Bisimulation and the reduction of Petri nets

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OUTLINE

- Reduction for Petri nets:
  known approaches
- Reductions correct w.r.t. bisimulation
- Place bisimulation:
  an extended notion
- Conclusions:
  place bisimulation in practice

Reductions for Petri nets

Goal: optimisation (reduction of the size of a net)

Related works:

C. Autant and Ph. Schnoebelen.
Place bisimulations in Petri nets. ICATPN'92

G. Berthelot.
Checking properties of nets using transformation.
Advances in Petri Nets 1985.

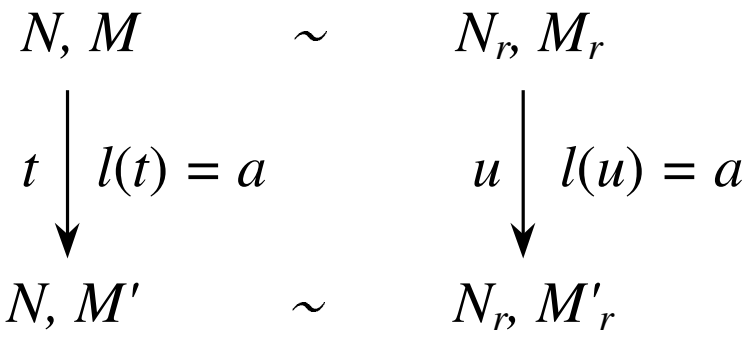
M. Nakagawa, S. Kumagai, T. Miyamoto, and D.-I. Lee.
Equivalent net reduction for firing sequence preservation.
IEICE Trans. on Fundamentals E.

S. M. Shatz, S. Tu, T. Murata, and S. Duri.
An application of Petri net reduction for Ada tasking deadlock analysis. IEEE Trans. Parallel and Distributed Systems, 1996.

M. Voorhoeve.
Structural Petri net equivalence.
Tech. report, Eindhoven Technical University, 1996.

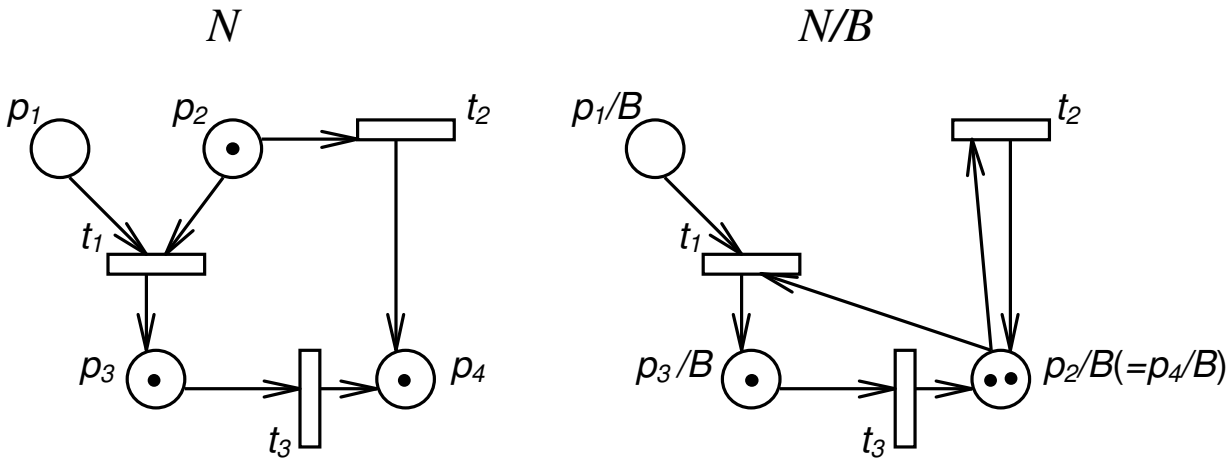
Our approach to reductions

Goal: develop structural reductions of Petri nets preserving the behaviour of a net in sense of bisimilarity, taking into account only relevant markings.



What are the correct ways of defining an equivalence on places?

