Soundness and *Separability* of Workflow Nets in the Stepwise Refinement Approach

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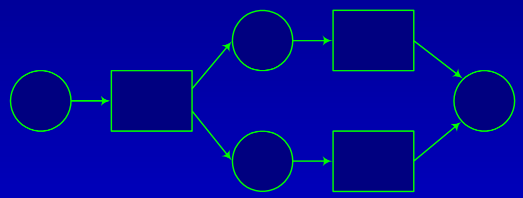
Workflow nets

A Petri net N is a Workflow net (WF-net) iff:

- N has two special places (or transitions): an initial place (transition) i: ${}^\bullet i = \emptyset$, and a final place (transition) f: $f^\bullet = \emptyset$.
- For any node $n \in (S \cup T)$ there exists a path from i to n and a path from n to f.



Applications: business process modelling, software engineering, ...

Soundness

Desired property: proper completion

Classical definition of soundness for WF-nets ([vdAalst]):
A WF-net N is sound iff:

- For every marking M reachable from $[i]$, there exists firing sequence leading to $[f]$.
- Marking $[f]$ is the only reachable from $[i]$ with at least one token in $[f]$.
- There are no dead transitions in $(N, [i])$.

TU/e

Refinement of Workflow Nets

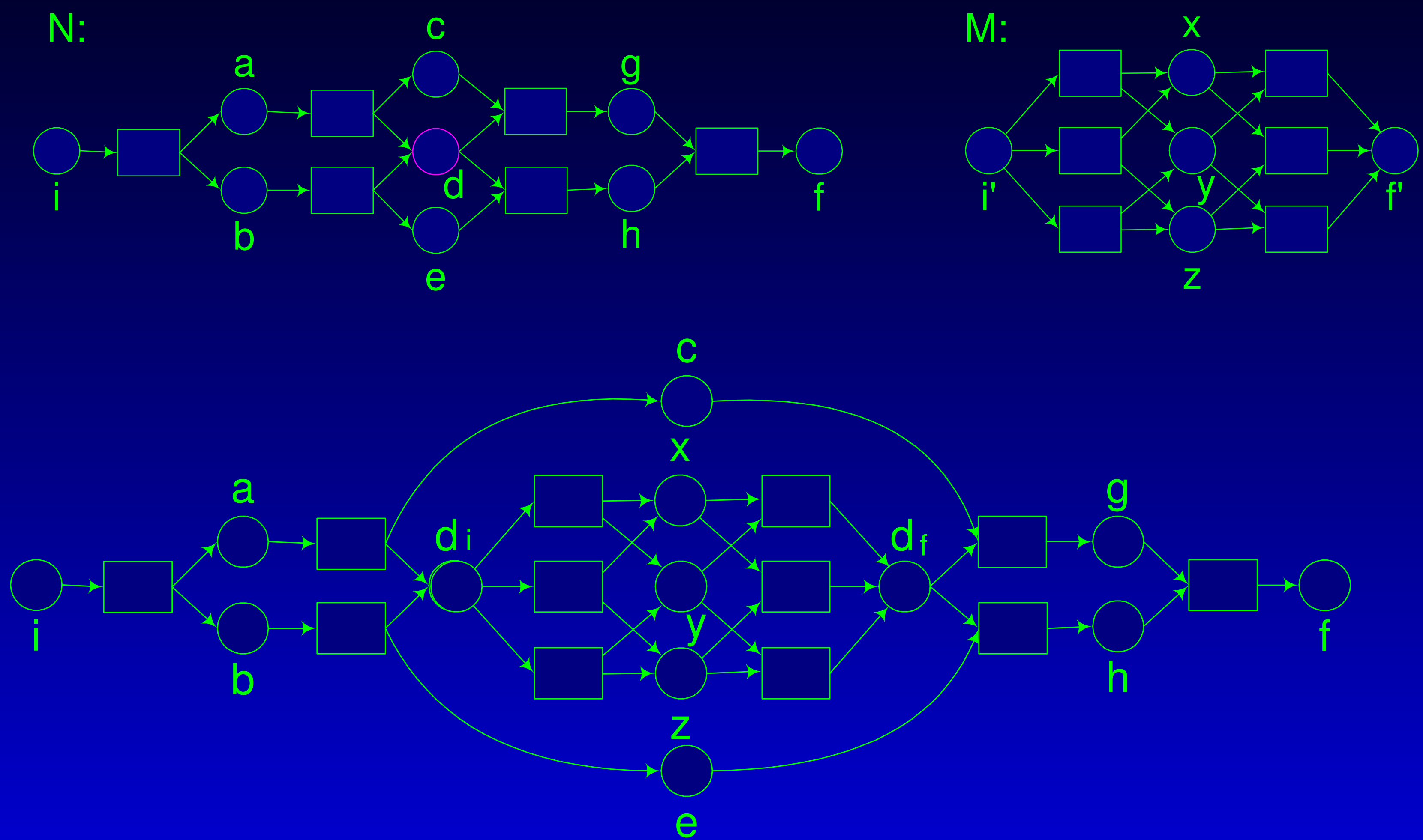
Place refinement: $N = L \otimes_p M$

Being at some location (place of the net) resources (tokens) undergo a number of operations.

Transition refinement: $N = L \otimes_t M$

A single task on a higher level becomes a sequence of subtasks also involving choice and parallelism.

Refinements and soundness



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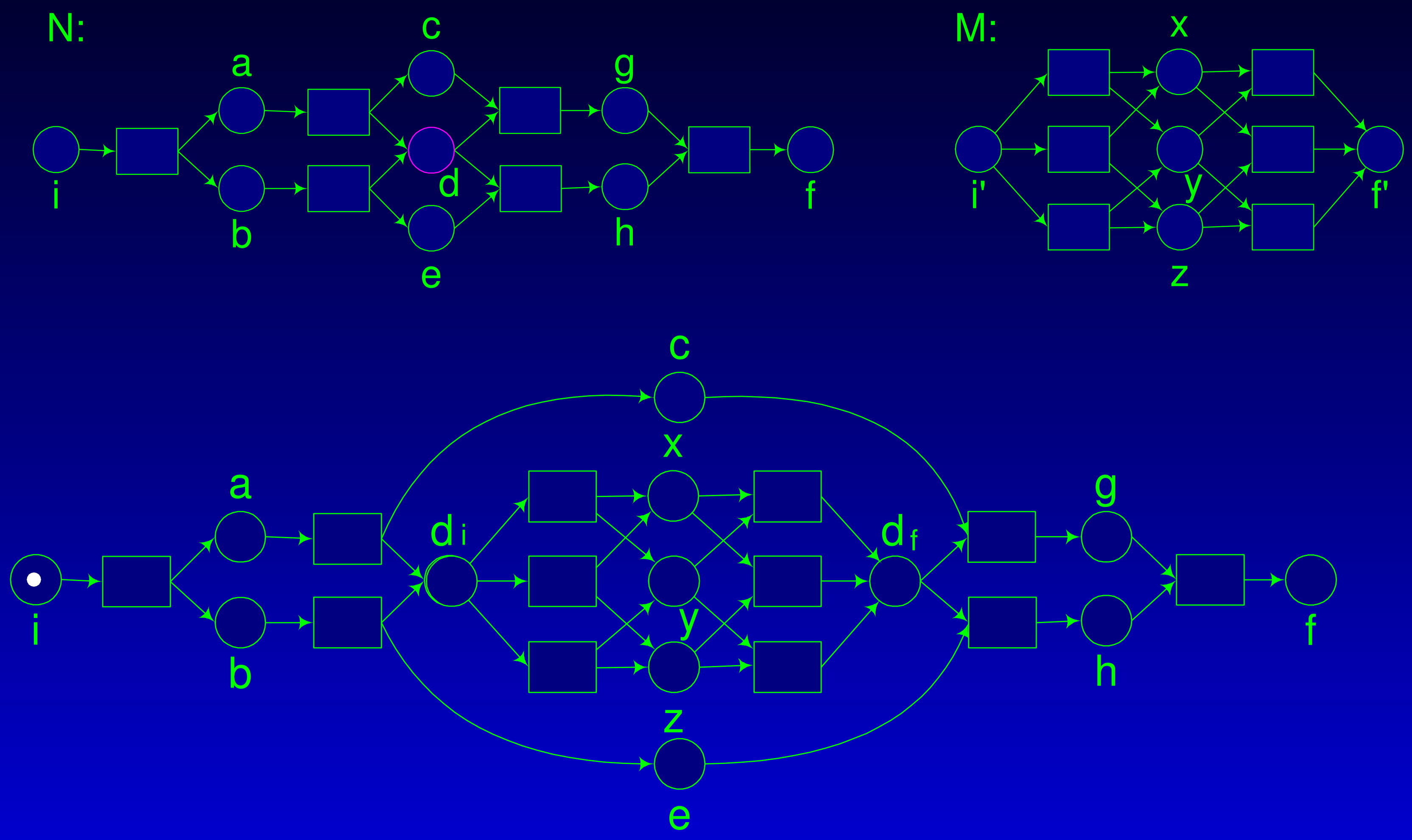

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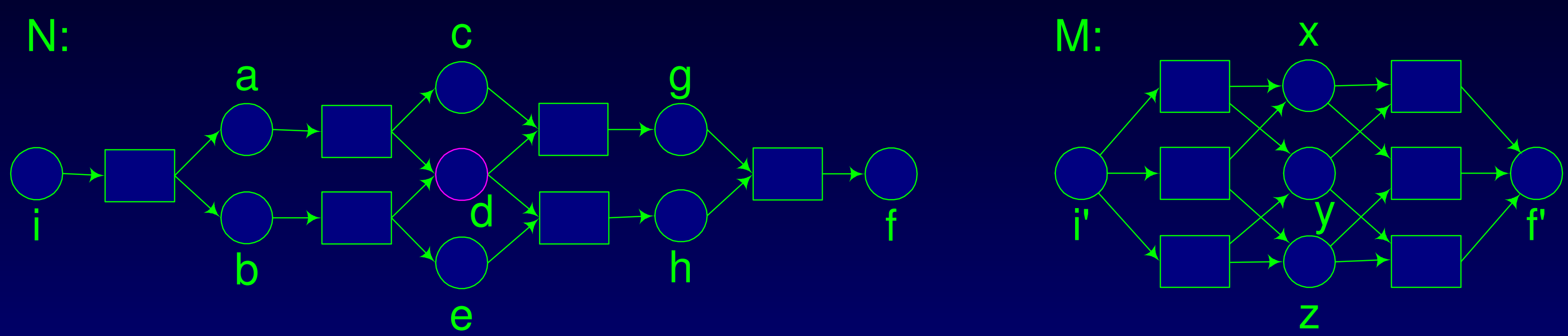

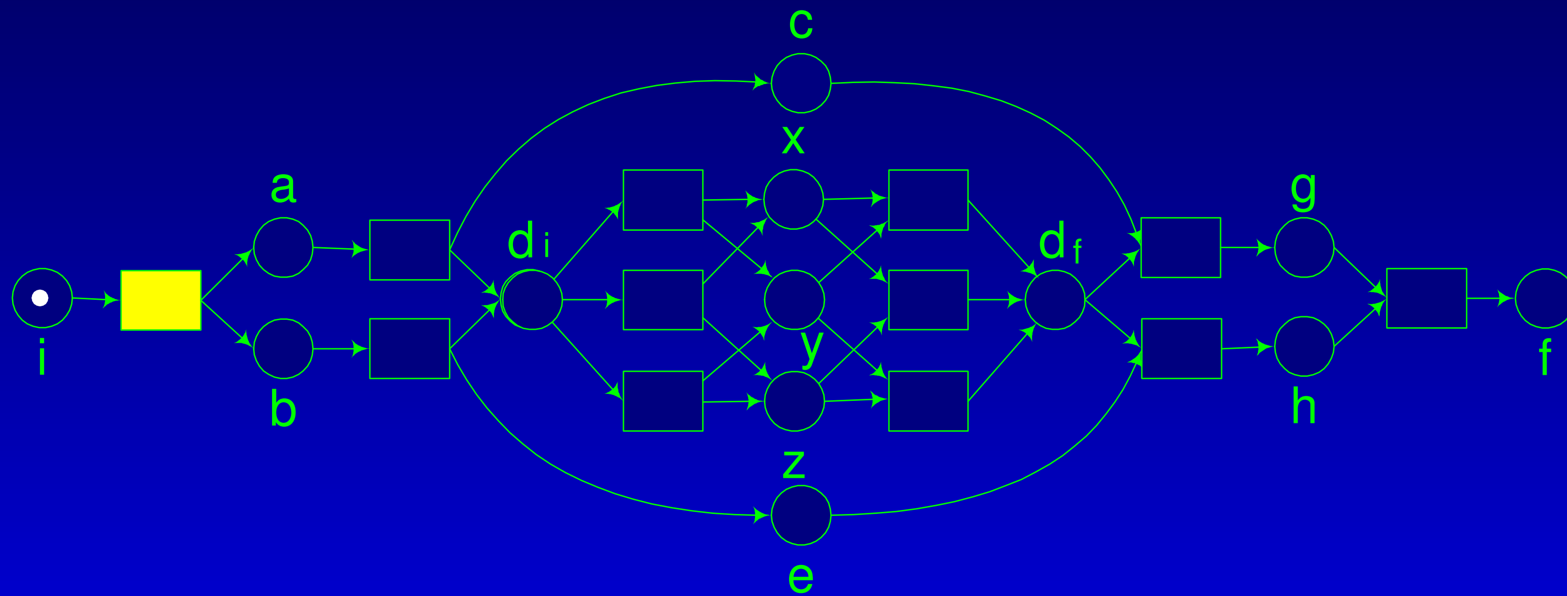

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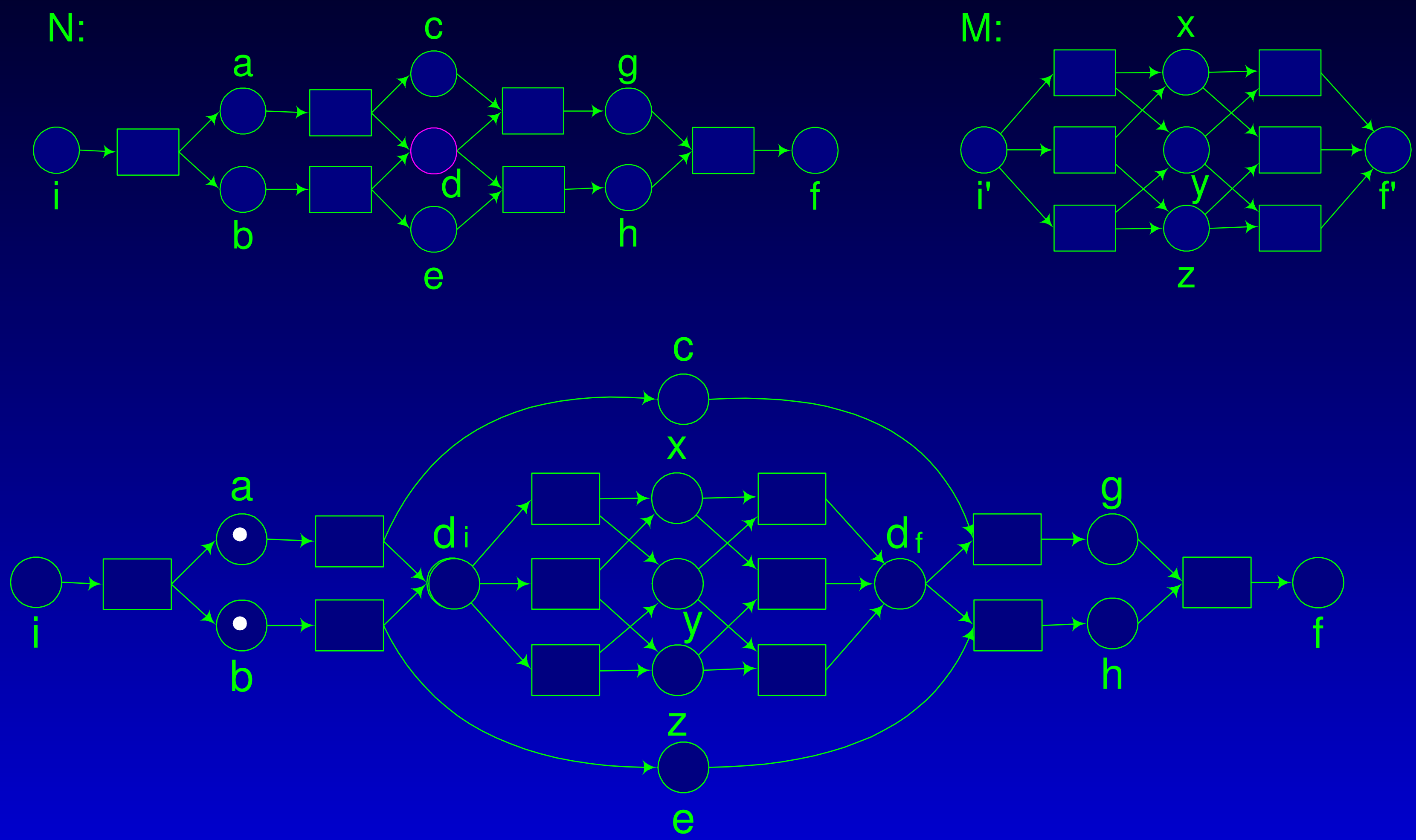

