

# Modelling projects for Kansrekening 2WS20

Remco van der Hofstad, Britt Mathijzen, Jaron Sanders  
Translator: Gianmarco Bet

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## 1 Scratch cards

The Eindhoven Dagblad (ED) intends to organize a crackling, stimulating scratch promotion for its newspaper deliverers. Figure 1 explains the promotion. You can also find a translation below.

### Scratch promotion - Rules of the game

- From delivery week 51 to week 12 you will receive 14 scratch cards, one for every promotion week;
- For every day without complaints, you can scratch open any scratch box from the current week's card;
- If you scratch 3, 4 or 5 matching symbols, you win a prize!
- One card with the same six symbols also means you won a prize and you have a shot at the main prize;
- The seventh box gives you a bonus chance. You can only scratch this if you have gone 6 days without complaints, and you still did not win a prize. With this you have a chance to have more accumulation points for the savings promotion.
- Send your winning scratch card with the corresponding complaints card to the office staff;
- On the reverse side of the ticket write down the prize that you would like to receive from the correct category;
- Never scratch open more boxes than the number of delivery days without complaints during that week, otherwise the card is invalid;
- Make sure the dates of the complaints cards match the week number on the card;
- The scratch cards are not exchangeable, they are personal.
- The rules can be found in the letter that you have already received, read it carefully once more;
- Every deliverer may win several prizes;



Figure 1: The scratch promotion rules.

The newspaper deliverers deliver the newspaper for 14 weeks, 6 days a week in the morning. They are provided a stack of newspapers to deliver and a card. This is a printout which indicates at what addresses they must deliver the newspapers and whether there were complaints the day before in their neighborhood or if they had no complaints. Usually the deliverer saves accumulation points for each delivery day without any complaints (similarly to air miles/fuel points) and with these they can order gifts from a catalogue. This accumulation points scheme keeps running during the promotion. For the special scratch promotion the deliverer receives 14 scratch cards in mid-December, one card for each week, with six scratch boxes in each card. For every day without any complaint they may scratch open one box at random. There are a total of six possible different symbols below the scratching layer: a star, a newspaper, a mailbox, a bicycle, a newspaper bag and a dog. The deliverers choose by himself which boxes they scratch each day without complaints. There may not be more scratched open boxes than there are complaints-free days (when a deliverer requests a price, he/she must send his/her complaints-card together with the scratch card). Three matching symbols indicate a prize of 10 Euro, four of the same symbols indicate a prize of €25 and the same five symbols indicate a prize of €50. The starting idea of the newspaper is that there will be 140 prizes of €10, 100 prizes €25 and 48 prizes of €50. With six equal symbols you automatically enter the final draw of the promotion. Here, 12 main prizes will be drawn: 4 of €250, 4 of €500 and 4 of €750.

Besides the six scratch boxes there is a seventh ‘consolation prize’ scratch box. This applies to anyone who has not received any complaints for six days and has not found any winning combination in the six boxes he has scratched already. As a consolation prize he may scratch the seventh box to earn accumulation points. These accumulation points can still be used for prizes from the catalogue. The seventh ‘consolation prize’ box can contain one of the following: 25 accumulation points, or €2.50, 50 accumulation points, or €5, 75 accumulation points, or €7.50. The ED employs 3000 deliverers. The percentage of the deliveries which run without complaints is 75% per day. It is expected that during this promotion this percentage can rise up to 90%.

The newspaper will give away a total of about €12300 in prizes. It cannot be exactly predicted how much this operation will cost as this depends on how many complaints will be received. The newspaper wants to know how this promotion needs to be implemented further, i.e. which symbols must be printed on the cards and in what quantities, so that the approximate total budgeted amount for the prizes is given away?

## 2 Newspaper boy

To make money a newsboy is selling daily newspapers on the corner of a street. Early in the morning, he buys a number of newspapers at a certain price. However, nobody wants to read yesterday’s news, so newspapers of a certain day are purchased only on the same day they are sold. The newspapers that remain at

the end of the day are thrown away.

The problem is that the boy does not know in advance how many papers he will sell that day. He obviously does not want to buy too few newspapers, but also not too many. The newspaper boy asks you for advice on a buying strategy.

Suppose now that the supplier offers to buy back the remaining newspapers (at a price lower than the purchase price). The boy wants to know how your analysis and advice changes.

