Week 1b

Recommended exercises

Grimmett & Stirzaker

Chapter 1.4	6	
Chapter 1.5	1	9
Chapter 1.8	5	14

1. Someone rolls five fair dice. What is the probability of a full house? (A full house is a combination of dice where three dice show the same number and the other two dice show the same number but different from the previously mentioned three)

2. Someone flips six fair coins and notes the number of tails. After that he throws a fair die once and notes the number. Find the probability that the two results sum up to 8.

3. At a party, all n attendees have brought a present. They play a game to redistribute their presents. The names of all players are written on different tickets except for the name of the host. The host draws the first ticket and gives his present to the person whose name is written on the ticket, call him/her G_1. After that, the name of the host will be put on a ticket as well. Then G_1 will draw a new ticket at random and hand his/her gift to G_2. Tickets that are drawn will not be replaced.
   The game ends as soon as someone draws the host, as he has already given his present away. Note that the sequence of receivers can be expressed as $(G_1, G_2, ..., G_i, H)$. What is the probability that everyone has received a new gift? In other words: find the probability that every player occurs in the sequence of receivers: $(G_1, G_2, ..., G_{n-1}, H)$

Extra exercises

Grimmett & Stirzaker

Chapter 1.5	5	
Chapter 1.7	1	
Chapter 1.8	11	12

1. In a lecture room 24 students are present.
   (a) Gene is one of the previously mentioned students. What is the probability that at least one student shares a birthday with him? You may assume every year has 365 days and that each day is equally likely to be the birthday of a person. Also, birthdays of different students are independent.
   (b) What is the minimum number of students such that this problem in (a) is at least 0.5?
   (c) What is the probability that at least one pair of students share a birthday?
   (d) What is the minimum number of students such that the probability in (c) is at least 0.5?
   (e) Can the probability in (a) and (c) ever become 1?

Exam question

A and B are two events such that $0 < \mathbb{P}(A) < 1$ and $0 < \mathbb{P}(B) < 1$.

(a) If $A \subseteq B$, can A and B be independent?

(b) If A and B are independent, can A and $A \cap B$ be independent?