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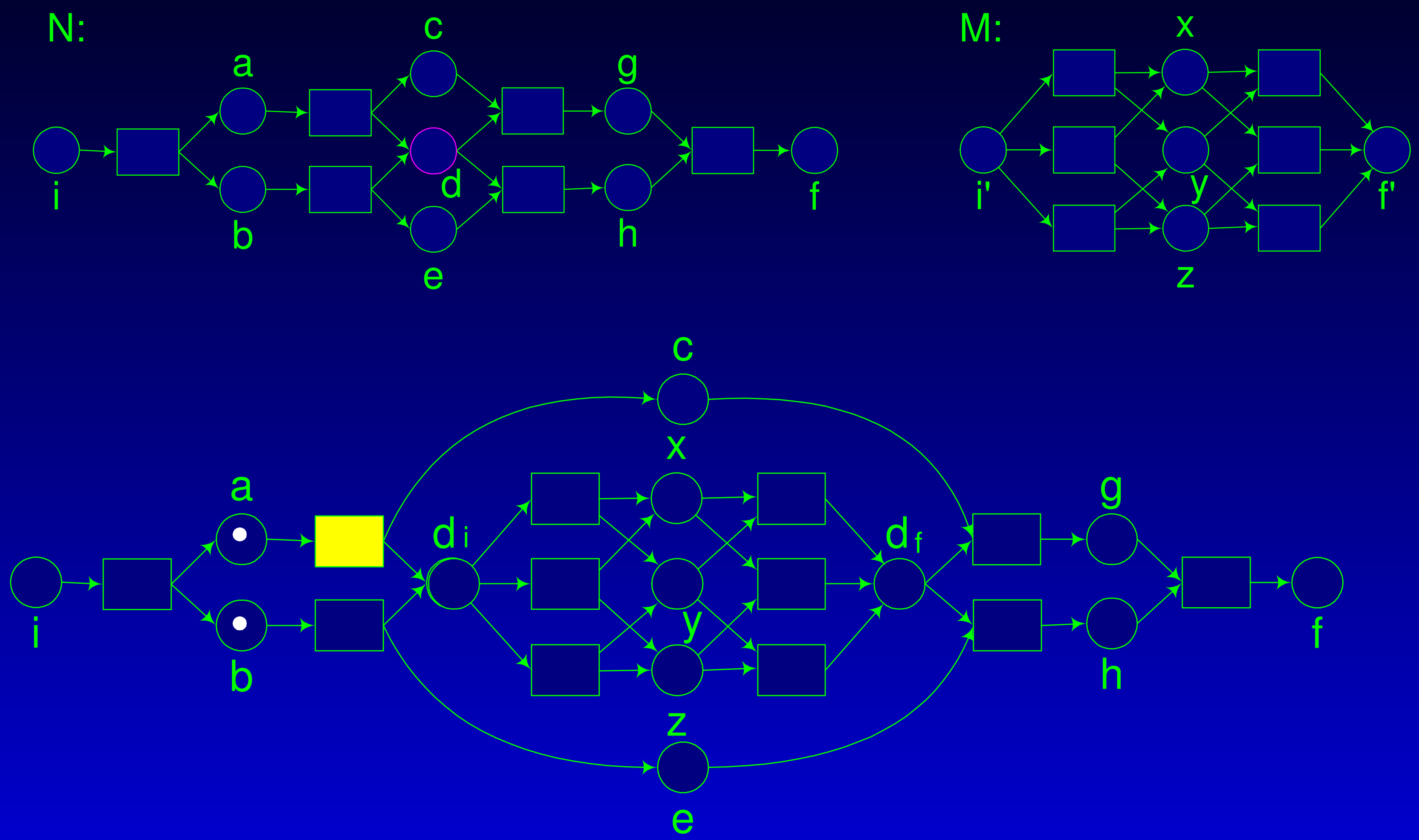

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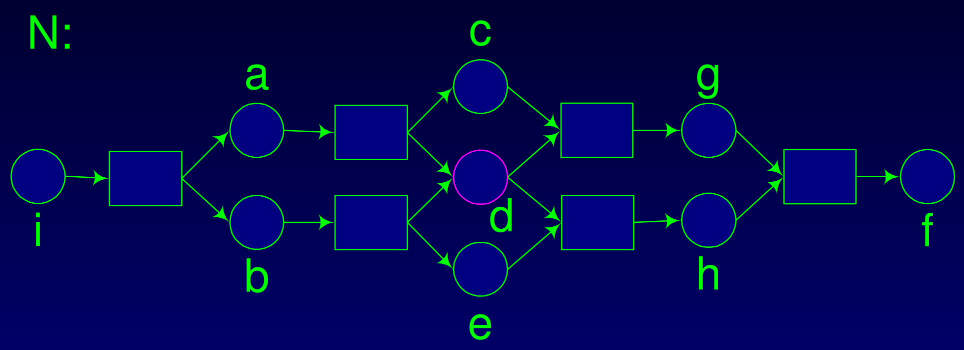

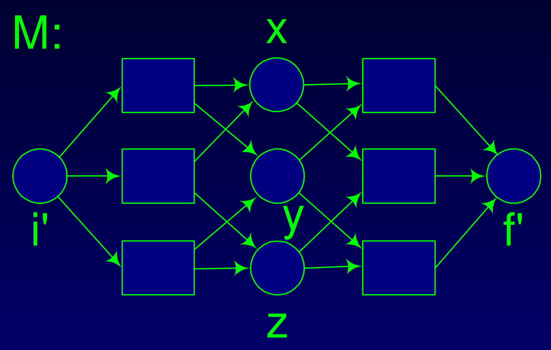

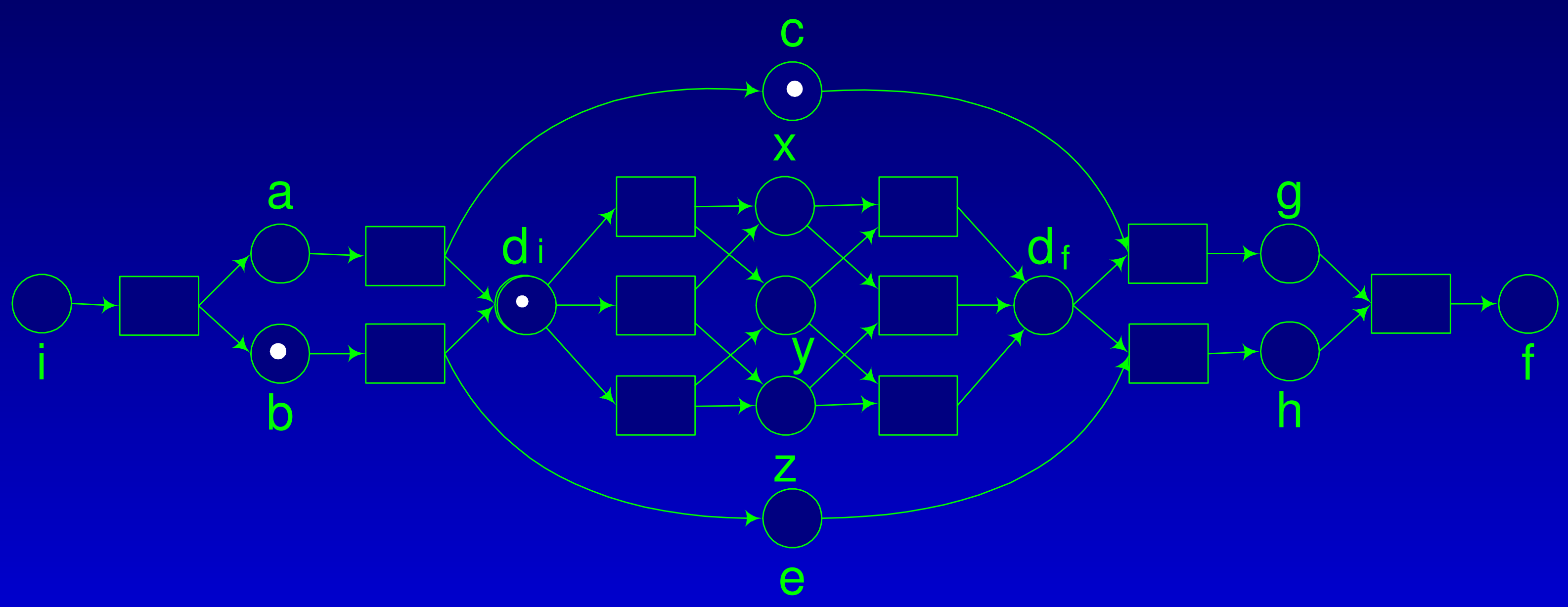

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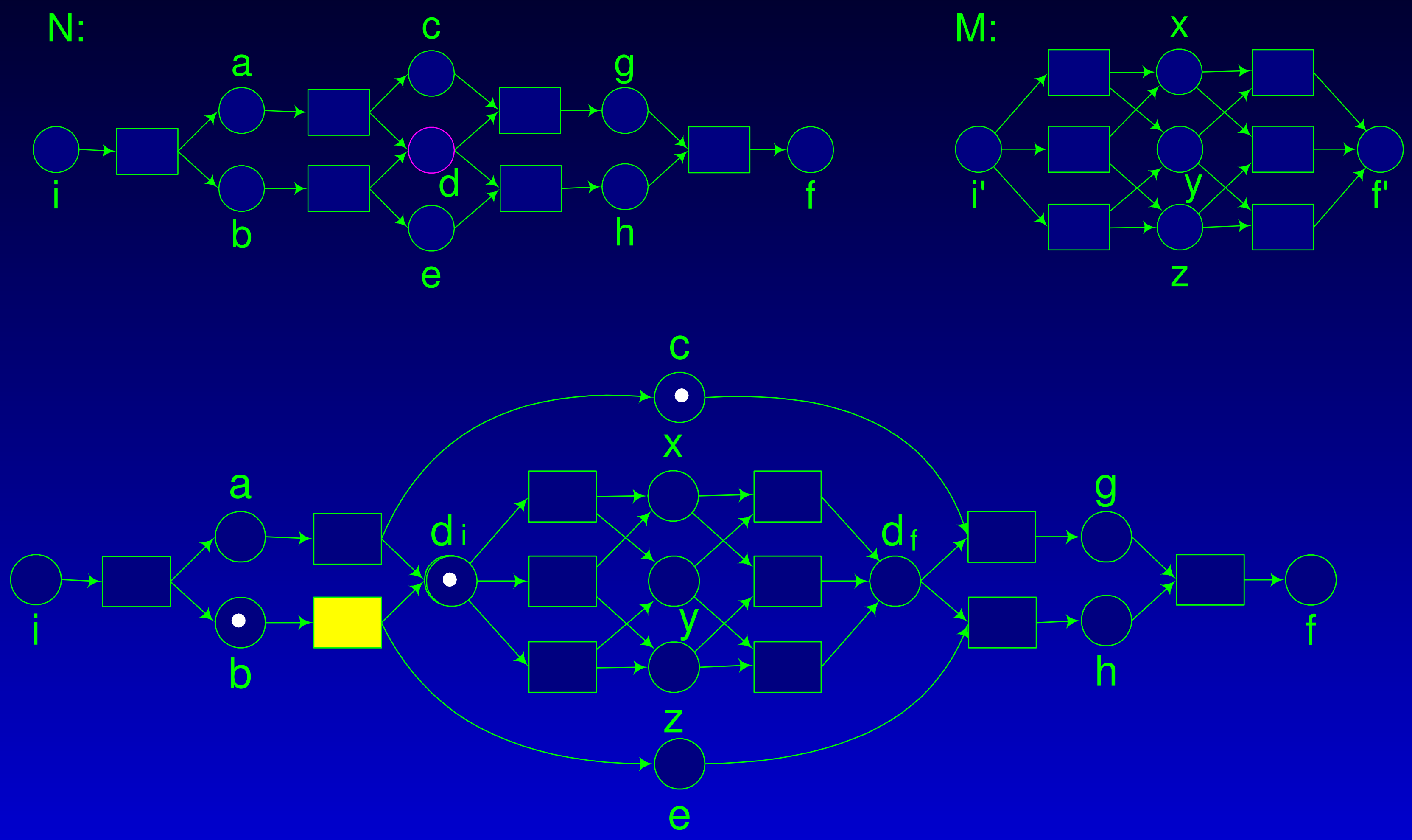

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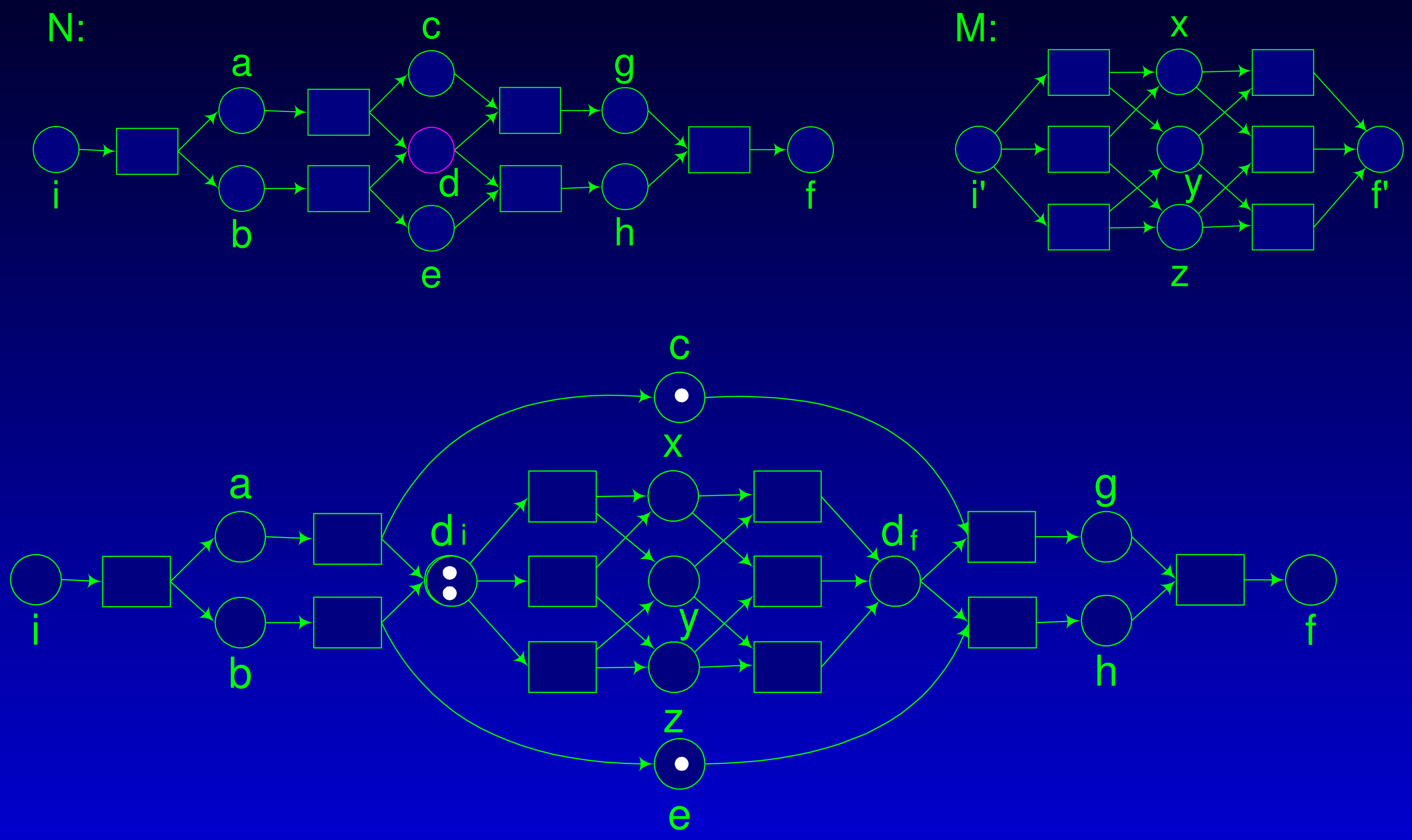

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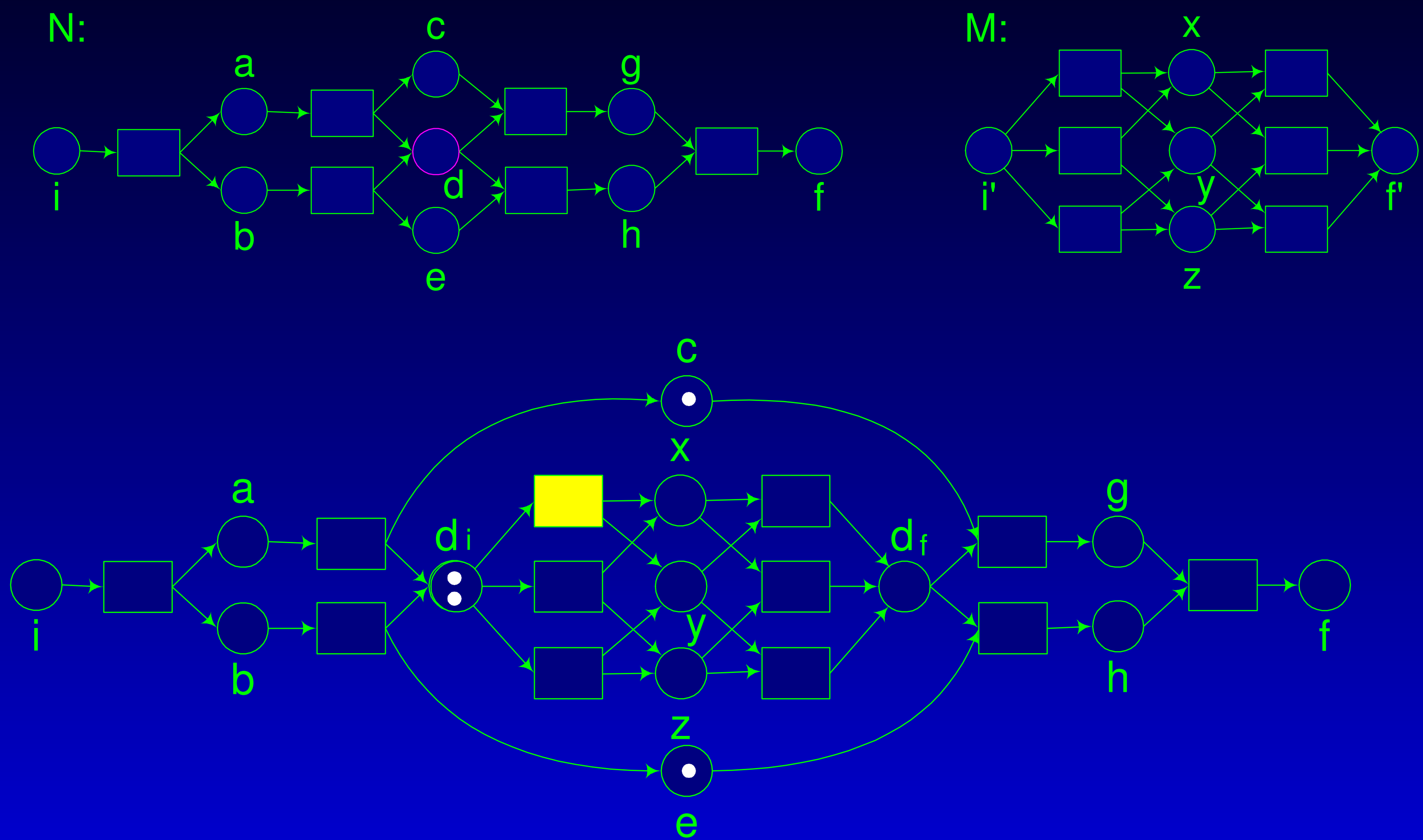