### 3 Cracking old Dutch bingo

A music association from Eindhoven organizes a bingo evening every Sunday. Each bingo night always consists of 41 rounds, and each time there is a guaranteed price of €1100, regardless of the number of players. To be more precise, in the first twenty rounds, the association pays the winner €25, and in the last round €100. If two players win simultaneously, the prize money is shared fairly among them.

Each round works as follows. A presenter pulls a stone from a jute bag. The stones are numbered from 1 up to and including 90, and the number on the drawn pebble is broadcasted. Each player covers the drawn number on all of his or her bingo cards. The stone is then discarded. A player wins if he or she is the first to completely cover one of the six smaller cards on a bingo card.

Players can purchase bingo cards on two occasions. They can purchase bingo cards at the start of evening for €10 each. From the moment of purchase, a player will play with the same card throughout the whole evening. After the twentieth round there is a break, and during the break new players can buy cards for €3.75 each. Current players can buy an extra bingo card, for €3.75.

Each bingo card is numbered to prevent fraud. There are 150 bingo cards in total. Players regularly ask for specific bingo cards, because from their previous experiences they believe that certain cards win more frequently than others. The board tolerates this because they know that the police have checked each individual card.

Every week the number of bingo cards which have been sold fluctuates around 110. The board of the music association wants to gain more insight in their bingo evening, and turns to you. The majority of the board is interested in the chance of winning of players, and the expected payoff. A single committee member asks whether players can be at an advantage by purchasing specific cards, or by applying purchasing strategies.

### 4 Enhancement of uncertain information

Our society is connected, now more than ever. As a result, in many Western countries secret services, but also companies like Google, eBay and Marktplaats have access to hundreds of pages of information about us. They know where we

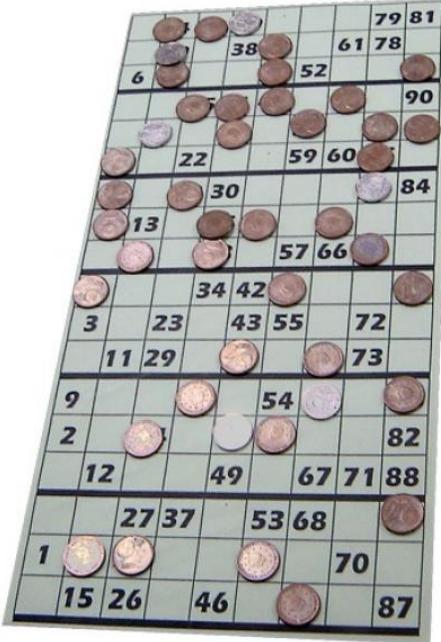


Figure 2: A bingo card.

live, what our phone number is, what is in our emails and what we write on our online profile pages.

The amount of personal information that people leave behind is incredible. If someone searches through a search engine the keywords ‘tropical island’, the company behind the search engine estimates that it is more likely that this person would go on vacation. Behind the screens, the company forms a profile of characteristics of each user (such as gender, age, height, interests, hobbies), together with the probability that these are correct. Eventually, the company sells these profiles to third parties.

This happens not only on the internet. If, for example you use a navigation system, the producer stores where you drive and how fast you drive. They also form a profile of you, and sell this information to third parties. For example, some time ago this type of information was sold to the police so that they could strategically place speed cameras.

Secret services and businesses collect as many of these types of profiles as possible. By combining information from different profiles and by also comparing different characteristics, they hope to get a clear profile of a person.

It is unclear which authority hired you, but a contact in a suit has offered you big money to make a model and an analysis about the usefulness (or uselessness) of combining uncertain information.

As a concrete first problem you can think of combining information about

reliability in various internet sales sites. How do you determine how reliable a person is on the basis of his / her purchase and sale data from the past? If Marketplace and eBay have both made a profile of a person and his / her reliability have been determined, how do you combine this information? Does it matter whether you know how many transactions the person has performed in both places?

## 5 Bankruptcy and risk

An insurer provides car insurance. Customers must pay €50 upfront, and then pay a monthly premium of €25. By purchasing this product, customers are entitled to compensation for damage to their car. The insurer has 1193 customers, and each month, both the premiums of all customers and the claims arrive at the office. Claims are on average about €937.85. This is because, for example, replacing a mirror is less expensive than replacing an engine. The insurer knows from experience that there are on average 30.9 valid claims per month.

The insurer fears that if multiple claims follow one after another, the insurance company could bankrupt. The insurer therefore wants a risk analysis from you, and if necessary, advice about their product.

