2WAB0 Calculus: Common Math Errors

- **Wrong:** $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$
  Right: $\sqrt{xy} = \sqrt{x}\sqrt{y}$ (for $x \geq 0,\ y \geq 0$)
- **Wrong:** $x\,(x-2) = 1 \ \Rightarrow\ x = 1$ or $x - 2 = 1$
  Right: $x\,(x-2) = 0 \ \Rightarrow\ x = 0$ or $x - 2 = 0$
- **Wrong:** The solution is $2x + 4$, so $x + 2$ (you cannot simply divide something by 2)
  Right: The solution is $2x + 4$, so $2\,(x+2)$
- **Wrong:** $x^2 = 3x \ \Rightarrow\ x = 3$
  Right: $x^2 = 3x \ \Rightarrow\ x = 3$ or $x = 0$ since $x\,(x-3) = 0$
- **Wrong:** $\sqrt{x^2} = x$
  Right: $\sqrt{x^2} = |x|$
- **Wrong:** $x^2 = a \ \Rightarrow\ x = \sqrt{a}$
  Right: $x^2 = a \ \Rightarrow\ x = \pm\sqrt{a}$
- **Wrong:** $\dfrac{2}{x+3} = \dfrac{2}{x} + \dfrac{2}{3}$
  Right: $\dfrac{x+3}{2} = \dfrac{x}{2} + \dfrac{3}{2}$
- **Wrong:** $e^{x+y} = e^x + e^y$
  Right: $e^{x+y} = e^x \cdot e^y$
- **Wrong:** Leaving out one of the branches when graphing $xy = 1$ (or $y = 1/x$)
  Right: The graph has two branches
- **Wrong:** $f^{-1}(y) = \dfrac{1}{f(y)}$
  Right: $x = f^{-1}(y)$ is the inverse function of $y = f(x)$
- **Wrong:** For an "arbitrary" limit considering $\lim\limits_{x \to a^+}$ and $\lim\limits_{x \to a^-}$ as separate cases
  Right: You only need to do this for a limit of type $\dfrac{\neq 0}{0}$
- **Wrong:** Thinking that you are not allowed to intersect the horizontal asymptote when graphing a function
  Right: The horizontal asymptote is only relevant for how the function behaves $\to \infty$

- **Right:** $\int e^{ax}\,dx = \frac{1}{a}e^{ax}$

  The derivative of e^{ax} is ae^{ax}, so to find the primitive function we need a (constant) factor $\frac{1}{a}$ to compensate

  Wrong: $\int e^{x^2}\,dx = \frac{1}{2x}e^{x^2}$

  The reasoning here is as follows: the derivative of e^{x^2} is $e^{x^2}\cdot 2x$, so to find the primitive function we need a factor $\frac{1}{2x}$ to compensate. However: $\frac{1}{2x}$ is not a constant. It depends on x. Hence we need to use the product rule: $\left(\frac{1}{2x}e^{x^2}\right)' = -\frac{1}{2x^2}e^{x^2} + \frac{1}{2x}e^{x^2}\cdot 2x = -\frac{1}{2x^2}e^{x^2} + e^{x^2}$
  and this is not equal to e^{x^2}.

  Note: This problem does not have an easy solution: $\int e^{x^2}\,dx$ cannot be "solved by hand". For example, $\int xe^{x^2}\,dx$ and $\int x^3e^{x^2}\,dx$ can be solved, both using the substitution $u = x^2$.