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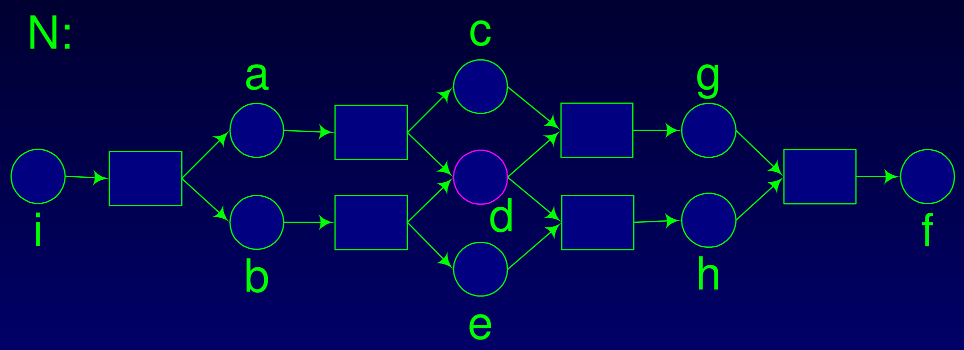

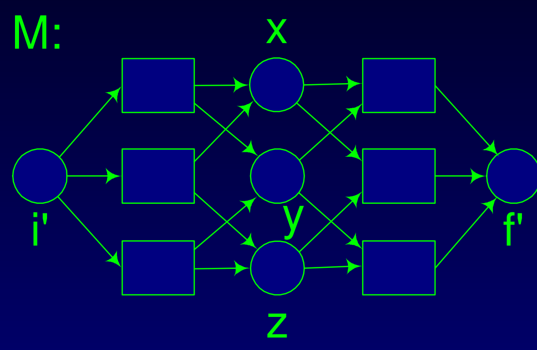

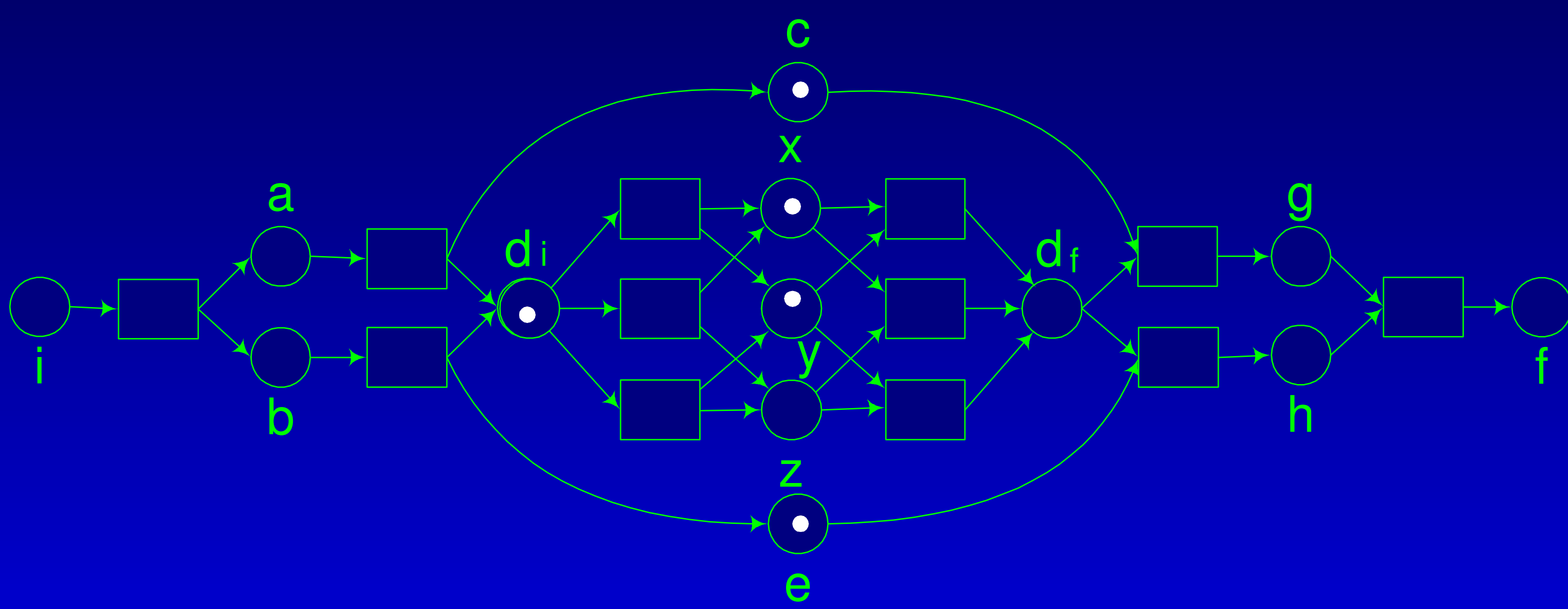

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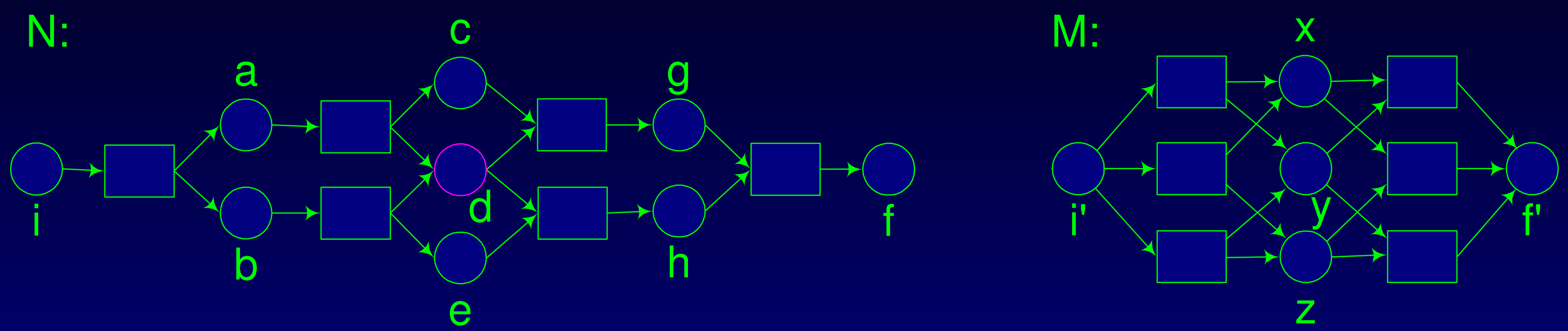

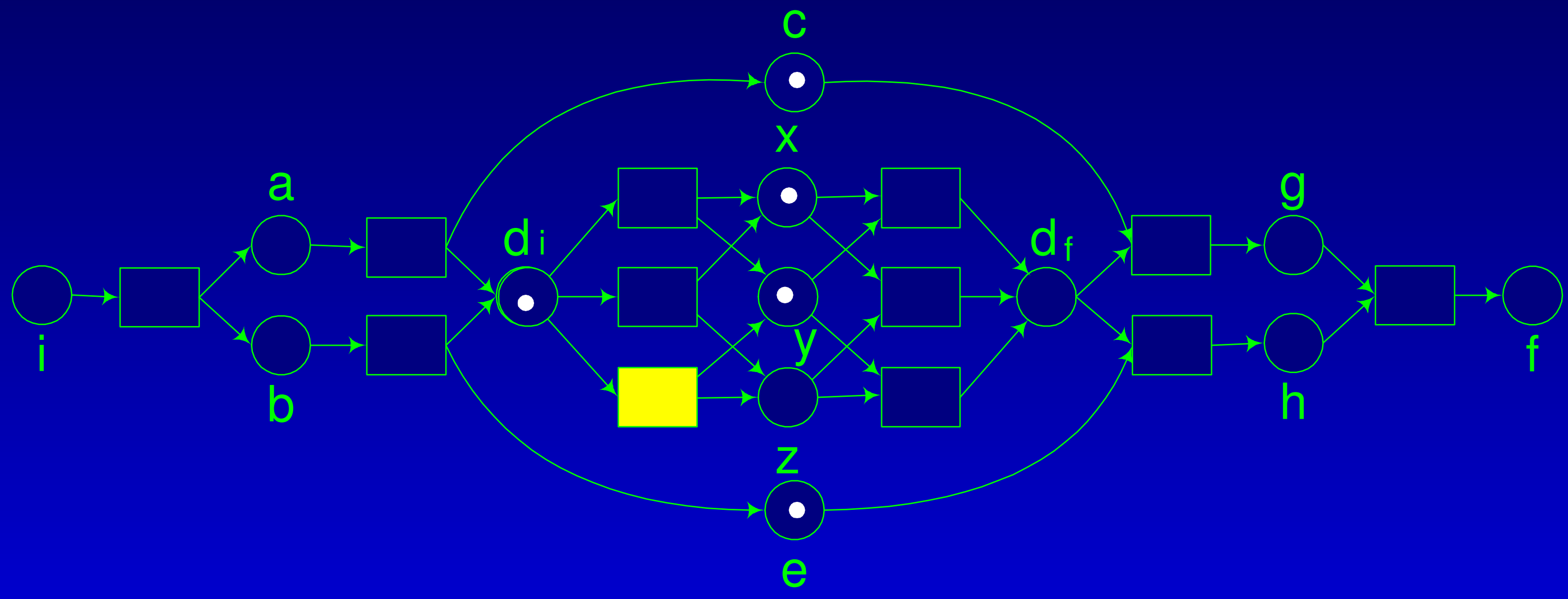

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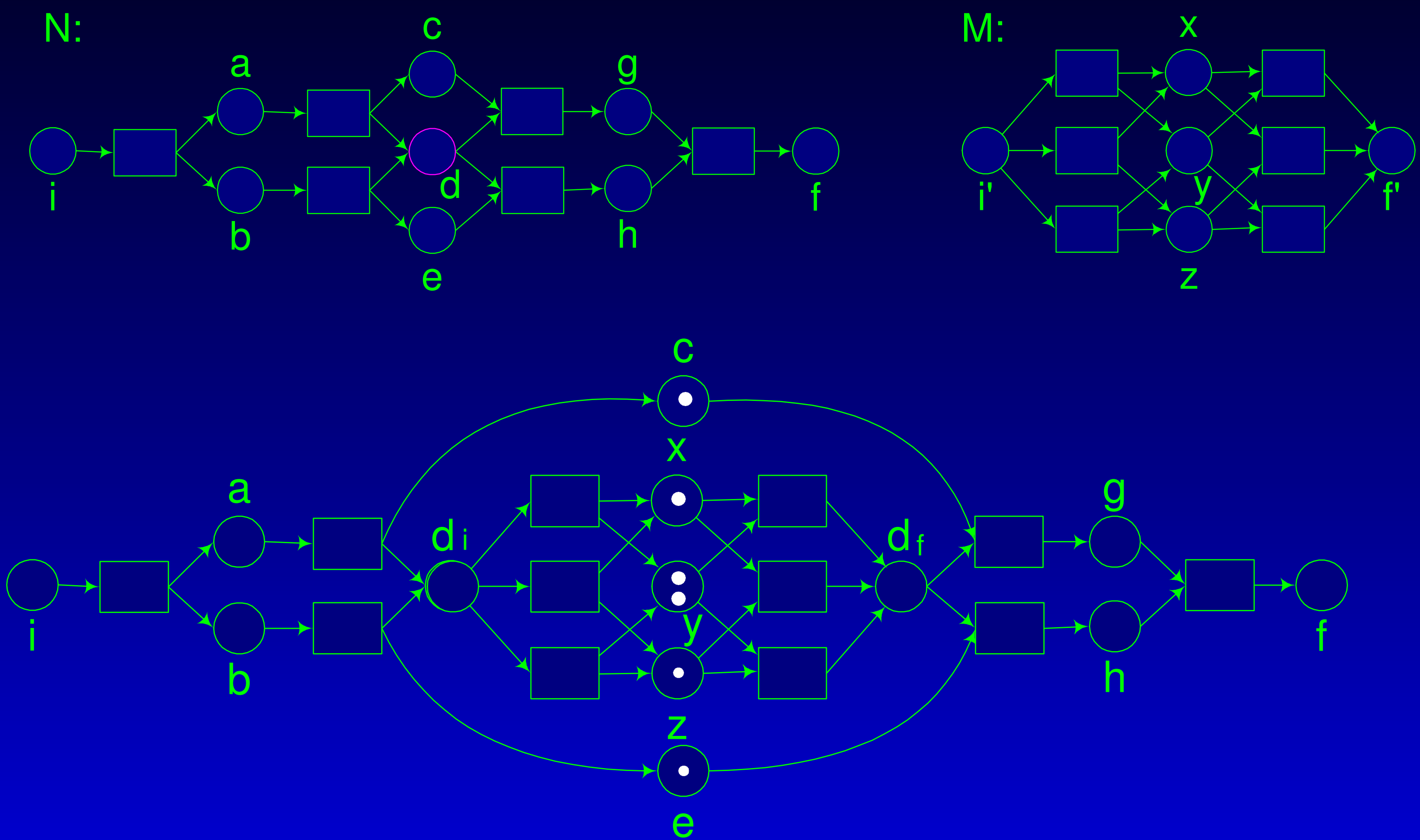

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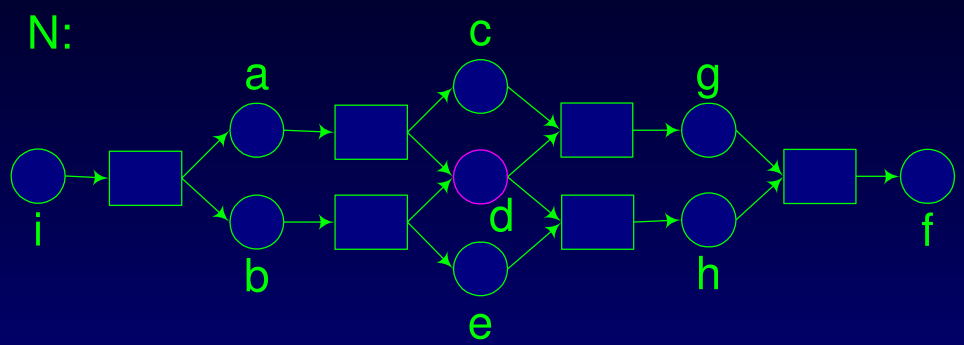