## 6 Optimum inventory

A caterer works in the background of each hospital. This caterer must meet strict quality and diversity of supply. This leads to severe problems for the inventory. In practice the number of patients fluctuates from day to day, and the choices made by patients from the menu vary from day to day. The ingredients which are needed in order to meet the demand, therefore, are intrinsically stochastic.

The caterer addresses this by storing ingredients. However, food spoils after some time, and this is very dangerous for patients with weakened immune systems. To prevent deaths, the caterer is obliged to destroy all the food that is over four days old, regardless of quality.

The destruction of food leads to losses for the caterer. On the other hand, the caterer must store enough food to meet the demand. The caterer wants to understand better the process from the potato to the casserole, or to the trash, and asks for your expert opinion. Your appointment has been made possible by a recent talk about whether there is an optimum storage capacity.

## 7 Dynamics of politics

The Lower House has 150 members of parliament who each have a casting vote. Members of the Parliament (MPs) belong to different political parties and political movements. Opinions are varied and coalitions often need to be formed.

Treating controversial bills can therefore lead to lengthy discussions. At the beginning of a discussion a MP always has an opinion about the bill, and he/she defends this view fervently. MPs are naturally open to other opinions, and they can be persuaded by someone with another point of view. Whether an MP is convinced or not depends on the amount of MPs who defend the competitive position and the quality of their arguments.

The General Court therefore decides to use probability theory to model and analyze consensus, and hires you to write an enlightening report.

## 8 Flu epidemic

An influenza pandemic can be seen as a virus which extends across a network (of a moving crowd). When an infected person comes into contact with an healthy person, the disease can be carried over. Important characteristics are the length and the size of the epidemic. Develop a model for the dissemination of the virus. Of course, the vaccination of both infected and healthy persons helps.

The Center for Disease Control and Prevention (CDC), located in Atlanta, asks you examine how vaccination can prevent the spread of a deadly virus.

## 9 Poor music playing to quite good

Orchestras consist of about 40 musicians, and each musician has his own expertise and qualities. Musicians play woodwind instruments such as clarinets and saxophones, brass instruments such as trumpets and trombones, and percussion instruments such as drums and xylophones. Some musicians can even play different instruments, but may not play each instrument equally well.

In front of the orchestra there is a conductor, which indicates when each musician must play a note. The conductor does this by rhythmically striking a baton in the air. Musicians are not able to respond immediately (in any case they need a short reaction time), and therefore they anticipate (unconsciously) the duration of the each subsequent stroke. This can be perhaps called a sense of rhythm. Aspiring musicians have more difficulty anticipating the next stroke (counting), and often play by accident just before or just after the stroke. The better the musician, the better their sense of rhythm, and the closer to the stroke they play.

Besides playing rhythmically, musicians must also play cleanly. Each note played can be false. A measure of how false a tone is the cent. Professional musicians can hear a difference of 5 to 6 cents and then correct the tone. Aspiring musicians have more difficulty, and their tone therefore often differs further from the correct. The audience can typically identify falsehood starting from 25 cents.

If you are listening to individual musicians, you can clearly hear if someone plays well or not. In a joint performance however the audience hears several musicians at the same time, and it is more difficult to identify individual voices. The conductor of the prestigious National Orchestra wants to better understand

the effect of interaction, and for instance wonders whether the quality of a performance scales with the size of an orchestra. He asks you for a report.

## 10 Solar machine manufacturer

A small company in Brabant develops and sells machines that produce solar cells. These machines make solar cells on round discs called wafers, through the process of chemical deposition. This process is occasionally incorrect, with the result that the wafer contains non-working solar cells.

European legislation requires that such a machine may be sold only if the machine produces wafers with working solar cells at least 95% of the time. Because this is a strict requirement, and each machine has a price tag of €119000 (that the company would rather not have to give back after the sale), the company decides each to test each machine in detail. The machine will now be delivered only if the test shows that it meets these requirements.

The machines are designed so that they can run fully automatically. A deposition takes an average of 9 minutes. The machine performs the diagnosis of a wafer in 2 minutes. If the wafer is faulty, one additional minute is necessary. Each wafer can be reused about 35 times, before it must be replaced with a new one. Every wafer costs about €87 each, and each deposition is costing the company about €2.

The company hires you to design a test method, with which to tell whether a machine meets European directives or not with probability bordering on certainty. The company is asking you to take into account the cost and duration of the test and also demands a report that convinces their customers the test procedure is appropriate.

