First read the entire examination. There are 5 exercises in total. Grades are included between parentheses at all parts and sum up to 9 points. Good luck!

1. The hyperbolic bound $HB(n) : \prod_{1 \leq i \leq n}(U^T_i + 1) \leq 2$, where $U^T_i$ denotes the processor utilization of the task $\tau_i$, is an example of a sufficient schedulability test.

   (a) (0.5) What can be concluded concerning schedulability of a set $T$ of $n$ task when the hyperbolic bound does not hold for $T$?
   **Answer:** The task set $T$ may but need not be schedulable. Hence, nothing can be concluded yet...

   (b) (1.0) Construct an example set of three tasks for which the left-hand side of $HB(3)$ is equal to 2.
   **Answer:** See RTA.Rehearse-080918.

   (c) (0.5) Is the task set that you constructed schedulable according to the Liu & Layland bound? Motivate your answer.
   **Reminder:** $LL(n) : U^T \leq n(2^{1/n} - 1)$, where $U^T$ denotes the processor utilization of the set $T$.
   **Answer:** The Liu & Layland bound will also hold if and only if the utilizations of all three tasks are the same; see RTA.Exercises-2 "Solutions LL-bound". Because the Liu & Layland bound is also a sufficient condition, just like the hyperbolic bound, nothing can be concluded when the bound does not hold.

2. Consider two periodic tasks $\tau_1$ and $\tau_2$, with characteristics as given in the following table, which are scheduled by means of fixed-priority scheduling with deferred pre-emption (FPDS), where $\tau_1$ has a higher priority than $\tau_2$.

<table>
<thead>
<tr>
<th></th>
<th>$T = D$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>3</td>
<td>$1\frac{1}{2}$</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>5</td>
<td>$1 + 1\frac{1}{4}$</td>
</tr>
</tbody>
</table>

   (a) (1.0) Determine the worst-case length $WL_2$ of the level-2 active period.
   **Answer:** Draw a timeline with a simultaneous release of both tasks at time $t = 0$. Because the pending load becomes zero at time $t = 9$, the worst-case length $WL_2 = 9$.

   (b) (1.0) Determine the worst-case response times $WR^D_1$ of $\tau_1$ and $WR^D_2$ of $\tau_2$ using
   
   \[
   WR^D_{ik} = \begin{cases} 
   WR^P_i(B^i + kC_i - F_i) + F_i - (k - 1)T_i & \text{for } i < n \\
   WR^P_i(kC_n - F_n) + F_n - (k - 1)T_n & \text{for } i = n 
   \end{cases},
   \]

   and the following recursive equations for $WR^P_i(C)$ and $WO^P_i(C)$, respectively.

   \[
   x = C + \sum_{j < i} \left[ \frac{x}{T_j} \right] C_j, \quad (2)
   \]

   \[
   x = C + \sum_{j < i} \left( \left[ \frac{x}{T_j} \right] + 1 \right) C_j. \quad (3)
   \]
**Answer:** For task $\tau_1$, it is only necessary to consider the first job: $WR^D_1 = 2\frac{3}{4}$. Because the worst-case length $WL_2$ of the level-2 active period contains $\left\lceil \frac{WL_2}{T_2} \right\rceil = 2$ jobs, we need to consider two jobs for task $\tau_2$. $WR^D_{2,1} = 3\frac{3}{4}$ and $WR^D_{2,2} = 4$, hence $WR^D_2 = 4$.

3. Consider four periodic tasks $\tau_1$, $\tau_2$, $\tau_3$ and $\tau_4$ (having decreasing priority), which share five resources, $A$, $B$, $C$, $D$, and $E$. Compute the maximum blocking time $B_i$ for each task for the following three protocols, knowing that the longest duration $D_i(R)$ for a task $\tau_i$ on resource $R$ is given in the following table (there are no nested critical sections).

<table>
<thead>
<tr>
<th></th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1$</td>
<td>6</td>
<td>7</td>
<td>0</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>4</td>
<td>14</td>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\tau_4$</td>
<td>0</td>
<td>11</td>
<td>9</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

(a) (1.0) Priority Inheritance Protocol (PIP).
**Answer:** Similar to Exercise 7.5 of the book of Buttazzo. Compared to that exercise, the columns have been exchanged ($A \rightarrow E \rightarrow B \rightarrow A$ and $C \leftrightarrow D$), and all non-zero values have been increased by 1.

(b) (0.5) Priority Ceiling Protocol (PCP).
**Answer:** Similar to Exercise 7.6 of the book of Buttazzo.

(c) (0.5) Highest Locker Protocol (HLP).
**Answer:** Same as for PCP.

4. The following questions concern presentations of assignments.

(a) (0.5) Hierarchical scheduling of independent applications: Two approaches were presented for two-level hierarchical scheduling of independent applications, one termed "Hierarchical Fixed-Priority Pre-emptive Scheduling" by Davis and Burns and the other "Periodic resource model for compositional real-time guarantees" by Shin and Lee. Although the analysis for the 2nd approach is more pessimistic than the former, the analysis for 2nd approach also has a major advantage. Explain that advantage.
**Answer:** Given the parameters of its server, the schedulability of a subsystem can be analyzed in isolation, i.e. without any knowledge about periods or capacities of other servers.

(b) Hierarchical scheduling of dependent applications: Two approaches were presented for global resource access, one approach termed H-SRP and the other termed SIRAP. The approaches differed with respect to their behavior when the amount of remaining capacity of a reservation was less than the amount of time needed for accessing the global resource.

i. (0.5) Explain the difference between both approaches.
**Answer:** H-SRP assumes budget overrun (optionally with pay-back) and SIRAP assumes (self-) blocking.

ii. (0.5) What are the consequence of SIRAP for a supporting mechanism?
**Answer:** Upon entering a critical section, a check is needed whether or not there is still enough remaining capacity to use the global resource, i.e. to execute the critical section.
5. The following questions concern guest-lectures.

(a) (0.5) Prof. Christian Hentschel gave a guest lecture entitled *Scalable video algorithms*. Briefly describe his motivation for scalable video algorithms. 

**Answer:** See his slides.

(b) (0.5) Mike Holenderski gave a guest lecture entitled *Swift mode changes*. He considered two scheduling approaches for mode changes, fixed-priority pre-emptive scheduling (FPPS) and fixed-priority scheduling with deferred pre-emption (FPDS). Which approach is better and why?

**Answer:** Assuming that processing a mode corresponds with a single sub-job, the worst-case lead-time of a mode-change is larger for FPPS than for FPDS, because for FPDS we need to wait for at most 1 sub-job to complete.

(c) (0.5) Alina Weffers-Albu gave a guest-lecture entitled *Behavioural Analysis of Real-Time Systems with Interdependent Tasks*. She explained that the behaviour of a chain of components scheduled by means of fixed-priority pre-emptive scheduling (FPPS) assumes a repetitive pattern after a finite amount of time. How can that amount of time for the so-called initial phase be minimized?

**Answer:** By giving the first task in the chain the lowest priority and/or by minimizing the size of the buffers prior to the lowest priority component in the chain.