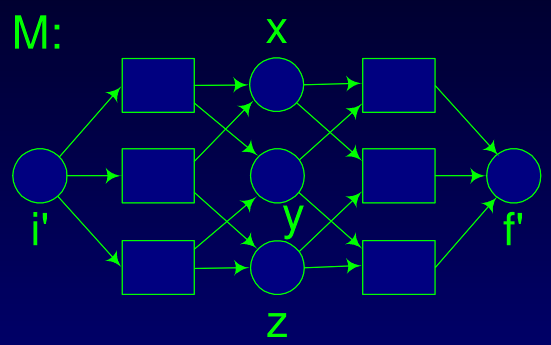

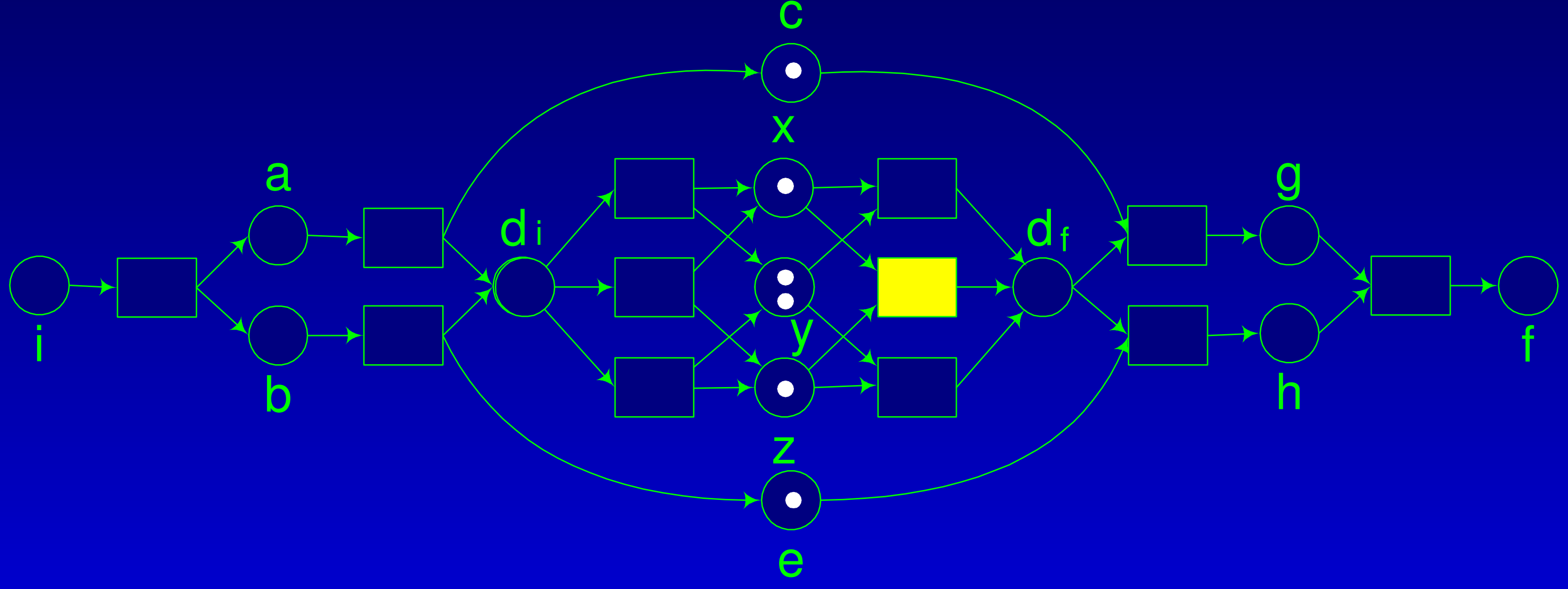

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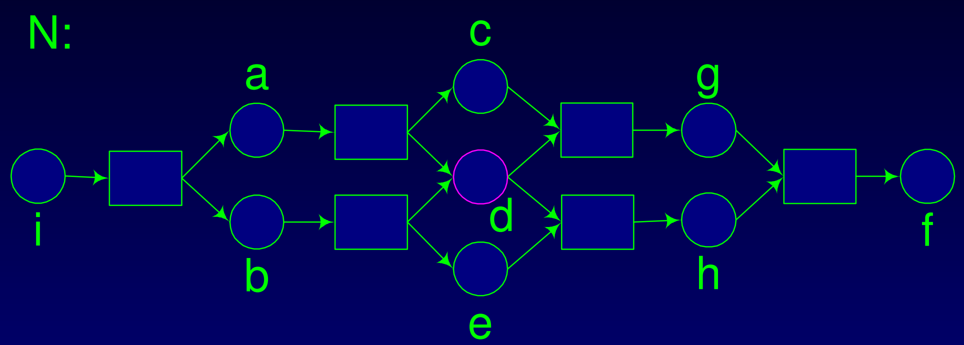

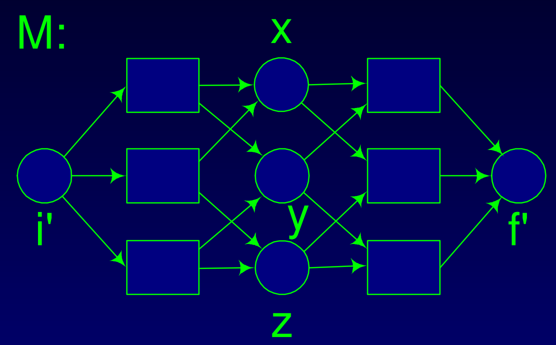

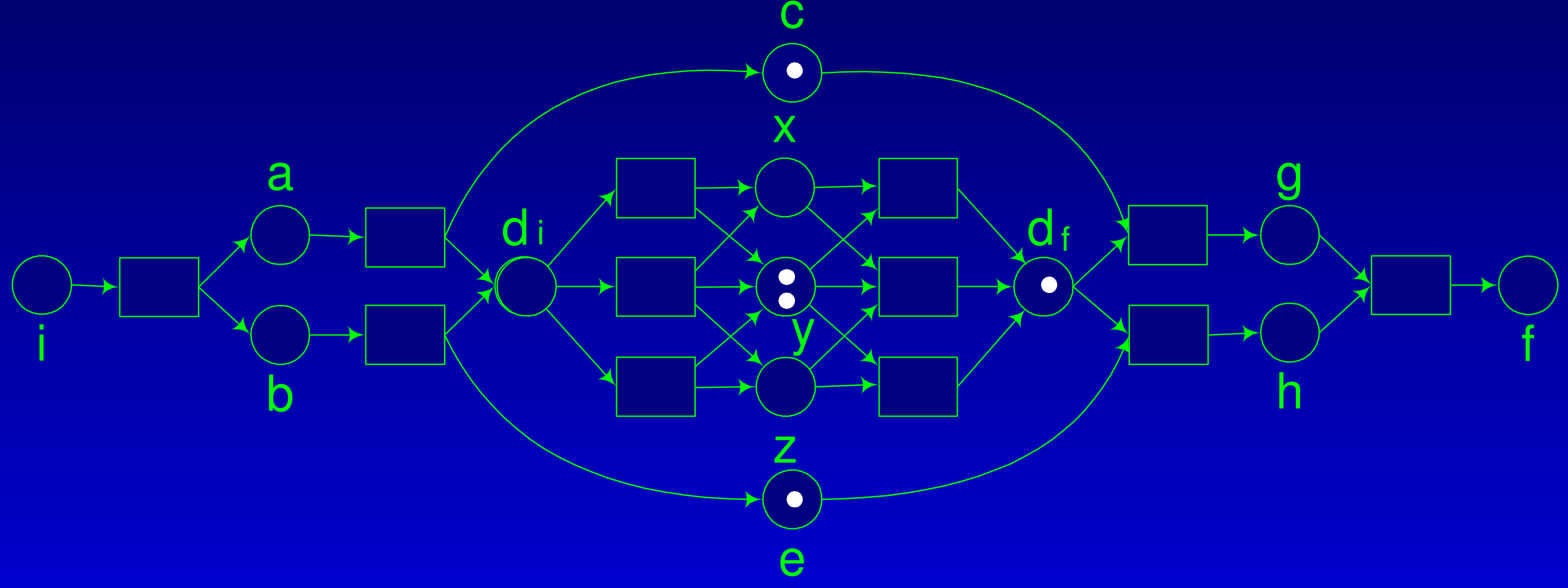

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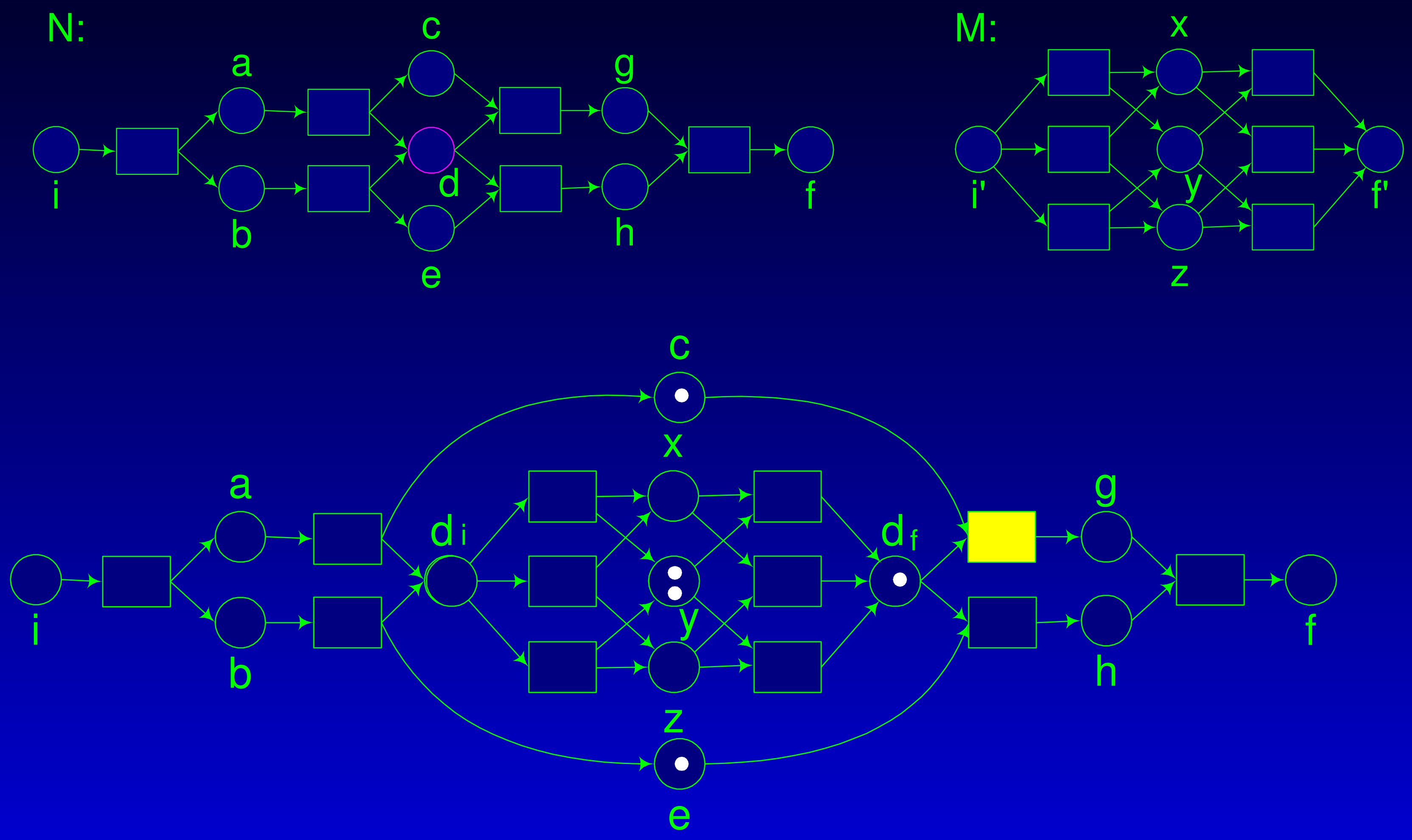

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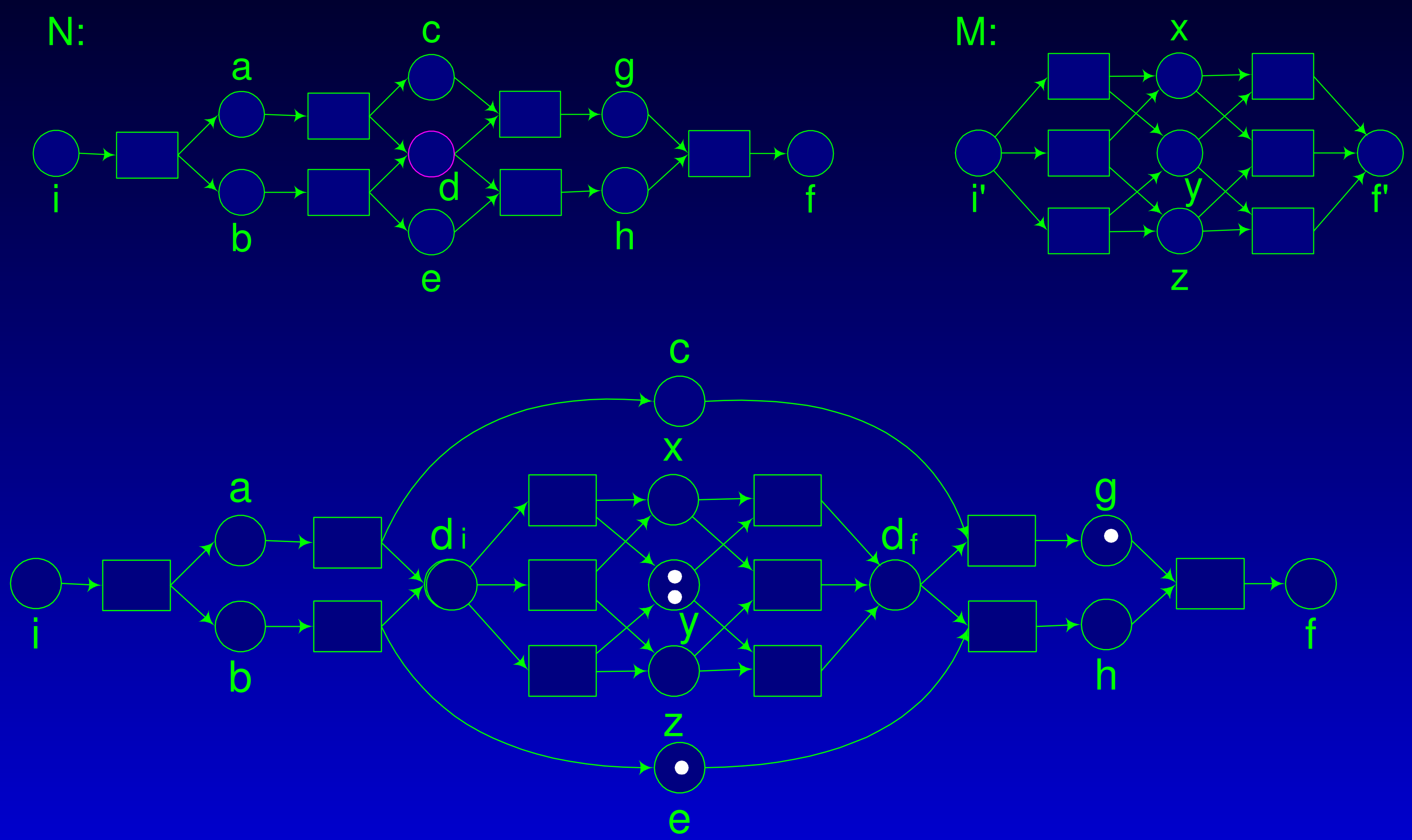

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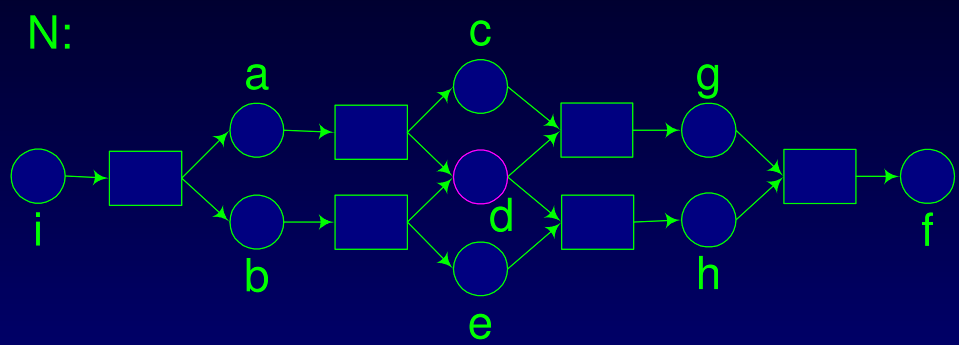
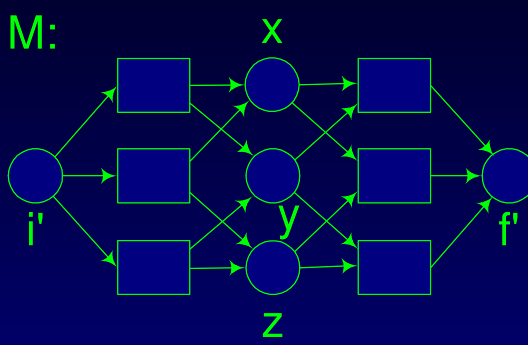
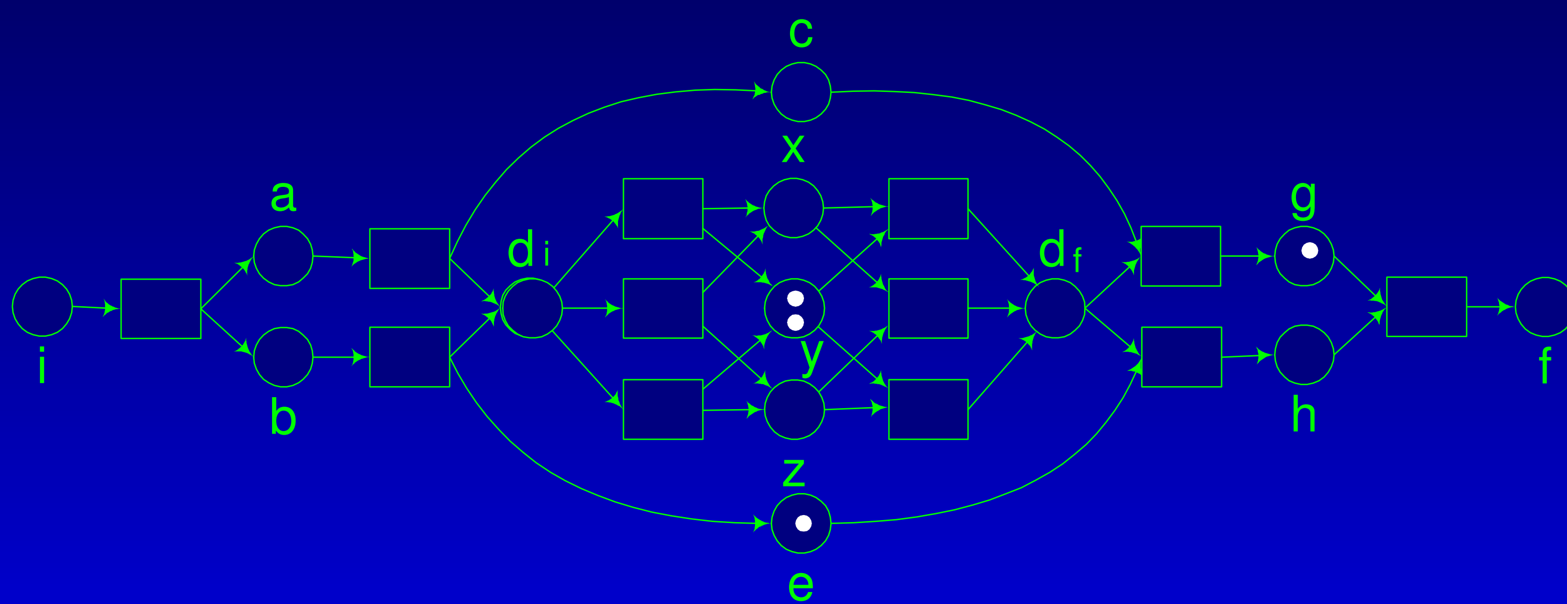






N and M are “sound”, but $N \otimes_d M$ is not!