## 11 Virology

The field of virology is concerned with the infectivity of viruses. To measure this, virologists use the plaque-forming units (PFUs). One PFU stands for number of virus particles that can form a so-called plaque per unit volume. For example, given a solution of 1,000 PFU / mL of Fruhsommer-Meningoenzephalitis, 1 mL of this solution contains enough virus particles to form on average about 1,000 infectious plaques in a mono-layer of cells. A plaque is a circular infection in a mono-layer of cells, which is caused by one initial infection.

In order to determine the PFU a *plaque assay* is made. In it, a virus solution is diluted a number of times, and each dilution is poured over a two-dimensional cell culture. When one or more virus particles infect a cell, they reproduce, and thereafter only spread laterally, forming a circular plaque. The number of plaques (and thus the number of initial infections) can then be counted, and from this follows the number of PFU / mL. It is important to note that not every virus particle in a solution leads to an infection.

A virologist from Wageningen asks you to develop a mathematical analysis

of the PFU. The virologist is interested in this because little is known about aspects such as the distribution of the virus particles and plaques on the cell cultures. The virologist wonders whether or not there exists an optimum number of dilutions; namely, if the dilution is too weak too many plaques are formed, which therefore can not be distinguished, and if the dilution is too strong, plaques are not formed. Finally, the virologist recently analysed the blood of a Saimiri oerstedii, and the virologist suspect the monkey is infected with a special virus that can infect only after three different components of the virus have come in a cell together. Rigorous insights about the effect of such collisions on the PFU may help the virologist to achieve breakthroughs.

## 12 Blood analysis

An independent laboratory receives daily vials of blood of patients submitted by various health institutions to test for the presence of a specific kind of virus particles. The lab has the capability to analyze each tube individually, but this is time-consuming and therefore very costly. This is why many laboratories normally combine the blood of different patients. This is done as follows: the blood of a predetermined number of individuals is mixed with one another, after which the test is carried out on the mixture. If the result is negative, then it can be concluded that none of the patients whose blood is present in the mixture carry the virus. However, if the test is positive, it will then have to be determined to who the infected blood belongs. The laboratory technician can choose any of the blood samples to be tested separately, or once again to mix a group of the samples. Of course, for each test a little amount of blood is required, reducing the number of tests that may be carried out with a tube. In addition, the blood has a limited shelf life.

The management of the laboratory would first like your opinion on a strategy with regard to the so-called batching of blood samples.

Often care institutions that supply the blood provide additional information about the health of the patient. This has the result that patients can be classified in different risk groups, with different risks of infection. The client would also like to know how this changes the above strategy.

## 13 Auction game in Wie is de Mol

The famous Dutch TV-show *Wie is de Mol* (Who is the Mole), is a game show with Dutch celebrities as contestants. Most contestants are candidates, who want to earn as much money as possible for a joint jackpot. Besides the candidates, there is one mole, whose identity is unknown to the candidates. His objective is to keep the jackpot as low as possible. The contestants play various games where they can earn or lose money from the joint jackpot. At the end of each episode, the contestants make a test. The one who performs the worst in the test has to leave the show. The mole never has to leave the show.

In the season from 2016, one of the games is an auction. The one who wins the auction, can remove some questions from this week's test. This is very valuable for a candidate, since it reduces his or her chances of having to leave the show. The nine contestants place their bids anonymously, without knowing each others bids. The game master looks at all bids, and declares the one who placed the second highest bid as the winner. The winner pays his winning bid from the joint jackpot.

A smart candidate wants to optimize his reward from this auction by using a probability model. The other candidates are not as smart, and do not take into account the strategy of the other candidates while placing a bid. The smart candidate asks you to make a probability model which models the bids of the other candidates and the mole, to find out what his best strategy is. Furthermore, the candidate would like to know how this strategy changes if the number of players in the auction game changes.

## 14 Lotteries and Jackpots

In the Dutch Staatsloterij (or State Lottery), one can participate in the Jackpot, which is held each 10th of the month as an extra on top of the regular lottery. For this, there are 7 balls, 6 blue balls and one orange ball, one of which is drawn each week. When the orange ball is drawn, the Jackpot is given to a lucky participant. When a blue ball is drawn, it is removed from the collection of balls, and the next 10th of the month again a ball is drawn from the smaller collection. The Jackpot starts with a pot of 7.5 M€, and each time that it is not awarded, the amount goes up. See <https://www.staatsloterij.nl/spel/jackpot> for more information.