Reduction by fusion



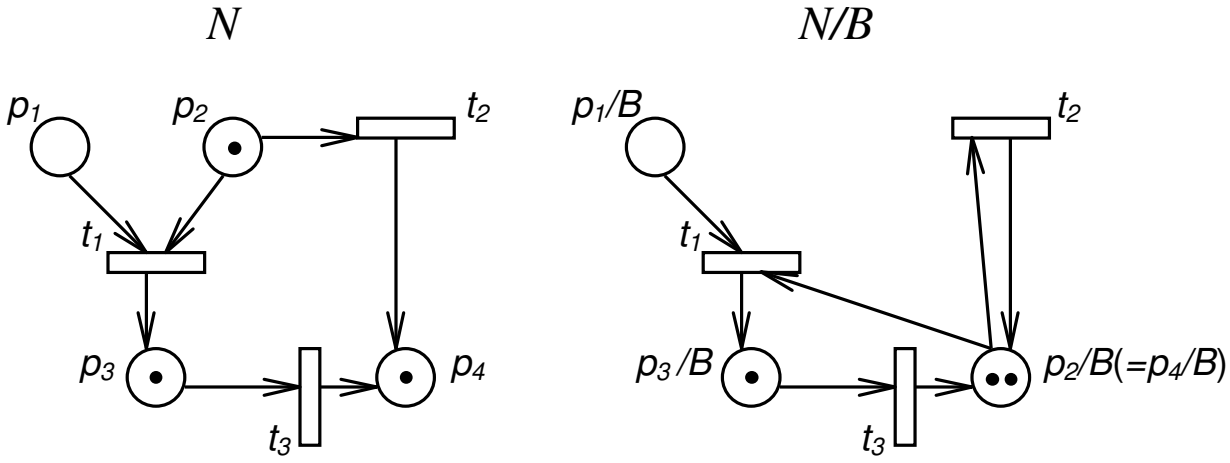
If $M \xrightarrow{t} M'$ in N, then $M/B \xrightarrow{t} M'/B$ in N/B.

If $m \xrightarrow{t} m'$ in N/B, then there exist

two markings M and M' in N

s.t. $m = M/B$, $m' = M'/B$ and $M \xrightarrow{t} M'$.

Reduction by fusion (continued)

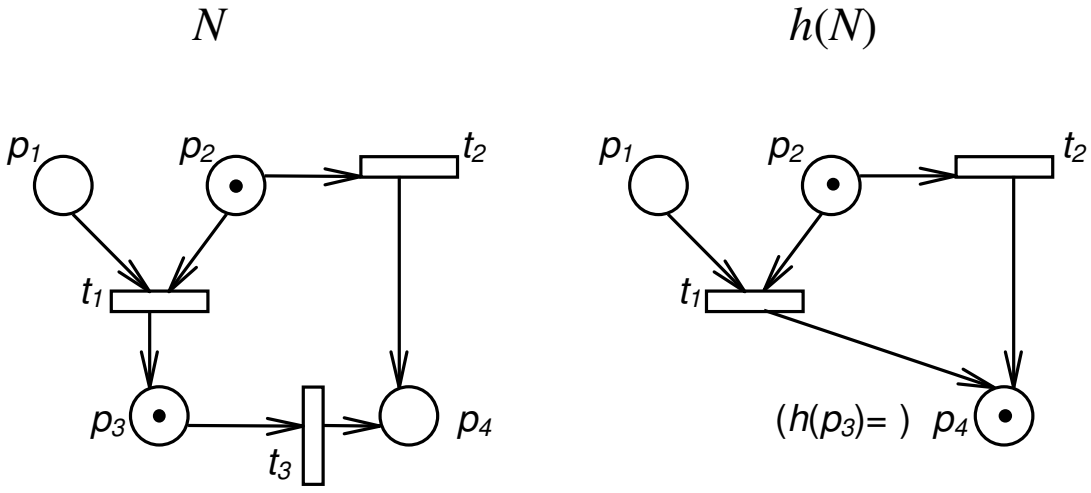


$\overline{B}$: if $p_1 B p'_1, \ldots, p_n B p'_n$ then $\{p_1, \ldots, p_n\} \overline{B} \{p'_1, \ldots, p'_n\}$.

$M \overline{B} M'$ iff $M/B = M'/B$.

When $\overline{B}$ relates bisimilar markings only, then $(_/B)^{-1}$ has the transfer property up to bisimulation.

Reduction by replacement



If $m \xrightarrow{t} m'$ in $h(N)$, then $m \xrightarrow{t} M'$ in N for some M' such that $h(M') = m'$.

If $M \xrightarrow{t} M'$ in N and $h(M) = M$, then $M \xrightarrow{t} h(M')$ in $h(N)$.

Correct reductions

$N = \langle P, T, F, l \rangle$ – a net,
$\mathcal{A} \subseteq \mathbb{N}^P$ – a set of relevant markings.