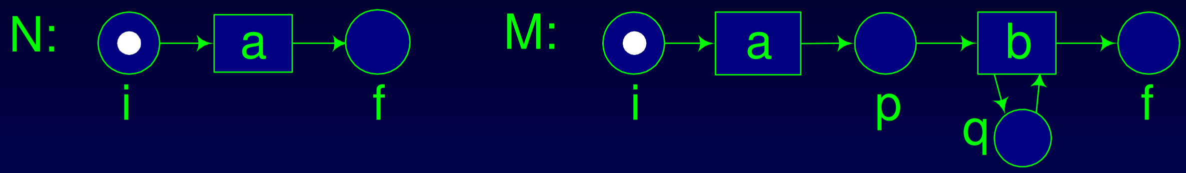
New definition of soundness

A sWF-net N with initial and final places i and f resp. is *k-sound* for $k \in \mathbb{N}$ iff $[f^k]$ is reachable from all markings m from $\mathcal{M}(N, [i^k])$.

A tWF-net N with initial and final transitions t_i, t_f respectively is k-sound iff the sWF-net formed by adding to S_N places p_i, p_f with $^\bullet p_i = \emptyset, p_i^\bullet = [t_i], {}^\bullet p_f = [t_f], p_f^\bullet = \emptyset$ is k-sound.

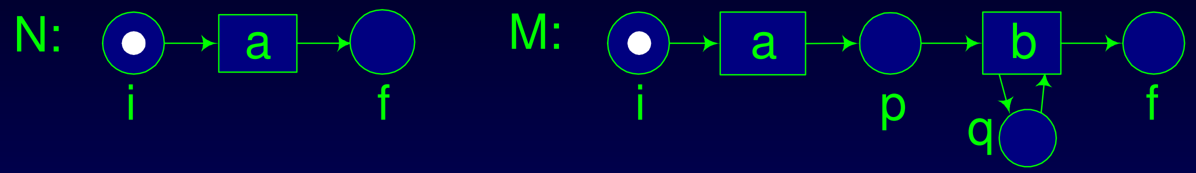
A WF-net is *sound* iff it is k-sound for every natural k.

Bisimulation of WF-nets



$(N, [i_N])$ and $(M, [i_M])$ are bisimilar.
But they should not be WF-bisimilar!

Bisimulation of WF-nets



$(N, [i_N])$ and $(M, [i_M])$ are bisimilar.
But they should not be WF-bisimilar!

Extra-condition for strong bisimulation:
$\forall k, x : [i_N^k] \; R \; x : x = [i_M^k]$ and
$\forall k, x : [f_N^k] \; R \; x : x = [f_M^k]$ and
$\forall k, x : x \; R \; [i_M^k] : x = [i_N^k]$ and
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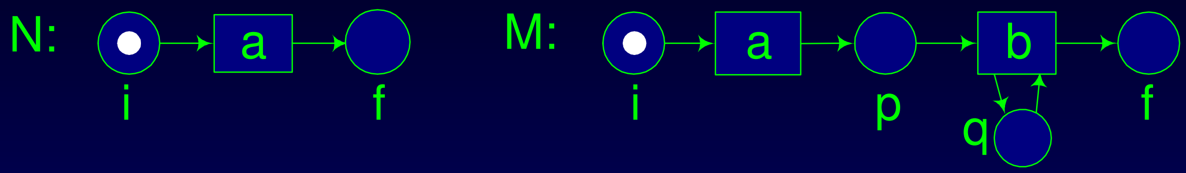


$(N, [i_N])$ and $(M, [i_M])$ are bisimilar.
But they should not be WF-bisimilar!

Extra-conditions for weak bisimulation:
$\forall k, x : [i_N^k] \; R \; x : [i_M^k] \Rightarrow_M x$ and
$\forall k, x : [f_N^k] \; R \; x : x \Rightarrow_M [f_M^k]$ and
$\forall k, x : x \; R \; [i_M^k] : [i_N^k] \Rightarrow_N x$ and
$\forall k, x : x \; R \; [f_M^k] : x \Rightarrow_N [f_N^k]$.

$\forall k : [i_N^k] \; R \; [i_M^k] \wedge [f_N^k] \; R \; [f_M^k]$.

Bisimulation and refinements

Place refinement

Let L be a WF-net with a place p and
M be a sound sWF-net with all transitions τ-labelled.
Then L and $N = L \otimes_p M$ are weakly WF-bisimilar.

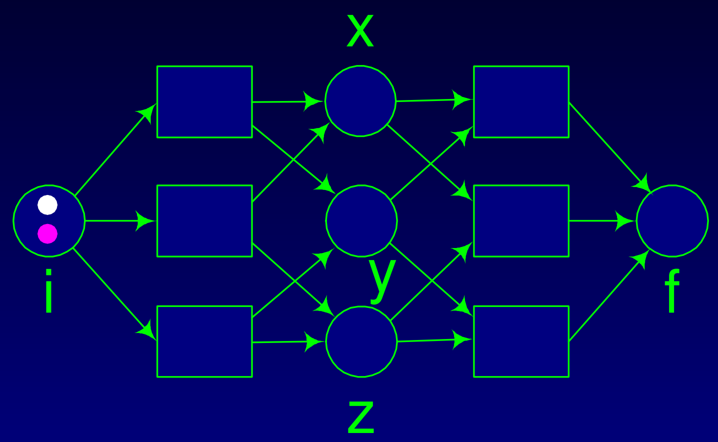
Transition refinement

Let L be a WF-net with transition t and
M a sound tWF-net with all transitions except i_M
labelled with τ.
Then L and $N = L \otimes_t M$ are weakly WF-bisimilar.

Soundness preservation

Let $N = L \otimes_n M$ be a refinement built of sound
WF-nets L, M. Then N is sound.

Workflow nets with id-tokens



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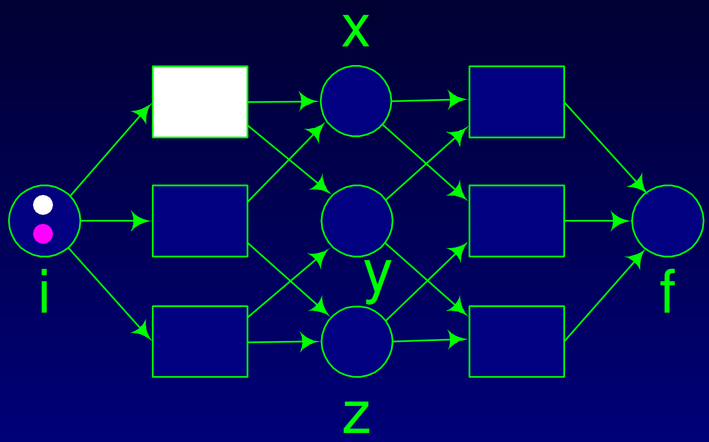

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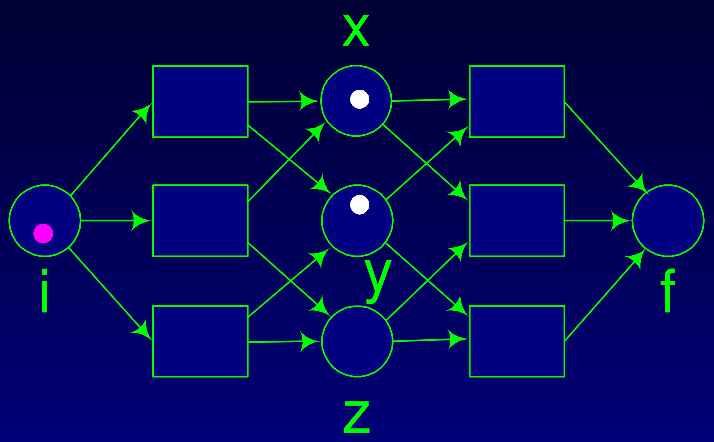

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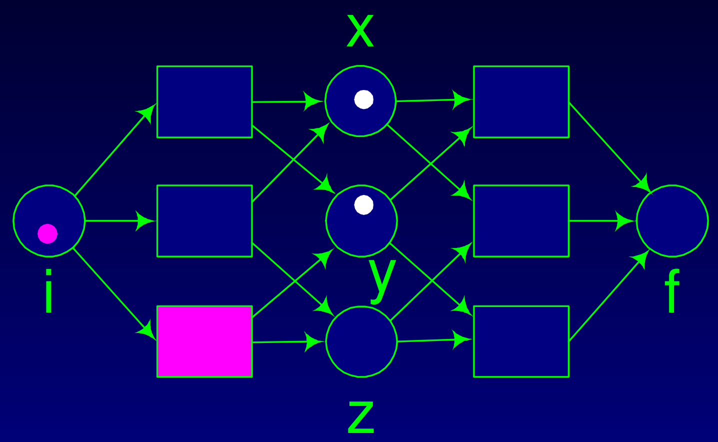

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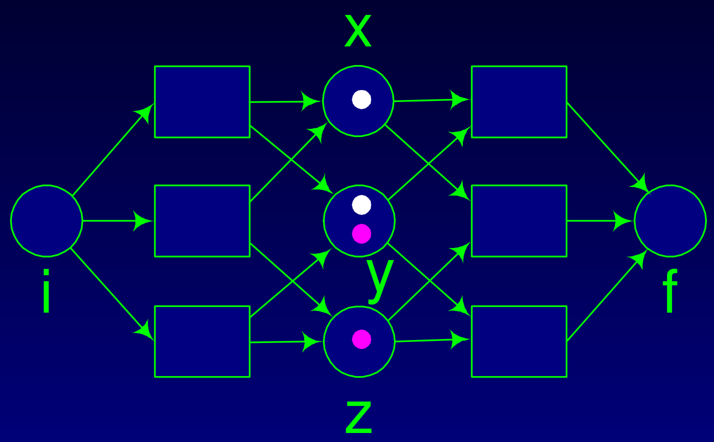

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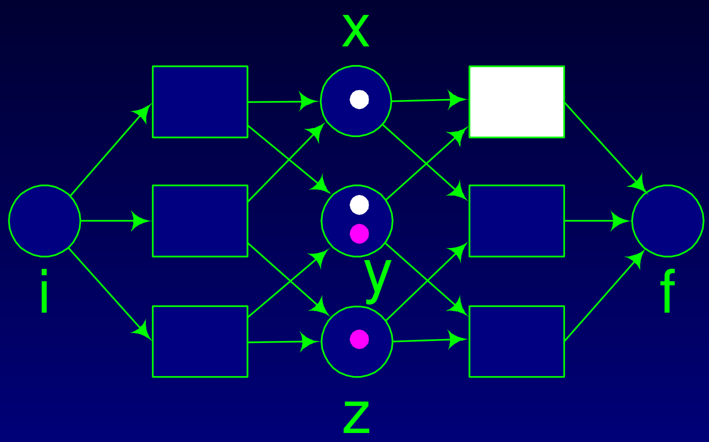

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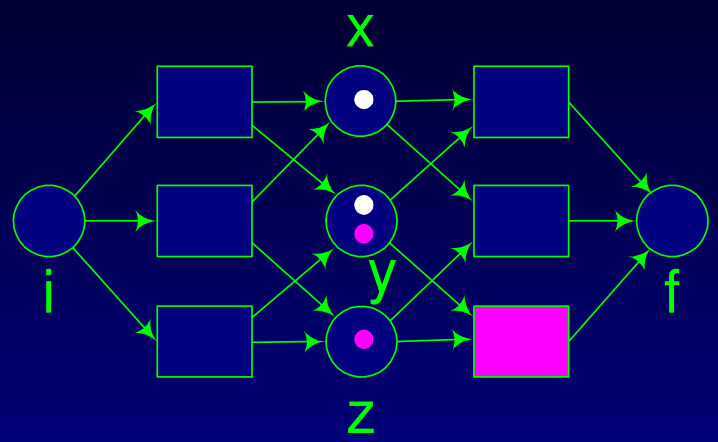

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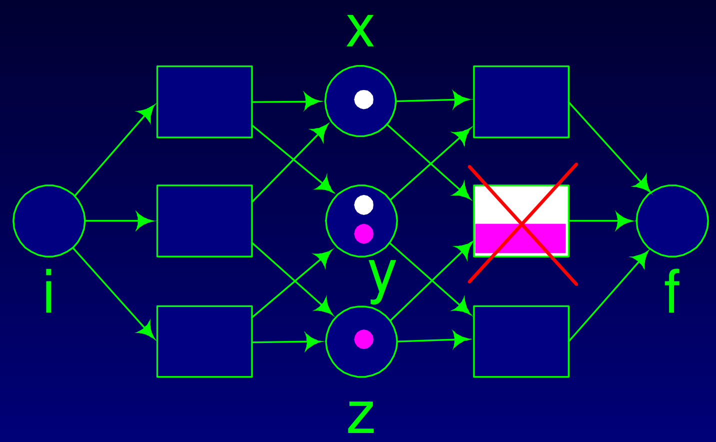

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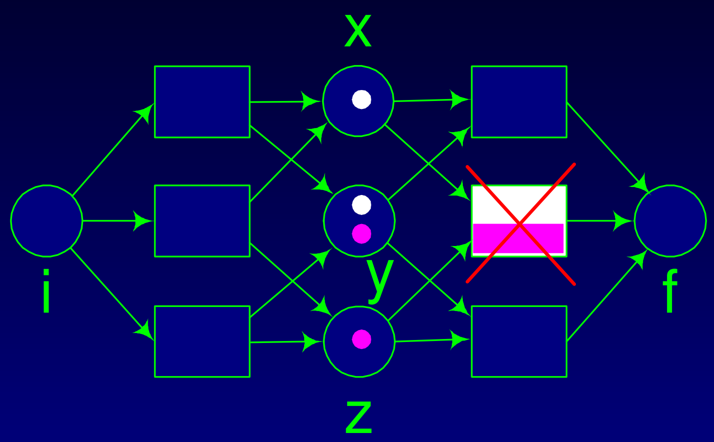


Let N be a Petri net, m an id-marking, and $\alpha(m)$ its uncoloured abstraction. Then there exists a simulation relation between (N, m) and $(N, \alpha(m))$.
The reverse does not hold.

Serialisability

Is there a class of nets whose behaviour is *trace equivalent* to the behaviour of nets with id-tokens?

An sWF-net N is serialisable iff for any $k \in \mathbb{N}$, any firing sequence σ such that $[i^k] \xrightarrow{\sigma}$ there exist firing sequences $\sigma_1, \ldots, \sigma_k$ such that $[i] \xrightarrow{\sigma_1}, \ldots, [i] \xrightarrow{\sigma_k}$ and $\sigma \in (\sigma_1 \| \ldots \| \sigma_k)$.

Theorem. An sWF-net N is serialisable iff for any id-marking M s.t. $\alpha(M) = [i^k]$ for some $k \geq 0$, we have $\{\sigma \mid [i^k] \xrightarrow{\sigma}_N\} = \{\sigma \mid M \xmapsto{\sigma}_N\}$.

Serialisable WF-nets

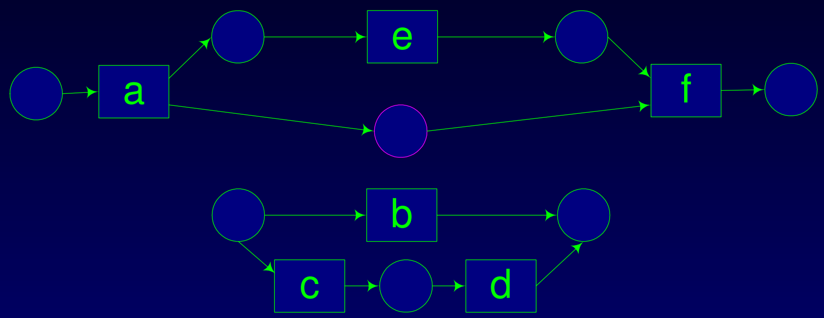
A Petri net $N = \langle S, T, F \rangle$ is a *state machine* (SM) iff $\forall t \in T : |{}^\bullet t| \leq 1 \wedge |t^\bullet| \leq 1$.

A Petri net $N = \langle S, T, F \rangle$ is a *marked graph* (MG) iff $\forall p \in S : |{}^\bullet p| \leq 1 \wedge |p^\bullet| \leq 1$.

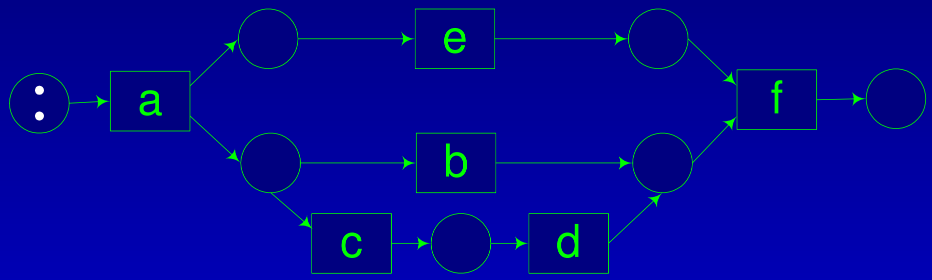
SMWF-nets are sound and serialisable.