A client wishes to investigate how this game can be played cleverly. She is interested in the Jackpot, since the money that can be won is enough to be able to stop working altogether. As a result, she is willing to put some money into it. However, it is not obvious to her what the best possible playing strategy is. Please advise her on this and explain her what options she has and which playing strategy you would recommend. Also advise her on whether there are any other Jackpots in competitors of the Staatsloterij that might be interesting for her. It would already help her to know how the Jackpot works precisely, for example, what is the chance of winning each round, and how much is put into the Jackpot each time it is not awarded.

## 15 Playing the casino

A client is quite fond of playing in the casino, as it gives quite a kick when winning money there. She tends to go to Holland Casino. She plays several different games, including roulette, blackjack, etc. She notices that her winning chances are not always equal, and is looking for advice on what game to play to have the best chance of winning. Please advise. Also, for the game that you

recommend, what is a good playing strategy. Is it best to play all the money at once, or is it better to play many times and divide the bets into smaller pieces. Does this depend on the precise game that she plays?

## 16 Sports tournaments

When two sports teams are playing against one another, usually one of the two teams is the stronger team, and the other team is the underdog. Still, it is possible for the underdog to win a match between two teams (for example because the better team misses important players). An organizer of a sports competition would like to have a competition where the probability that the best team wins is fairly high. There are several ways to design this competition. One can for example think of a double round-robin (such as the *Eredivisie*), where each team plays a home game and an away game against every other team, or single round-robin, where each team plays against each other team just once. But alternatives such as a knock out tournament are also possible. On the other hand, since the athletes already have many different sporting events, the organizer does not want to ask the athletes to play too many matches. She would like to have your advice on what kind of a competition she should adopt.

## 17 A trip to Rome

Al, John and Jack just graduated at TU/e, and decided to travel around Europe. Before returning to Eindhoven, they decide to stop in Rome. Since they spent a lot of money during the trip, they want to visit only their favorite monuments in Rome: Colosseum, Vatican Museums, S. Peters cathedral, Pantheon and Trevis fountain.

As known, in Rome there is a lot of traffic, so busses, subway and trains are often delayed. Since the city is visited every day by thousands of tourists, it is not uncommon to queue for some time to enter museums and churches. The waiting time is not the same during the day. In addition, for some hours during the day, some museums have a special discount for young people.

The three friends think about in which ways they can visit all the monuments. What is the best plan if they want to minimize the cost of the visit, or they prefer just to make a quick tour? What is instead the best strategy if they want to make a quick and cheap tour?

## 18 The populist votes

It appears that in many recent elections and referenda, the populist vote was always underestimated. For example, in the Brexit campaign, the odds of the 'Leave' vote was under estimated, in the recent presidential election in the US, the votes for Donald Trump were estimated too low. An institute performing such polls wishes to investigate whether this is true, and, if so, how it could be

taken into account. Can you give advice to this institute? It would be helpful to verify whether indeed the populist vote is always under estimated in the (recent) past, and what could be reasons why this is the case. Some say that this is due to the fact that the polling institutes fail because they are themselves part of the ‘establishment’ (as a recent Volkskrant article) suggests. Others say that it comes from the fact that people are reluctant to disclose that they voted the populist vote. If needed, you may make assumptions that you know the sizes of certain subpopulations in society.

## 19 Pokemon GO

In the popular mobile game Pokemon GO, one of the goals of the players, divided in 3 different teams, is to compete for the control of some locations on the map, called gyms. Each gym is controlled by one team, and has a level from 1 to 10 that indicates how many players of that team have put a pokemon there. Players who pass next to a gym can perform one of two actions, depending on who is owning the gym:

If an opposing team controls the gym then they can remove all the enemy pokemons and put one of theirs, thus making it a level 1 gym of their team. If their own team controls the gym, they can add a pokemon thus increasing the level by 1. These actions require to fight against the pokemons defending the location, and are less likely to succeed if the level of the gym is higher. Since the game was released, periods were observed in which gym were changing team highly frequently and others in which they were kept by one team for long periods. Determine what causes these different behaviours.

## 20 Elo Rating

The Elo rating system was originally created to determine the world ranking of chess players and has become a popular way to build ranking for games in which different players compete mostly in single matches rather than in organized tournaments. All players start with the same base rating, and every time two of them play a match some points are transferred from the loser to the winner, depending on the difference between the ratings of the two players. The higher was the loser’s rating compared to the winner’s one, the more points will be transferred. Consider a system of players with different skill levels playing against each other, in each match obviously the better player is more likely to win. How do the ratings evolve when considering different ways to arrange the matches (completely at random, arranged based on rating, based on challenges that the other player should accept) and different functions to determine the number of points transferred?