Week 3a

Recommended exercises

Recommended exercises test the basic concepts of the material covered per lecture. Understanding of how these concepts work is key to solving more difficult exercises and exam questions. Therefore, we recommend that you try to solve and understand these problems first before moving on to the harder exercises.

Grimmett & Stirzaker

Chapter 3.3	6	
Chapter 3.4	6	
Chapter 3.5	2	3

1. An insurance company sells a policy where they pay out x euro when event E happens. The company estimates that E occurs with probability p. For how much should the company sell these policies such that their expected profit is 10%?

2. Players A and B play a game. A writes down a number: 1 or 2. If B can guess the correct number i he wins i euros from A. If B guesses incorrectly he gives 0.75 euros to A. B decides to guess 1 with probability p and to guess 2 with probability $1-p$. Calculate his expected profit if
(a) A has written down 1
(b) A has written down 2
Which value of p maximizes the minimal possible profit?

3. X is a stochastic variable, such that
$$p = \mathbb{P}(X = 1) = 1 - \mathbb{P}(X = -1).$$
Find $c \neq 1$ such that $\mathbb{E}(c^X) = 1$.

Extra exercises

Extra exercises test a more in-depth understanding of the material. These questions are often longer and more difficult. Even though these questions might require more effort, we still recommend you to give them a try.

Grimmett & Stirzaker

Chapter 3.3	8
Chapter 3.4	3

Show that for a stochastic variable N with values in $\mathbb{N}$ the following holds
$$\sum_{i=0}^{\infty} i\,\mathbb{P}(N > i) = \frac{1}{2}\left(\mathbb{E}(N^2) - \mathbb{E}(N)\right).$$
Hint: Start with $\sum_{i=0}^{\infty} i\,\mathbb{P}(N > i) = \sum_{i=0}^{\infty} i \sum_{k=i+1}^{\infty} \mathbb{P}(N = k)$ and change the summation order

Exam question

Sometimes, we will add an exam question so that you can test your skills. Keep in mind that we limit these question (so that you will have enough exercises when preparing for the exam) and might therefore not fully cover the material of this week.

1. Let $X_n \sim \text{Binom}(n, p)$ and $X_m \sim \text{Binom}(m, p)$. What is the probability mass function of $X_m + X_n$?

2. Show that for a stochastic variable X with values in $\mathbb{N}$ the following holds:
$$\mathbb{E}[X] = \sum_{i=0}^{\infty} \mathbb{P}(X > i).$$