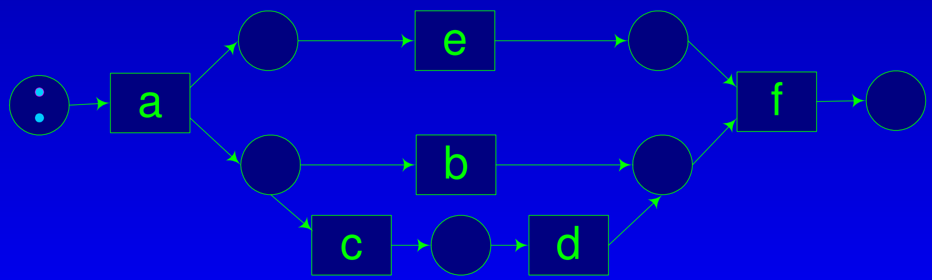
Cycle-free MGWF-nets are sound and serialisable.

Is serialisability compositional?



Consider trace $aecabf$.





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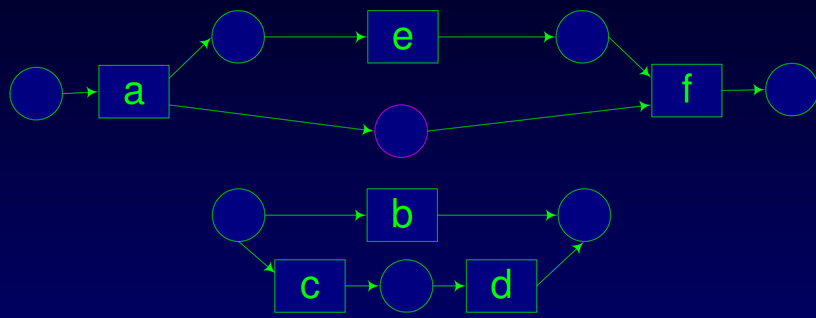

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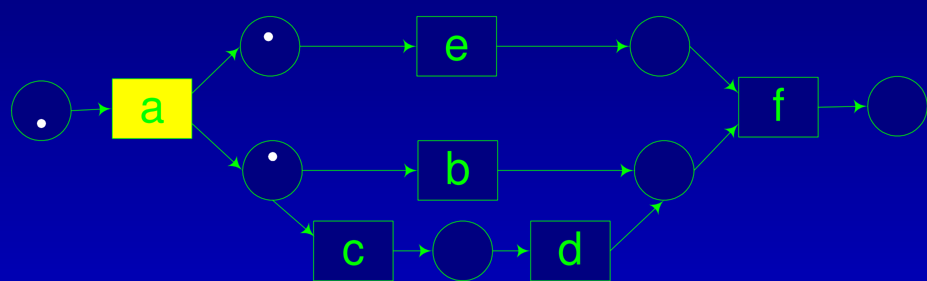

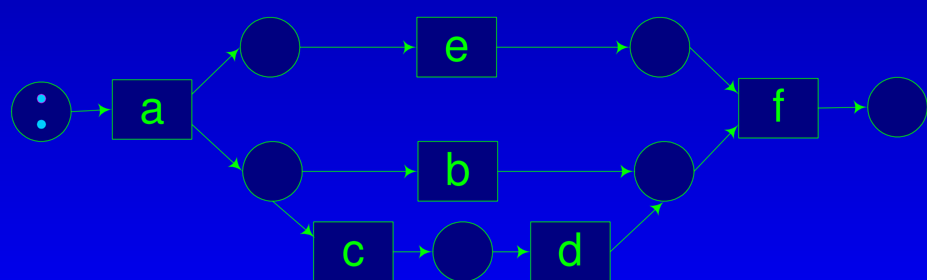

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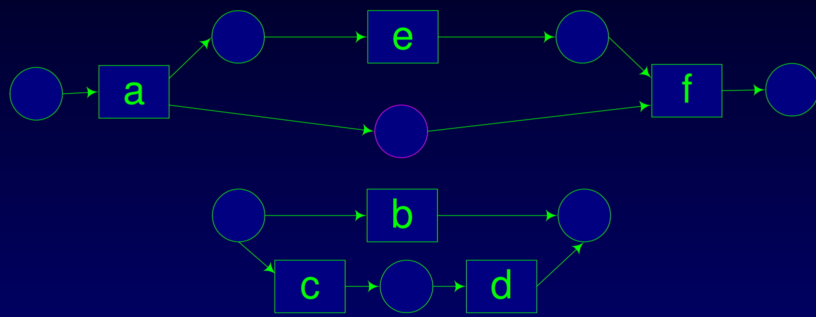

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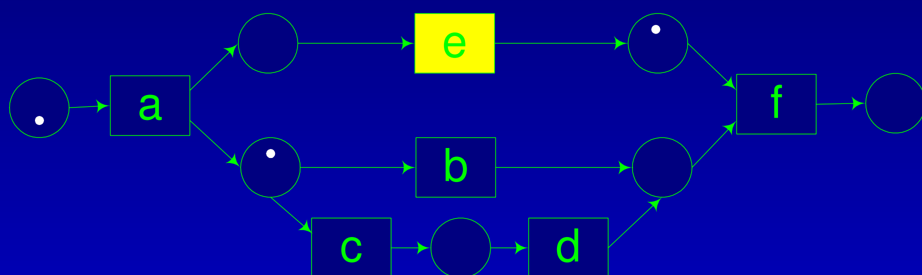

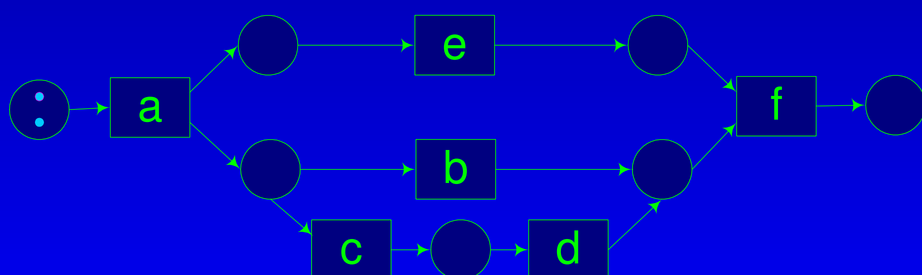

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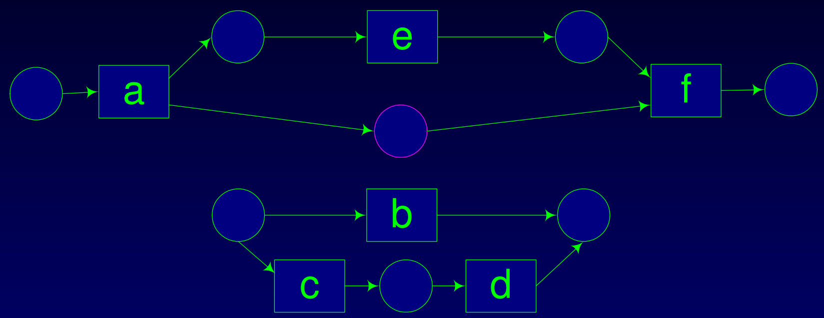

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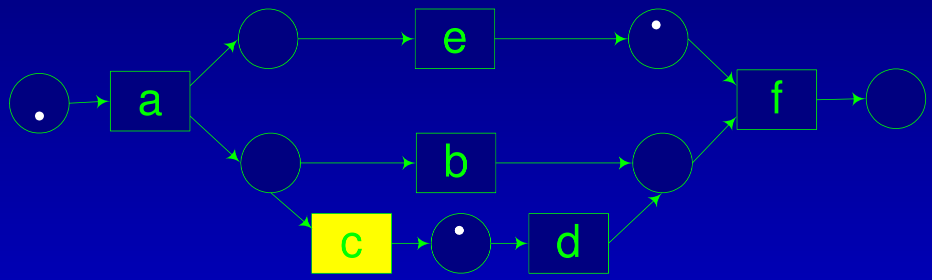

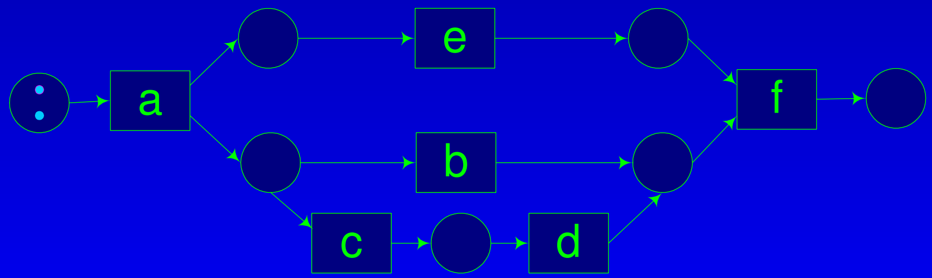

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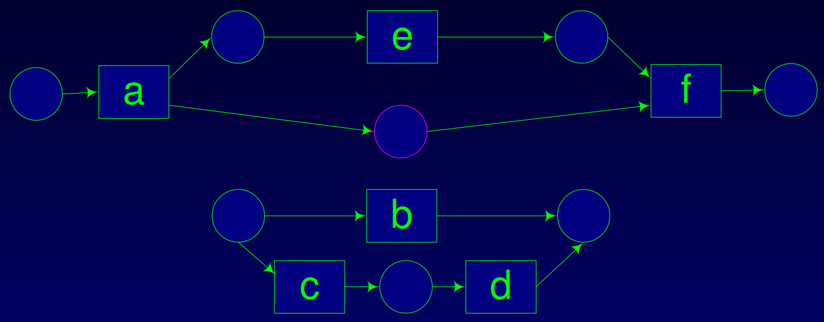

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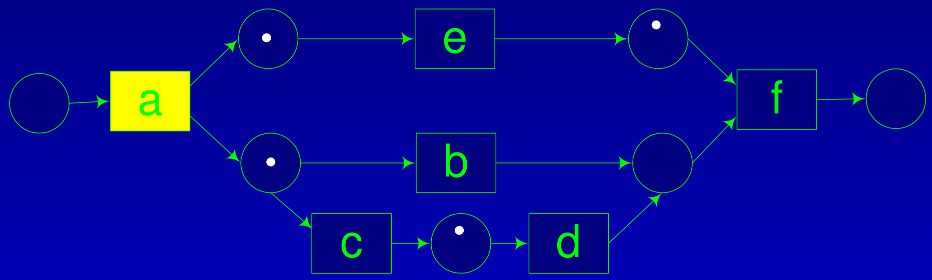

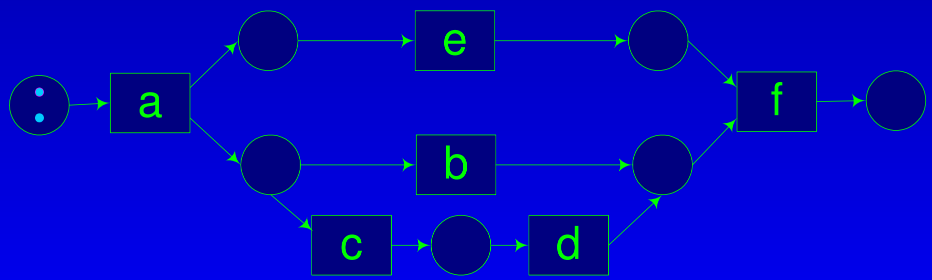

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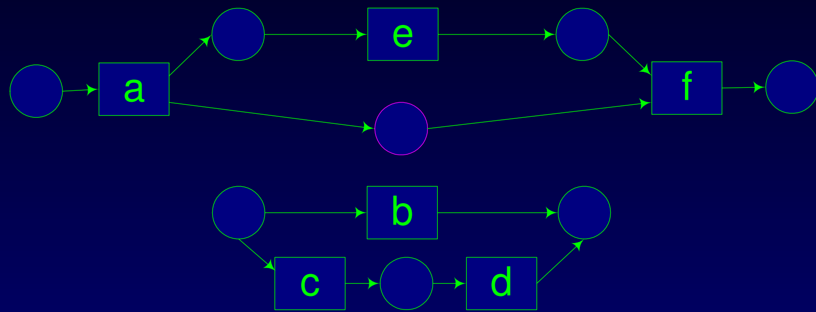

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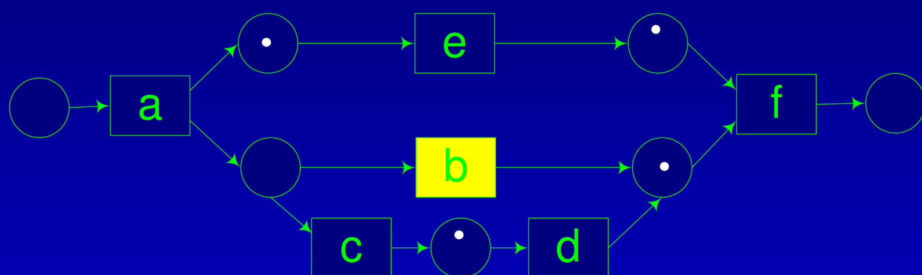
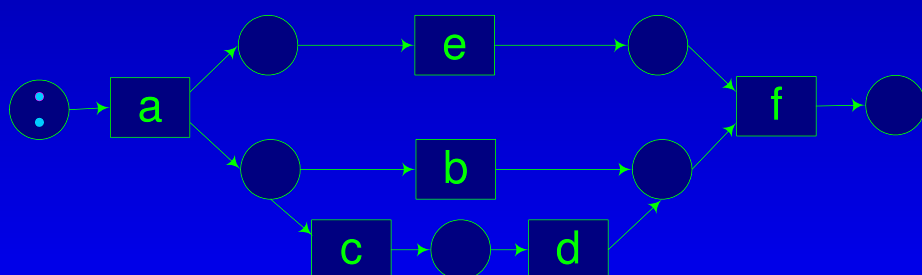
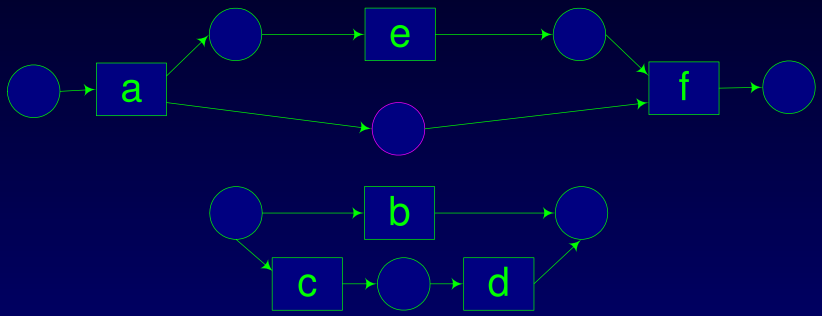




Is serialisability compositional?



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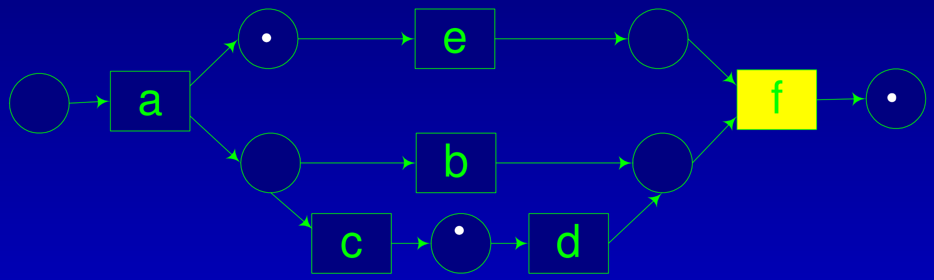

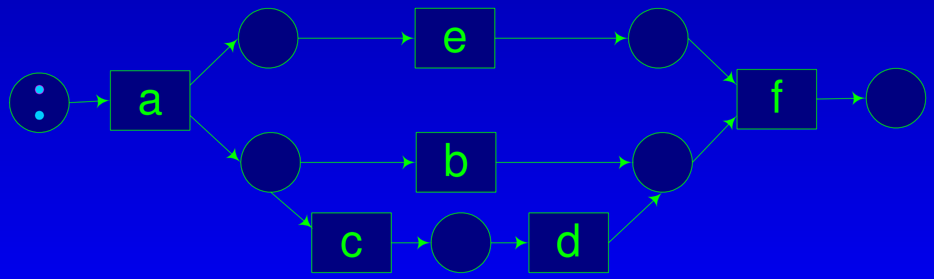

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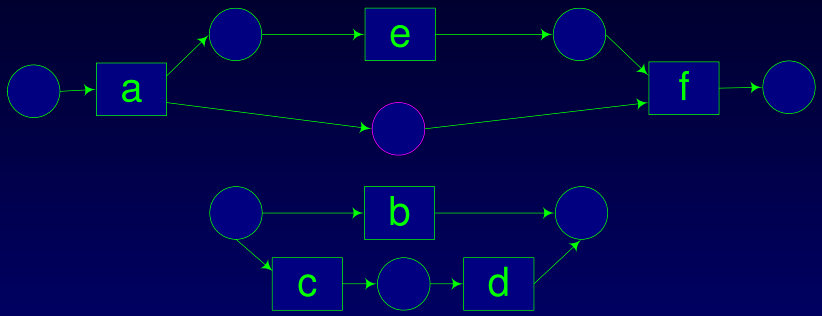

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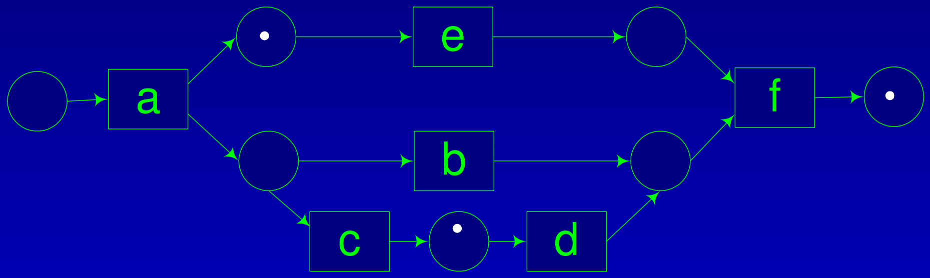

