Week 4a

Recommended exercises

Recommended exercises test the basic concepts of the material covered per lecture. Understanding of how these concepts work is key to solving more difficult exercises and exam questions. Therefore, we recommend that you try to solve and understand these problems first before moving on to the harder exercises.

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Chapter 3.7	1(a)	7	
Chapter 4.1	1(b)	2	3

1. For what value of C and m is $f(x) = Cx^{-2m}$ (for x >1) a probability density function?
2. Find the expectation and variance of the following probability density functions:
   (a) $f(x) = \frac{1}{20}$ for $-10 < x < 10$.
   (b) $f(x) = \frac{1}{2}e^{-|x|}$ for $-\infty < x < \infty$.
   (c) $f(x) = \frac{1}{20}$ for $0 < x < 10$ and $f(x) = \frac{1}{2}e^{-|x|}$ for $-\infty < x < 0$.
3. At a station, every quarter of an hour, a train leaves to destination A (the first train leaves at 7:00) and every quarter of an hour a train leaves to destination B (the first train leaves at 7:05). A passenger arrives, uniformly distributed, between 7:00 - 8:00 at the station and takes the first train that leaves.
   (a) What is the probability that he arrives at destination A?
   (b) Does this probability change when he would arrive uniformly between 7:10 - 8:10?

Extra exercises

Extra exercises test a more in-depth understanding of the material. These questions are often longer and more difficult. Even though these questions might require more effort, we still recommend you to give them a try.

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Chapter 4.1	1(c)

1. The stochastic variables $X_1, X_2, \ldots, X_n$ are i.i.d. and geometrically distributed. Prove (with the use of induction) that $X_1 + \cdots + X_n$ is negative binomially distributed. Support this finding with an argument that does not depend on a calculation.

Exam question

Sometimes, we will add an exam question so that you can test your skills. Keep in mind that we limit these question (so that you will have enough exercises when preparing for the exam) and might therefore not fully cover the material of this week.

Let X be uniformly distributed on $[0, 1]$. Show that the probability density function of X^2 is given by:

$$f_{X^2}(u) = \begin{cases} \frac{1}{2\sqrt{u}} & \text{for } u \in (0,1) \\ 0 & \text{else.} \end{cases}$$