An equivalence $B \subseteq P \times P$ is a *correct fusion* for net N with relevant markings $\mathcal{A}$ iff $N, M \sim N/B, M/B$ for all $M \in \mathcal{A}$.

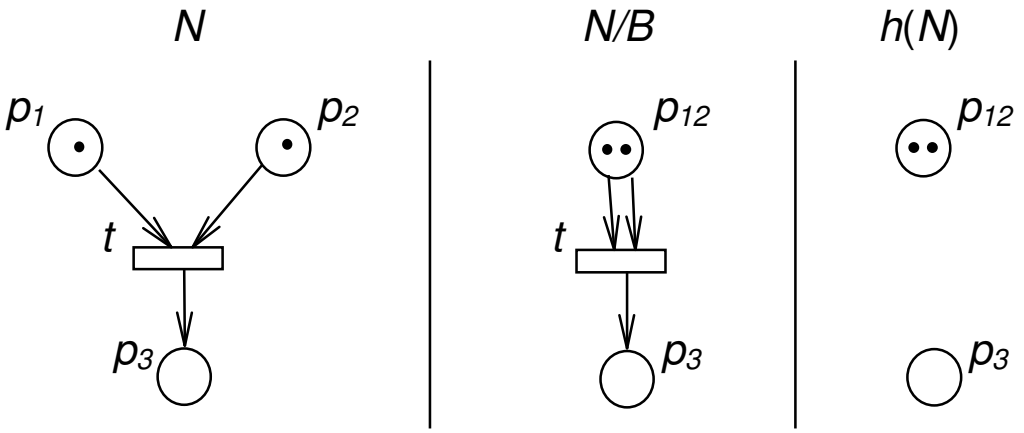
A projection $h: P \to P$ is a correct projection for net N with relevant markings $\mathcal{A}$ iff $N, M \sim h(N), h(M)$ for all $M \in \mathcal{A}$.

$$h: P \to P \quad \dashrightarrow \quad B_h: \; p \, B_h \, p'$$

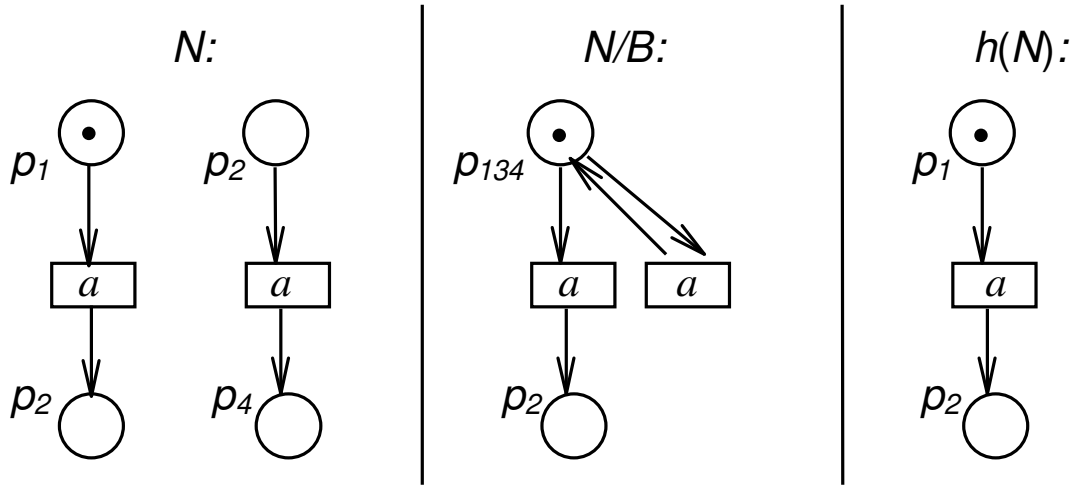
B_h is a correct fusion $\Leftrightarrow$ h is a correct projection?

Correct reductions (continued)

B_h is correct and h is not correct:



B_h is not correct and h is correct:



Correct reductions (continued)

$\mathcal{A}$ is *succ-closed* if $M \in \mathcal{A}$ and $M \to M'$ entail $M' \in \mathcal{A}$.

$\mathcal{A}$ is *B-closed* if $M \in \mathcal{A}$ and $M \,\overline{B}\, M'$ entail $M' \in \mathcal{A}$.

Assume $\mathcal{A}$ is succ-closed and B-closed.
Then B is a correct fusion iff $\overline{B} \cap (\mathcal{A} \times \mathcal{A})$ is included in $\sim$, the largest bisimulation between markings of N.