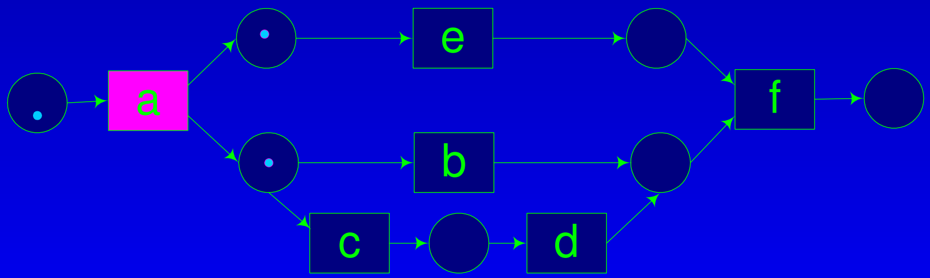

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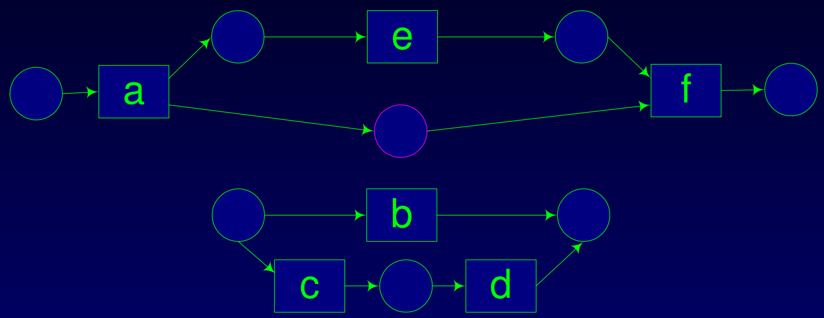

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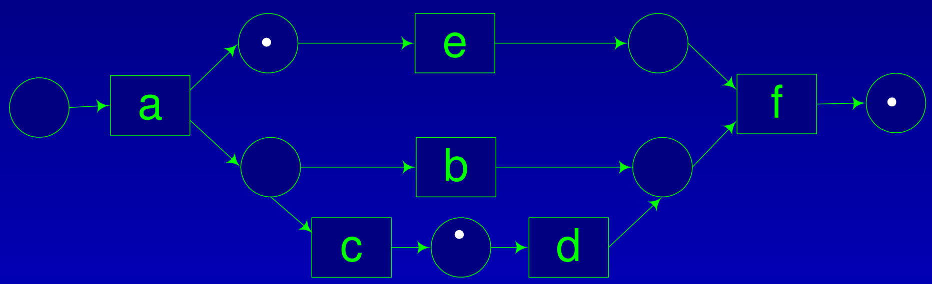

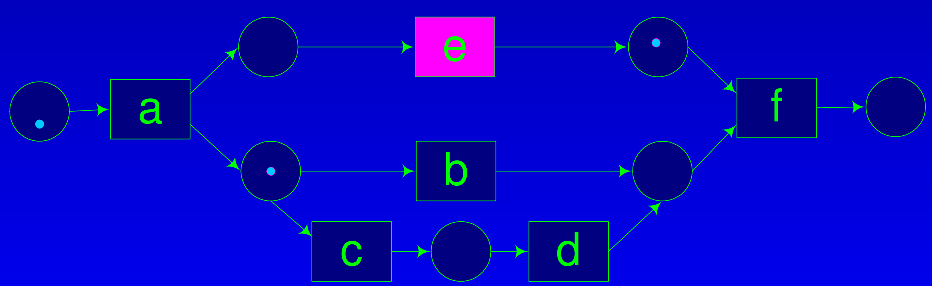

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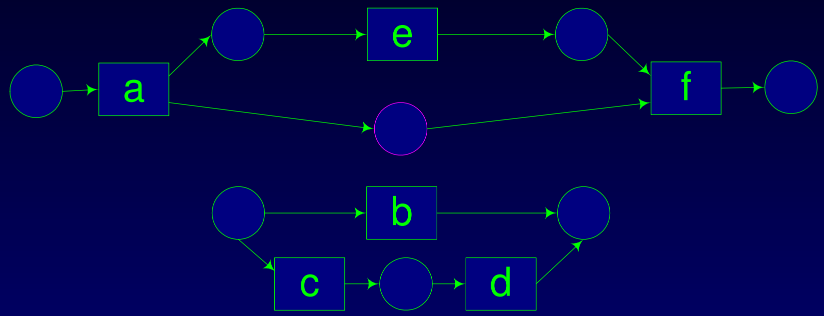

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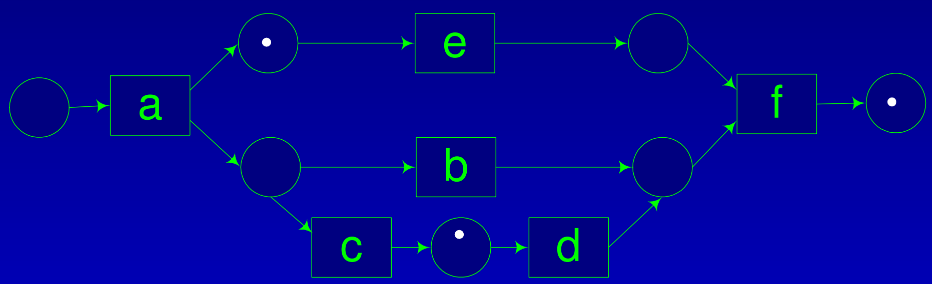

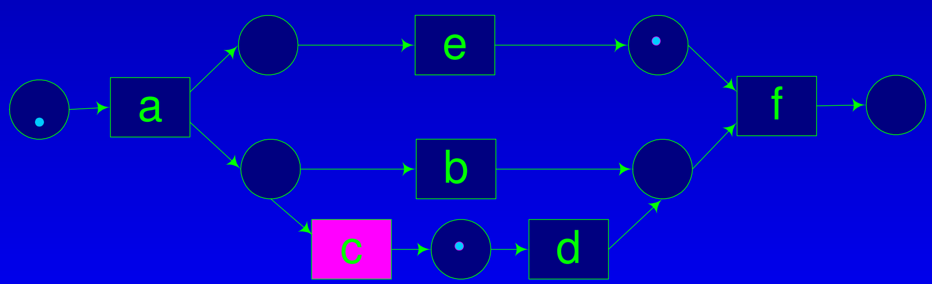

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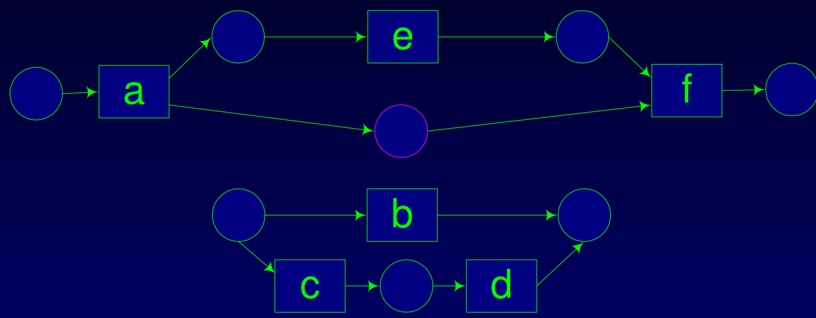

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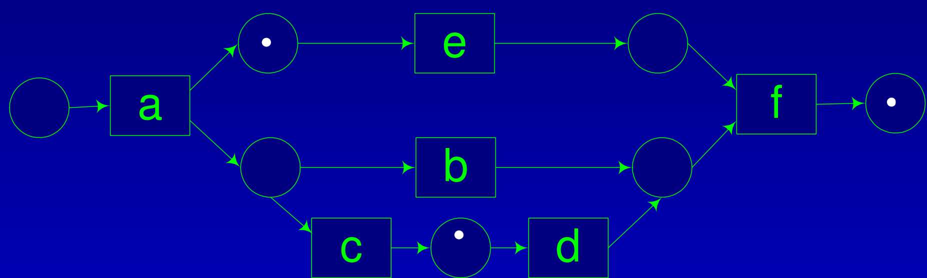

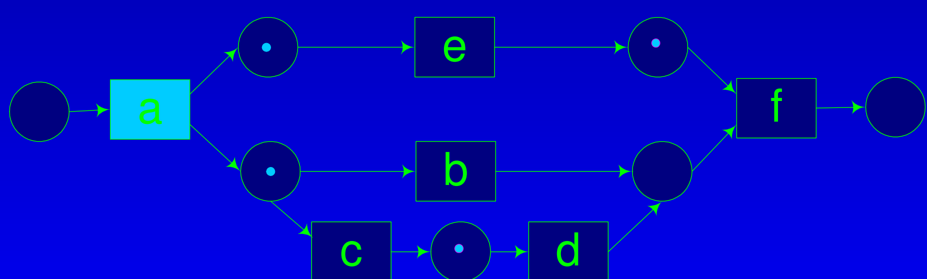

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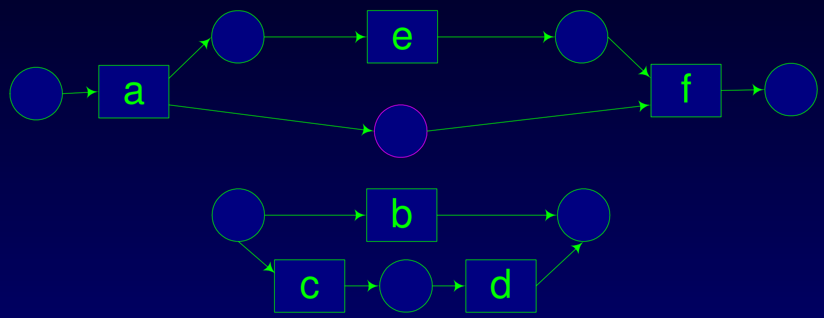

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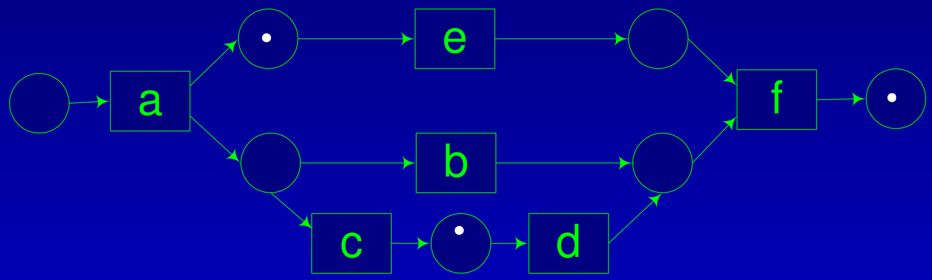

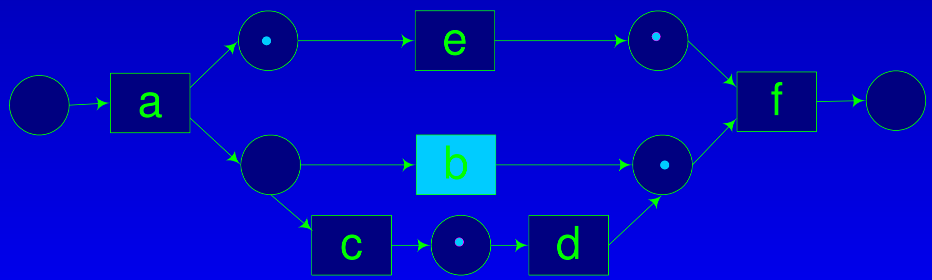

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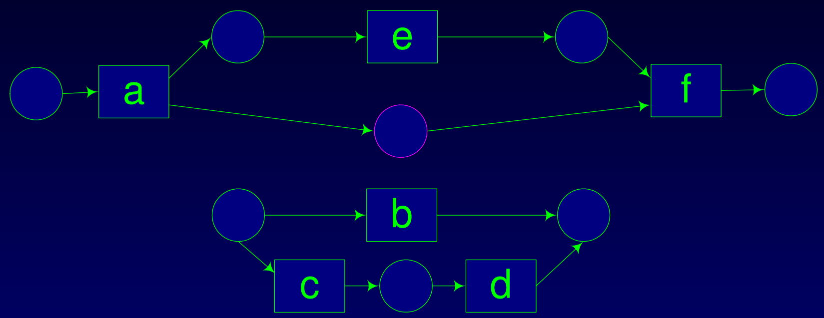

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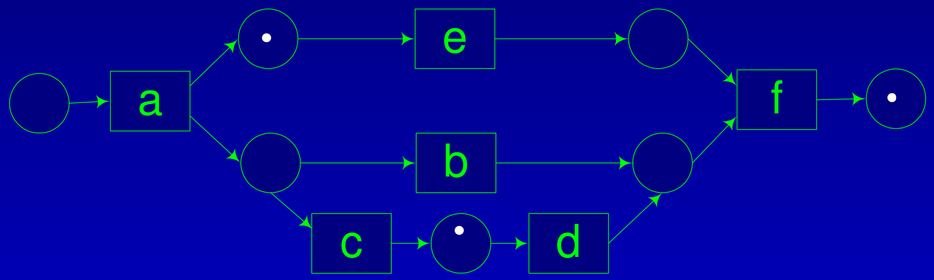
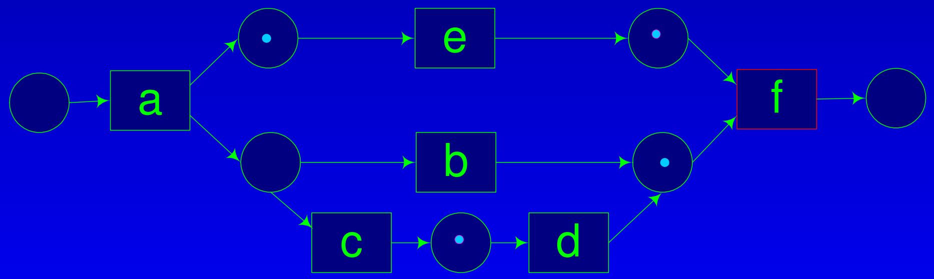




Weak separability

If we look at the markings of the net only:

An sWF-net N is *weakly separable* iff for any $k \in \mathbb{N}$ and any marking m, $[i^k] \xrightarrow{*} m$ implies that there exist markings $m_1, \ldots, m_k$ such that $m = m_1 + \ldots + m_k$ and $[i] \xrightarrow{*} m_j$ for $j = 1, \ldots, k$.

Serialisability implies weak separability.

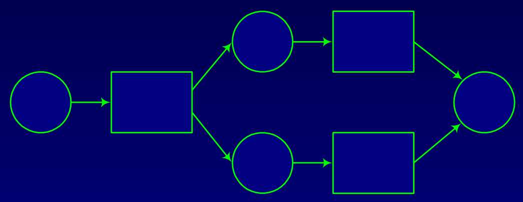
Weakly separable + 1-sound ⇒ sound

Consider a marking m reachable from $[i^k]$.
Since N is weakly separable, there exist $m_1, \ldots, m_k$ such that $m = m_1 + \ldots + m_k$ and $[i] \xrightarrow{*} m_1, \ldots, [i] \xrightarrow{*} m_k$.

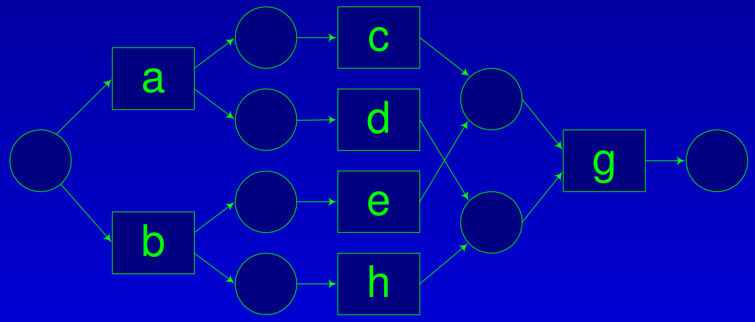
Since N is 1-sound, $m_1 \xrightarrow{*} [f], \ldots, m_k \xrightarrow{*} [f]$, which means that $m \xrightarrow{*} [f^k]$. So N is sound.

