvent along the imaginary axis. In recent years general characterizations have been obtained for bounded semigroups and general decay rates and applied to several classes of PDEs. In the talk I will give an overview on some of these results. Moreover, I will present an extension to semigroups which are not necessarily uniformly bounded, which was recently discovered in collaboration with Jan Rozendaal. Our proofs are based on new $L^p - L^q$ Fourier multiplier theorems for operator-valued symbols.

Stephan Wackerle – Fraunhofer ITWM.

Homogenization and dimension reduction for a textile shell

The wrinkling of textiles is an interesting behavior to investigate and optimize. In a first step a model of a woven and hence periodic textile as a three dimensional elasticity problem with contact between yarns is considered. On the microscopic scale we use the decomposition of displacement, possible for both the linear [1] and non-linear case [2][**?**], to obtain Korn-like inequalities for the single fibers depending on the small parameters. Upon these decomposed displacements we can define a plate-like displacement by using only the information in the contact nodes and interpolation in between them to arrive at estimates for the whole structure. These are then the basis for the technique of periodic unfolding to obtain the homogenized limit equation.

To give rise to the mentioned wrinkling of the textile, the plate-model requires some non-linear behavior. In particular the von-Karman-plate gives by its semi-linearity the desired coupling between the and out-of-plane or bending and in-plane or membrane displacements. For this we discuss the necessary order of the estimates and the energy to generate the von-Karman plate in the limit as the starting point for the optimization.

Joint work with: Georges Griso, Julia Orlik

[1] G. Griso. Decomposition of displacements of thin structures. J. Math. Pures Appl. 89 (2008), 199-233.
[2] Blanchard, Dominique and Griso, Georges. Decomposition of deformations of thin rods: Application to nonlinear elasticity. Analysis and Applications 07 (2009), 21-71.
[3] Blanchard, Dominique and Griso, George. Decomposition of the Deformations of a Thin Shell. Asymptotic Behavior of the Green-St Venant's Strain Tensor. Journal of Elasticity 101 (2010), 179-205.
[4] G. Griso. Asymptotic behavior of structures made of curved rods. Analysis and Applications. 6, 01 (2008), 11-22.