Assume h is a projection and $\mathcal{A}$ is B_h-closed and succ-closed.
Then h is a correct projection for $N, \mathcal{A}$ iff B_h is a correct fusion for $N, \mathcal{A}$.

Combining correct reductions

If h_1 is a correct projection for $N, \mathcal{A}$
and h_2 is a correct projection for $h_1(N)$,
where $h_1(\mathcal{A}) \stackrel{def}{=} \{ h_1(M) \mid M \in \mathcal{A} \}$,

then $h_2 \circ h_1$ is a correct projection for $N, \mathcal{A}$.

If B_1 and B_2 are correct fusions for $N, \mathcal{A}$, then $B \stackrel{def}{=} (B_1 \cup B_2)^*$, the smallest equivalence larger than B_1 and B_2, is a correct fusion for $N, \mathcal{A}$.

If $h = h_2 \circ h_1$ is a correct projection, then h_1 is a correct projection.

Let B be a correct fusion.
If B' is an equivalence such that $B' \subseteq B$ then B' is a correct fusion.

The largest correct fusion

If $\mathcal{A}$ is succ-closed, there exists a largest correct fusions for $N, \mathcal{A}$.

(only equivalences under which $\mathcal{A}$ is closed are considered).

The largest correct fusion $R(N, \mathcal{A})$ is not computable in general, even with $\mathcal{A} = \mathbb{N}^{\mathrm{P}}$.

[Jančar 95]

Place bisimulation

A reflexive symmetric relation $B \subseteq P \times P$ is a *place bisimulation* over $N, \mathcal{A}$ if $\mathcal{A}$ is B-closed and for all $M_1, M_2 \in \mathcal{A}$ such that $M_1 \overline{B} M_2$, for all steps $M_1 \xrightarrow{t} M'_1$, there is a step $M_2 \xrightarrow{u} M'_2$ with $M'_1 \overline{B} M'_2$ and $l(t) = l(u)$.

Any transitive place bisimulation over $N, \mathcal{A}$ is a correct fusion.

There exists a largest place bisimulation over $N, \mathcal{A}$ (denoted $B(N, \mathcal{A})$).

Clearly, $B(N, \mathcal{A}) \subseteq R(N, \mathcal{A})$.

How to compute $B(N, \mathcal{A})$?

Place bisimulation (continued)

How to reduce a number of checks to a finite number?

The set of all *minimal covers* for a transition t, denoted $C(t)$, is the set of all minimal markings $M \in \mathcal{A}$ such that ${}^\bullet t \subseteq M$.

$C(t)$ is always finite! (follows from Dickson's lemma)

Relation $B \subseteq P \times P$ has the weak transfer property if $\mathcal{A}$ is B-closed, and for all $t \in T$, for all $p \in {}^\bullet t$, for all $q \in P$ such that pBq, for all $M_1 \in C(t)$ there is a $u \in T$ s.t. $l(u) = l(t)$ and, writing M_2 for $M_1 - p + q$, we have $M_2 \xrightarrow{u} M'_2$ with $M_1 \,\overline{B}\, M_2$ (assuming $M_1 \xrightarrow{t} M'_1$).

The largest reflexive and symmetric B having the weak transfer property exists and this is $B(N, \mathcal{A})$.

Algorithm

input a labeled Petri net N,
a succ-closed set $\mathcal{A}$ of relevant markings.

output $B(N, \mathcal{A})$.

step1 Let $B = E(N, \mathcal{A})$, "the largest place equivalence under which $\mathcal{A}$ is closed".

step2 Check if B has the weak transfer property:

- If it has, then B is $B(N, \mathcal{A})$.
- Otherwise, there is a $t \in T$, a $M \in C(\mathrm{t})$, a $p \in M$, a $q \in P$ with pBq such that $M_1 \xrightarrow{t} M'_1$ can not be imitated from $M - p + q$. Then remove the pairs (p, q) and (q, p) from B and return to step 2.

Conclusions:
place bisimulation in practice

Open questions:

- What is an effective way of computing $C(t)$?
- How can $E(N, \mathcal{A})$ (the first approximation of $B(N, \mathcal{A})$) be chosen?

Easy if:

- $\mathcal{A}$ is finite.
- $\mathcal{A}$ is given by a Presburger formulas and it can be effectively computed.

Conclusions:
place bisimulation in practice

The source of difficulties: $\mathcal{A}$.

Possible ways of extending applicability:

- Learn how to define $\mathcal{A}$.
- Apply similar techniques to Petri nets with silent moves.
- Compute the largest possible fusion for a class of Petri nets, where it is computable.