Weak separability ? Soundness

Weakly separable but *not* 1-sound net:



Sound free-choice but *not* weakly separable net:



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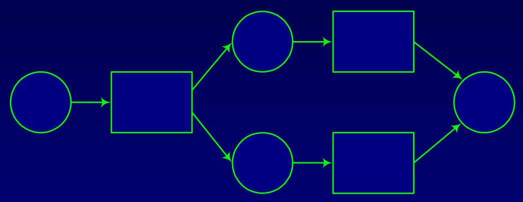

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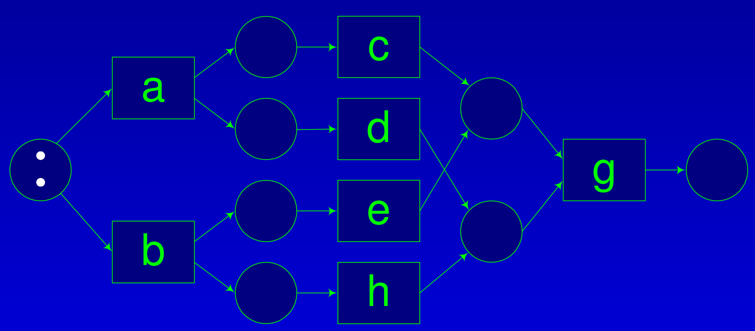

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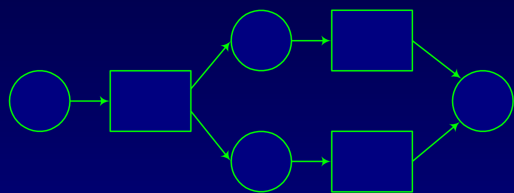

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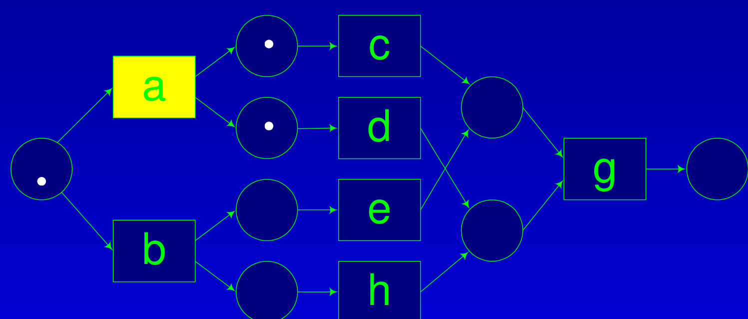

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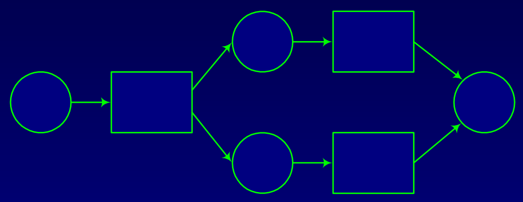

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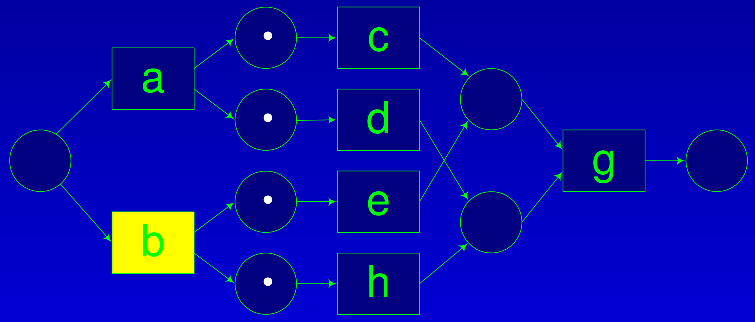

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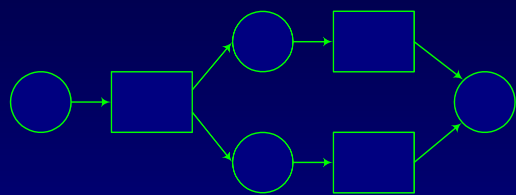

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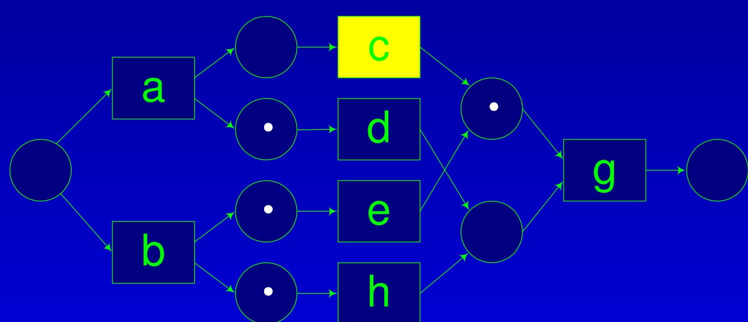

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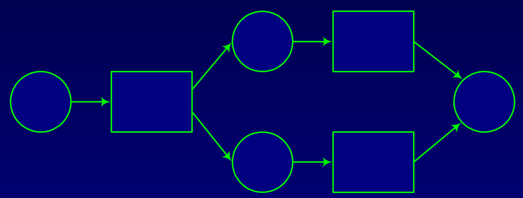

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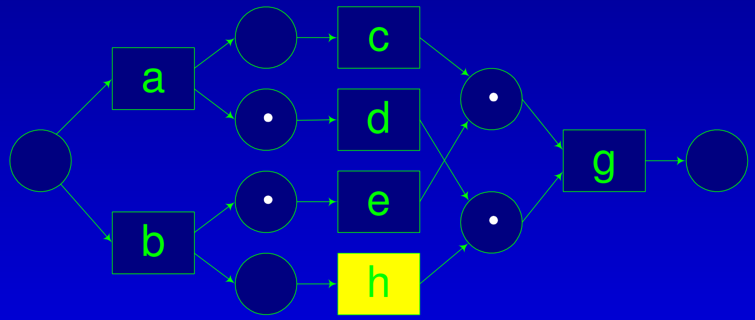

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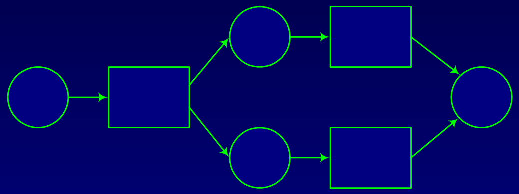

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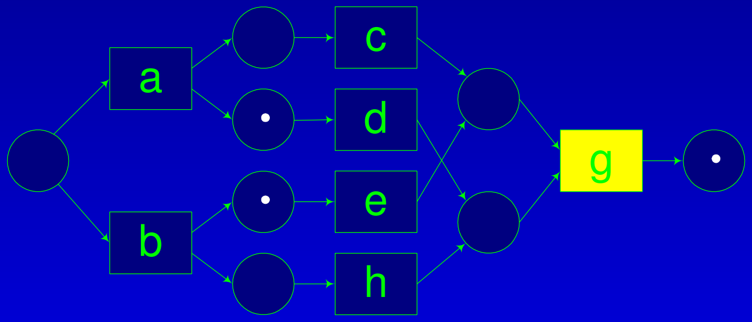

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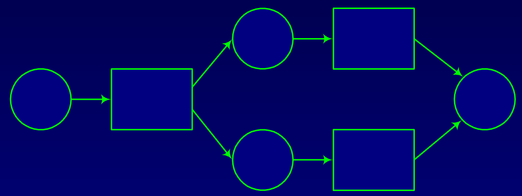

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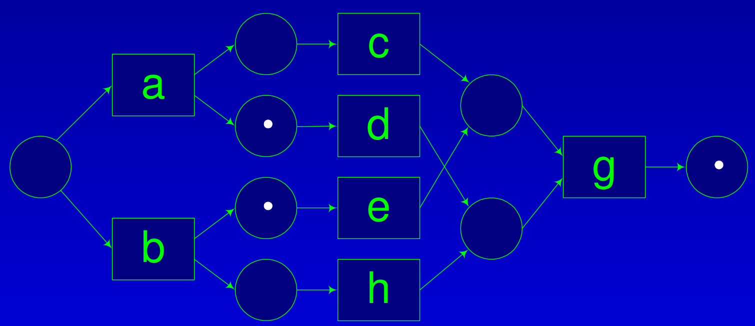
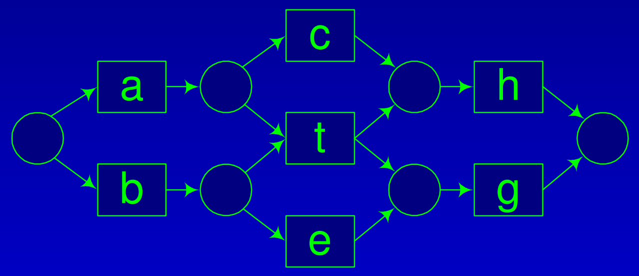


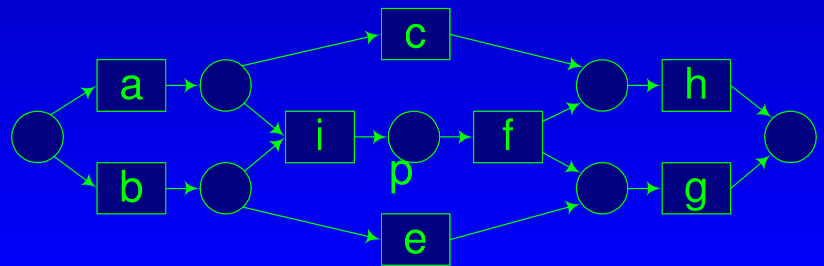
Is weak separability compositional?

Weak separability is a congruence with respect to the place refinement:

Let L, M be weakly separable WF-nets, M a sound sWF-net and $p \in P_L$. Then the net $N = L \otimes_p M$ is weakly separable.

Transition refinement does not necessarily result in a weakly separable net:





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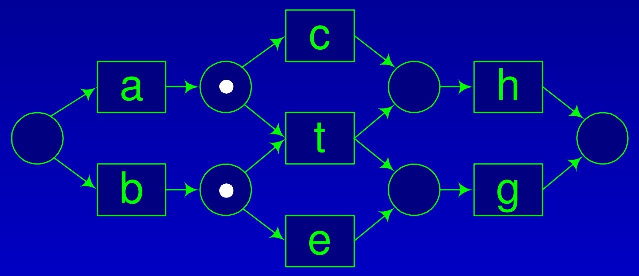
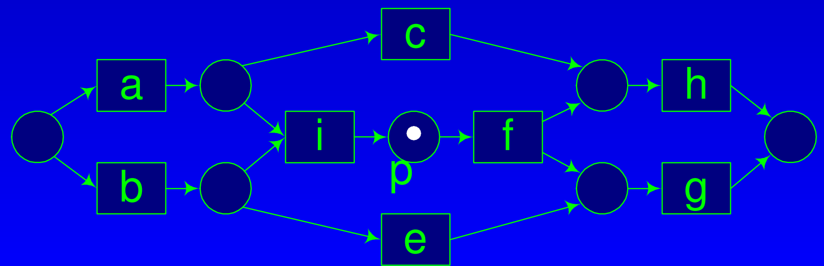




Separability

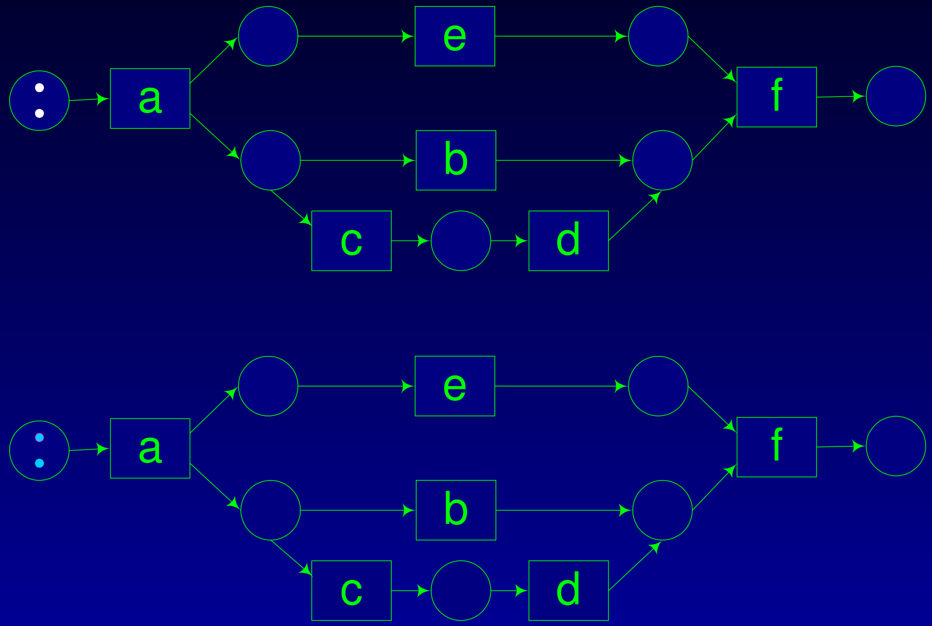
What would be a notion stronger than weak separability but not as restrictive as serialisability?

An sWF-net N is separable iff for any $k \in \mathbb{N}$, any firing sequence σ such that $[i^k] \xrightarrow{\sigma}$, there exist firing sequences $\sigma_1, \ldots, \sigma_k$ such that $[i] \xrightarrow{\sigma_1}, \ldots, [i] \xrightarrow{\sigma_k}$ and $\overrightarrow{\sigma} = \overrightarrow{\sigma_1} + \ldots + \overrightarrow{\sigma_k}$.

(1) Serialisability implies separability.
(2) Separability implies weak separability.

TU/e

Separability $\not\Rightarrow$ Serialisability

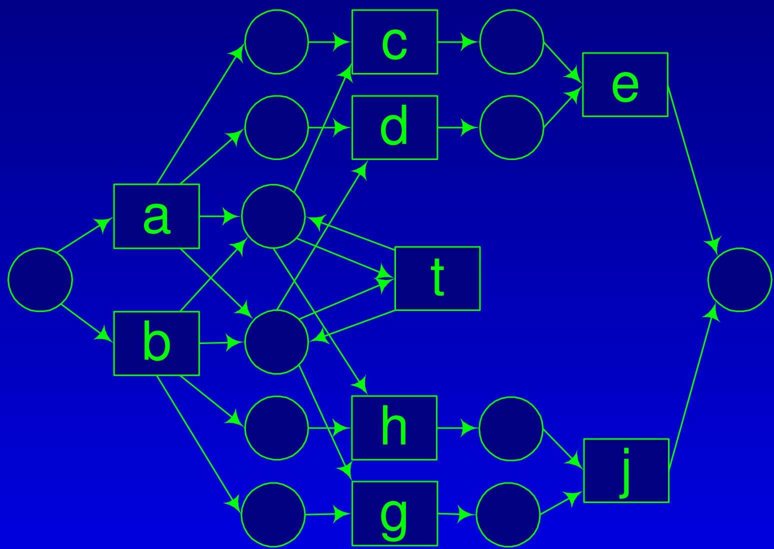


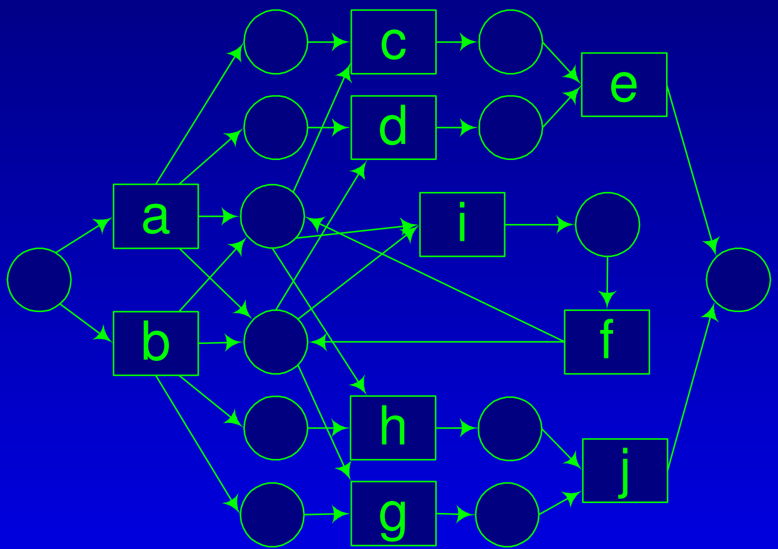
The problematic trace $aecabf$ can now be separated into $aebf$ and ac.

Is separability compositional?

Separability is a congruence w.r.t. the place refinement:
Let L, M be separable WF-nets. If $p \in P_L$ and M is a sound sWF-net then $L \otimes_p M$ is separable.

Transition refinement is still a problem:

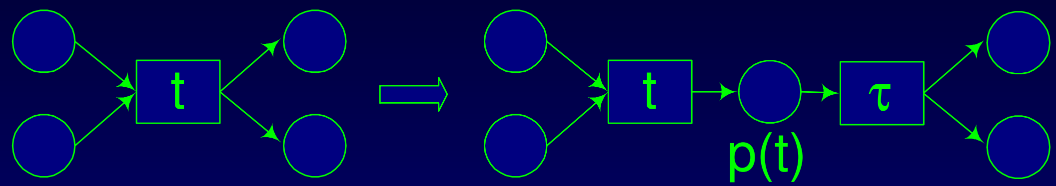




Trace $abicg$ cannot be separated.

Split-separability

A simple transition refinement —split refinement:



Transition t is replaced with the tWF-net Σ_t with places $\{p_t\}$ and transitions $\{i_t, f_t\}$ such that $^\bullet i_t = f_t^\bullet = \emptyset$, $i_t^\bullet = {}^\bullet f_t = [p_t]$.

An sWF-net N is split-separable iff
$\mathcal{S}(N) = (\ldots(N \otimes_{t1} \Sigma_{t1}) \otimes_{t2} \ldots) \otimes_{tn} \Sigma_{tn}$,
$T_N = \{t_1, \ldots, t_n\}$
(the net obtained by applying the split-refinement to every transition of N), is separable.

Split-separable nets

Split-separability implies separability.

Any split refinement of a split-separable net is split-separable.

Let L, M be split-separable WF-nets.
(1) If $p \in P_L$ and M is a sound sWF-net then $L \otimes_p M$ is split-separable.
(2) If $t \in T_L$ and M is a sound tWF-net then $L \otimes_t M$ is split-separable.

Split-separability is compositional!

ST-nets

Can we find classes of nets that are sound and (split-) separable by construction?

SMWF-nets and acyclic MGWF-nets are sound.

They are serialisable, hence separable.
The classes of SMWF-nets and acyclic MGWF-nets are closed under the split-refinement operation, hence, these nets are split-separable.

So, nets constructed from state machines and acyclic marked graphs by means of refinement are sound and split-separable.

We call these nets ST-nets.

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Conclusions

- Bisimilarity results speed up verification of composite nets.
- Separability can be used to provide cost-effective management.
  If costs are associated to every transition firing, the total cost of processing of k orders given by a trace in a WF-net is equal to the sum of costs of processing of k individual orders, each given by a trace with 1 initial token.
- Weakly-separable 1-sound nets are (strongly) sound.
- ST-nets are “sound by construction” and (split-)separable.

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Future work

- Are the problems of soundness and separability decidable in general?
- Can we identify other classes of sound (split-)separable WF-nets?
- Can we adapt this framework to deal with communicating WF-nets?
- k-separability?