Problem 8. (Totally dual integral systems)

a) [KV, Exercise 5.8]
Consider the following two systems of linear inequalities
\[
\begin{pmatrix}
1 & 1 \\
1 & 0 \\
1 & -1
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix} \leq
\begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix},
\begin{pmatrix}
1 & 1 \\
1 & -1
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix} \leq
\begin{pmatrix}
0 \\
0
\end{pmatrix},
\]
which define the same polyhedron. Show that the first system is totally dual integral whereas the second one is not.

b) [KV, Exercise 5.9]
Let \( a \in \mathbb{Z}^n \setminus \{0\} \) be a nonzero integral vector and let \( b \in \mathbb{Q} \) be a rational number. Show that the inequality \( a^T x \leq b \) is totally dual integral if and only if the entries of \( a \) are relatively prime, i.e \( \gcd(a_1, \ldots, a_n) = 1 \).